NUMERICAL INVESTIGATIONS ON THE QUASI-STATIONARY RESPONSE OF ANTENNAS TO WIDEBAND LFMCW EXCITATION

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ABSTRACT

In this contribution, we numerically investigate if the quasi-stationary (QS) response is a valid approximation for the response of antennas excited by wideband linear-frequency modulated continuous waveforms. We give results for two idealized example systems, showing how the validity of the QS response is dependent on the system's resonant behavior. It will be shown that the error between exact output and QS response is approximately linearly dependent on the sweep-rate of the linear-frequency modulated excitation. We then conduct our simulations for a realistic wideband radar system operating from 5 GHz to 8 GHz and using impulse responses extracted from electromagnetic simulations of a dipole and a biconical antenna.

Index Terms— Quasi-Stationary, FMCW, Antennas

1. INTRODUCTION

Wideband linear-frequency modulated continuous-wave (LFM-CW) radars are used in many state-of-the art systems for, e.g., local positioning [1] and imaging [2].

For high-accuracy and high-resolution systems, linear ramp reproduction is important and thus also non-ideal effects of the antennas used have to be taken into account [3–5]. Those antenna non-idealities are generally treated by assuming the QS model to simplify the expression for the response of an antenna, modeled as LTI system with impulse response $h(t)$, to LFMCW excitation [3–5]. Instead of the exact input-output relation for a system excited by a cosine with instantaneous phase $\phi(t)$

$$y(t) = h(t) * x(t) = h(t) * \cos(\phi(t)), \tag{1}$$

the output is approximated by $y(t) \approx \hat{y}(t)$, where the QS response $\hat{y}(t)$ is given by

$$\hat{y}(t) = |H(\jmath\omega(t))| \cdot \cos\left(\phi(t) + \arg\left\{H(\jmath\omega(t))\right\}\right), \tag{2}$$

and $\omega(t) = d/dt\,\phi(t)$ is the instantaneous radial frequency of the excitation signal. It is often stated that (2) is a valid approximation for (1) if the maximum rate of variation in $\omega(t)$ is slow compared with the speed of response of $h(t)$ [6], or if the bandwidth of $\omega(t)$ is much less than the signal itself [4].

However, those statements do not give precise bounds on the validity of the QS response and for a particular system it always remains questionable if those conditions are fulfilled or not. Indeed in the early years of frequency-modulated transmission this issue has been ardently discussed and some sophisticated analytic treatment has been proposed [6, 7], but the provided formulas are only suitable to investigate systems with a few poles. An application to systems with a large number of poles, which may result from extracting a pole/residue model from antennas [8], has not yet been proposed.

Hence in this contribution we make use of todays computational power to investigate this issue on a numerical basis. In particular, our research is driven by the following question: given a certain radar antenna modeled by the impulse response $h(t)$ and a LFMCW excitation with a fixed lower and upper sweep frequency, how large can the sweeprate, i.e. the rate of variation in $\omega(t)$, be selected such that (2) is a valid approximation for the exact response (1)?

2. THE LFMCW EXCITATION SIGNAL

We consider a single up-sweep, real-valued bandpass LFMCW signal $x(t) = \cos(\phi(t))$ with a instantaneous frequency $f(t) = \omega(t)/2\pi$ defined as a piecewise function

$$f(t) = \begin{cases} f_l & \text{for } t < 0 \\ f_l + \mu t & \text{for } 0 \leq t < T \\ f_u & \text{for } T \leq t \end{cases}, \tag{3}$$

where f_l is the sweep lower frequency, μ the sweeprate, T the sweep duration, and f_u the sweep upper frequency. Consequently, the instantaneous phase is given by integrating (3)

$$\phi(t) = \begin{cases} 2\pi f_l t + \phi_0 \\ 2\pi f_l t + \pi\mu t^2 + \phi_0 \\ 2\pi f_u (t-T) + 2\pi f_l T + \pi\mu T^2 + \phi_0 \end{cases}, \tag{4}$$

where the integration constants are selected such that the phase is continuous for all t and $\phi(0) = \phi_0$. Note that (4) is defined over the same intervals as (3), but the interval limits have been omitted in (4). For numerical evaluation, a finite-duration discrete-time representation $x[n]$ is obtained by shifting $x(t)$ by the guard-time T_g to the right and then taking N consecutive samples beginning at $t = 0$

$$x[n] = x(nT_s - T_g), n \in [0, N-1], \tag{5}$$

where $T_s = 1/f_s$ is the sampling period and the total number of samples is $N = f_s \cdot (T_g + T + T_g) = N_g + N_T + N_g$, i.e. the finite-duration series $x[n]$ contains the LFMCW ramp of duration T plus an additional CW component of duration T_g at the beginning and the end of the signal. The relationship between sampling rate and sweep- as well as guard-intervals is selected such that

$$N, N_g, N_T \in \mathbb{N}. \tag{6}$$

The series of instantaneous frequency can be obtained by

$$f[n] = f(nT_s - T_g), n \in [0, N-1]. \tag{7}$$

Due to (6), the difference between any two samples of the instantaneous frequency is an integer multiple of the LFMCW frequency step Δf_{LFMCW}

$$f[n] - f[m] = k\Delta f_{\text{LFMCW}}, k \in \mathbb{N}_0 \tag{8}$$

and Δf_{LFMCW} is given by

$$\Delta f_{\text{LFMCW}} = (T_s/T) \cdot (f_u - f_l). \tag{9}$$

3. THE ANTENNA AS LTI SYSTEM

We adopt a common method to characterize an antenna as LTI system with spatially-dependent impulse response [9]. Thus, given a plane-wave E-field $\vec{e}_{\text{rx}}(t, \Theta_{\text{rx}}, \Phi_{\text{rx}})$ incident from the spherical coordinates $\Theta_{\text{rx}}, \Phi_{\text{rx}}$, the voltage wave $u^-_{\text{rx}}(t)$ leaving the antenna terminals is given by

$$\frac{u^-_{\text{rx}}(t)}{\sqrt{Z_{\text{c,rx}}}} = \vec{h}^{\text{ant}}(t, \Theta_{\text{rx}}, \Phi_{\text{rx}}) * \frac{\vec{e}_{\text{rx}}(t, \Theta_{\text{rx}}, \Phi_{\text{rx}})}{\sqrt{Z_0}}, \tag{10}$$

where $(\cdot) * (\cdot)$ is a dual dot-product and time-domain convolution operator [8] taking into account the polarization of $\vec{h}^{\text{ant}}$ and $\vec{e}_{\text{rx}}$, and Z_0 as well as $Z_{\text{c,rx}}$ are the characteristic impedance of free space and the antenna reference impedance, respectively. Equivalently the farfield $\vec{e}_{\text{tx}}(t, r, \Theta_{\text{tx}}, \Phi_{\text{tx}})$ at distance r, generated due to a voltage wave $u^+_{\text{tx}}(t)$ incident on the antenna terminals is given by

$$\frac{\vec{e}_{\text{tx}}(t, r, \Theta_{\text{tx}}, \Phi_{\text{tx}})}{\sqrt{Z_0}} = \frac{1}{r}\frac{1}{2\pi c_0}\delta(t - r/c_0) * \frac{\partial}{\partial t}\vec{h}^{\text{ant}}(t, \Theta_{\text{tx}}, \Phi_{\text{tx}}) * \frac{u^+_{\text{tx}}(t)}{\sqrt{Z_{\text{c,tx}}}}. \tag{11}$$

Since $\vec{h}^{\text{ant}}$ is describes a LTI system, it can be represented as a sum of complex exponentials [8]

$$h(t) = R_1 e^{s_1 t} + \ldots + R_K e^{s_K t} = \sum_{k=1}^{K} R_k e^{s_k t}, \tag{12}$$

where s_k are the poles and R_k the corresponding residues of the system, with $s_k = \sigma_k + \jmath\omega_k$ and $\sigma_k < 0$.

The fundamental question motivating our investigations, as already outlined in the introduction, can now be stated more precisely: given that either the incident E-field $\vec{e}_{\text{rx}}(t)$ or the excitation voltage wave $u^+_{\text{tx}}(t)$ is an LFMCW signal, can we then represent the received voltage wave $u^-_{rx}(t)$ or the transmitted E-field $\vec{e}_{\text{tx}}(t)$ using the QS response, and are thus performance measures such as [3–5] valid?

To continue with our numerical evaluation, the antenna impulse response is sampled at M time-instants

$$h[m] = h(mT_s), m \in [0, M-1], \tag{13}$$

where the number of samples M is related to the impulse response length T_h via the sampling period T_s by

$$M = T_h/T_s, M \in \mathbb{N}. \tag{14}$$

For computing the system's QS response, the transfer function evaluated at each instantaneous frequency sample $f[n]$ is necessary. While it principally could be obtained by sampling the Laplace transform of (12) at $2\pi f[n]$, which would be numerically very efficient, in the context of our simulations a more accurate frequency-domain representation of the system is obtained by the FFT of $h[n]$, since the FFT includes the effects of sampling and truncation of $h(t)$. Hence we apply an FFT with sufficiently dense frequency resolution to $h[m]$ and then select the appropriate frequency bin for each sample of the instantaneous frequency $f[n]$

$$H_{\text{FFT}}[n] = \text{FFT}\{h'[m]\}\Big|_{m=\lfloor f[n]/\Delta f_{\text{FFT}}\rceil}, \tag{15}$$

where $\lfloor(\cdot)\rceil$ denotes rounding towards the nearest integer and $h'[m]$ is the zero-padded version of $h[m]$ defined by

$$h'[m] = \begin{cases} h[m] & m \in [0, M-1] \\ 0 & m \in [M, M_{\text{FFT}}] \end{cases}. \tag{16}$$

Since the frequency resolution of the FFT is given by

$$\Delta f_{\text{FFT}} = \frac{f_s}{M_{\text{FFT}}} \tag{17}$$

and the FFT frequency resolution should be fine enough to contain a distinct frequency bin for each distinct instantaneous frequency sample

$$\Delta f_{\text{FMCW}} \geq \Delta f_{\text{FFT}} \tag{18}$$

the necessary FFT length is given by

$$M_{\text{FFT}} = \frac{T}{T_s^2}\frac{1}{f_u - f_l} = \frac{1}{\mu T_s^2} = \frac{f_s^2}{\mu}. \tag{19}$$

4. EXACT, QUASI-STATIONARY, AND DISTORTION RESPONSES

Assuming a sufficiently small sample duration T_s as well as a sufficiently large evaluation time of the impulse response T_h (details will be discussed below), the exact response of the system (1) can be numerically evaluated at discrete-time samples by

$$y[n] = h[n-k] * x[k] = \sum_{k=0}^{n} h[n-k]x[k]. \quad (20)$$

The numerical representation of the QS response computed using $H_{\text{FFT}}[n]$ (15) is then given by

$$\hat{y}_{\text{FFT}}[n] = |H_{\text{FFT}}[n]| \cos(2\pi f[n] + \arg\{H_{\text{FFT}}[n]\}). \quad (21)$$

Followingly, the distortion term $d_{\text{FFT}}[n]$ capturing the deviation of the exact system response from the QS approximation using the FFT transfer function is given by

$$d_{\text{FFT}}[n] = y[n] - \hat{y}_{\text{FFT}}[n]. \quad (22)$$

5. SAMPLE RATE AND SIGNAL DURATION

On a first glimpse, one might expect that the LFMCW excitation signal is band-limited by f_l and f_u. But this is not true, since the sweep bandwidth $B = f_u - f_l$ increases depending on the sweeprate [10]. Since a detailed discussion would go beyond the scope of this contribution, we can merely state here that throughout our investigations a sample rate of $f_{s,\text{LFMCW}} = 10 \cdot f_u$ has been shown to be sufficient for precise results.

For representing the impulse response $h(t)$, at least a sample rate corresponding to twice the maximum pole frequency is necessary

$$f_{s,h(t)} \geq \max_k\{\omega_k\}/\pi. \quad (23)$$

Otherwise spectral aliasing will occur. Then, H_{FFT} and the frequency response of the analog system will differ to a great extent since poles with $\omega_k > \pi f_s$ are aliased into the first Nyquist zone and the numerical computations will not accurately model the analog system. Equation (23) can be satisfied by either selecting a sufficiently large f_s, by limiting the frequency of the extracted poles of $h(t)$, or by low-pass filtering $h(t)$. The sample rate is then determined by

$$f_s = \max\{f_{s,h(t)}, f_{s,\text{LFMCW}}\}. \quad (24)$$

The impulse response is sampled for a duration of T_h, where T_h should be selected such that the energy of the impulse response at the end of the sampling interval is almost zero. Otherwise, significant spectral leakage will occur.

Finally it has to be avoided that transients due to the finite-duration excitation $x[n]$ leak into the sweep interval and the finite-duration excitation needs to be long enough to be able to also observe the deviation from QS behavior, which leaks over the sweep duration T. Hence the guard intervals need a length of

$$T_g \geq T_h. \quad (25)$$

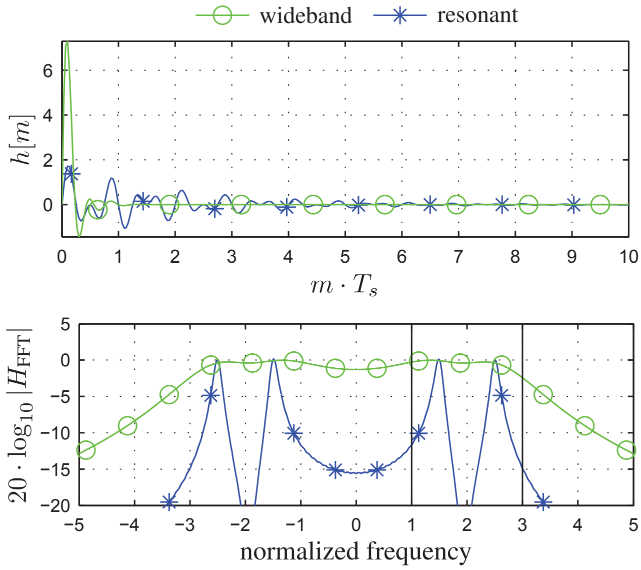

Fig. 1. Impulse response (top) and frequency response magnitude (bottom) of simple example systems. The resonant system has K=4 conjugate complex poles $s_k = \{-0.5 \pm \jmath 2\pi 1.5, -0.5 \pm \jmath 2\pi 2.5\}$ with $R_k = \{\mp \jmath 0.5, \mp \jmath 0.5\}$. The wideband system has K=4 conjugate complex poles $s_k = \{-5 \pm \jmath 2\pi 1.5, -5 \pm \jmath 2\pi 2.5\}$ with $R_k = \{\mp \jmath 2.5, \mp \jmath 4\}$. f_u and f_l are indicated by two solid black vertical lines.

6. EVALUATION FOR EXEMPLARY WIDEBAND AND RESONANT LTI SYSTEMS

Before evaluating the distortion term obtained from realistic antennas in a realistic radar scenario, the influence of sweeprate as well as system characteristics are examined for two example LTI systems, which mimic the behavior of a resonant and wideband antenna, in this section. Therefore, we restrict our attention to a resonant and a wideband LTI system, with impulse responses and frequency responses shown in figure 1, defined by the dimensionless set of poles and residues given in the caption of figure 1. The parameters of the excitation are also dimensionless: $f_l = 1$, $f_u = 3$, $f_s = 10 \cdot f_u$, $T_h = 10$, $T_g = 20$, $\mu = 1 \cdot 10^{-4} \ldots 1 \cdot 10^{-1}$.

The LFMCW sweep starts at the normalized frequency $f_l = 1$, ends at the normalized frequency $f_u = 3$, and hence crosses both poles. Consequently, the envelope of the QS responses shown in figure 2 and 3 reflect the shape of the system's frequency response magnitude. Further note that the

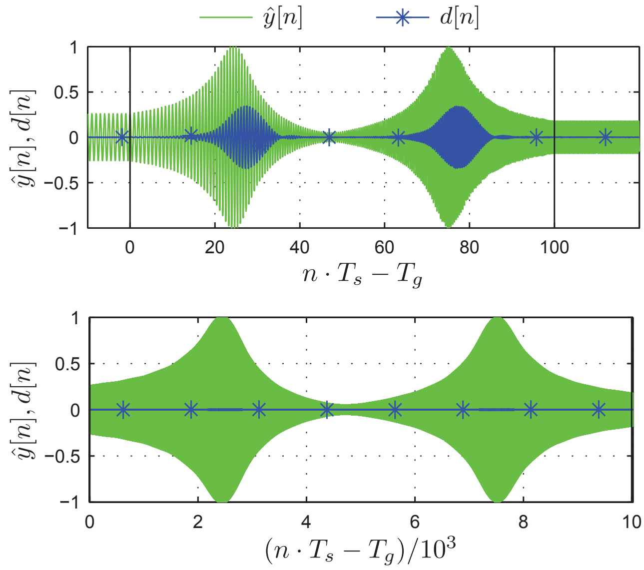

Fig. 2. Quasi-Stationary responses and distortion signals obtained for LFMCW excitation with (top) $\mu = 0.02$ and (bottom) $\mu = 0.0002$ applied to resonant example system. The fast sweep results in a significant distortion component.

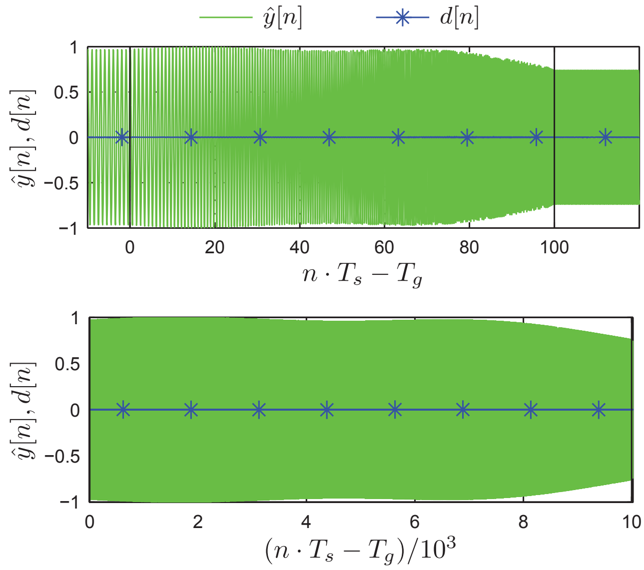

Fig. 3. Quasi-Stationary responses and distortion signals obtained for LFMCW excitation with sweeprates of (top) $\mu = 0.02$ and (bottom) $\mu = 0.0002$ applied to wideband example system. The distortion component is small for both μ.

envelope of the QS responses is independent of the sweeprate. The effect of changing sweeprate merely reflects into a warping of the time-axis of the QS response.

In the case of a low sweeprate, $\mu = 0.0002$, both systems are able to follow the LFMCW excitation signal, the distortion component is low, and hence the QS response is a valid approximation of the exact system response. However, for a fast sweeprate of $\mu = 0.02$ the resonant system is not able to follow the LFMCW excitation immediately. Especially when the instantaneousous frequency coincides with a pole frequency the resonant modes of the system are excited, a significant contribution of the distortion term to the total output signal is obvious and the QS response alone cannot be used to model the output of the system accurately.

Further insight into the dependence of distortion term magnitude on the LFMCW signal's sweeprate is given by figure 4, where the maximum of the distortion term $\max_m\{d[m]\}$ is plotted versus the sweeprate, for $0.0001 \leq \mu \leq 0.1$. Interestingly, the maximum of the distortion term scales down linearly with the sweeprate. In addition, the distortion term obtained from the wideband system is about 100 times smaller than the distortion term obtained from the resonant system.

7. REALISTIC RADAR SYSTEM AND ANTENNAS

We now consider a realistic radar system motivated by the authors antenna designs from [1]. Therefore, a wideband biconical antenna and a resonant dipole have been simulated using the finite integration technique in the frequency range from 0 GHz to 20 GHz, and the antenna impulse responses for $\Theta_{\text{rx}} = 90°$, $\Phi_{\text{rx}} = 0°$ have been computed by post-processing the simulation results, like shown in figure 5. The Dipole has a length of 13.77 cm, a wire thickness of 0.3 mm and resonances at 1 GHz, 3 GHz, 5 GHz, etc. The Biconical antenna has a radius of 20 mm, an opening angle of 90° and thus an $S_{11} \leq -10$ dB covering the range from 3 GHz to the upper simulation frequency. The parameters of the LFMCW excitation are: $f_l = 5\,\text{GHz}$, $f_u = 8\,\text{GHz}$, $f_s = 10 \cdot f_u$, $T_h = 10\,\text{ns}$, $T_g = 12\,\text{ns}$, $\mu = 1 \cdot 10^{18} \ldots 1 \cdot 10^{13}$ Hz/s.

Figure 6 shows the distortion term's maximum magnitude

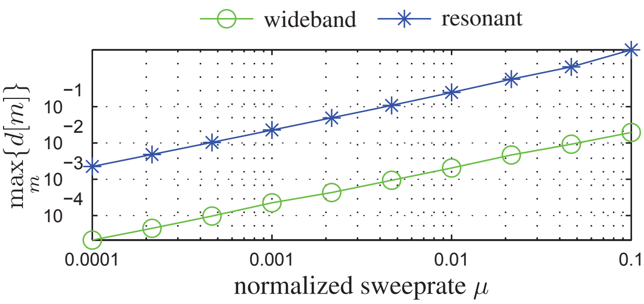

Fig. 4. Maximum of error $\max_m\{d[m]\}$ versus sweeprate for both, resonant and wideband systems. The error in the wideband system is approximately one percent of the error obtained for the resonant system. For both systems, the error linearly scales down with the sweeprate.

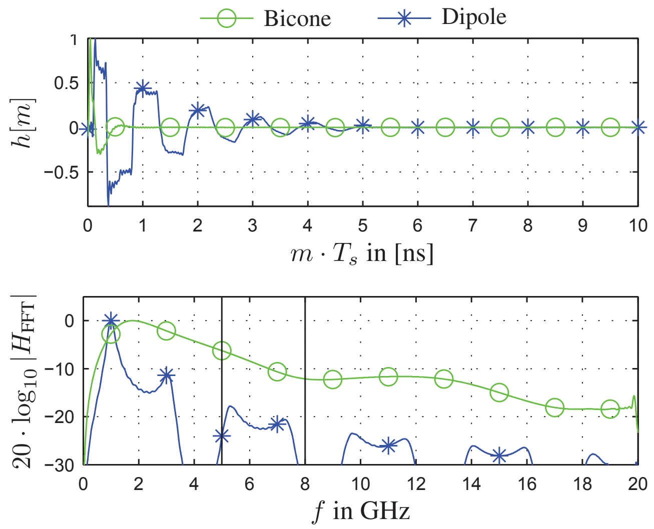

Fig. 5. Normalized impulse responses (top) and frequency response magnitudes (bottom) of Dipole and Biconical antennas. f_l and f_u are indicated by two solid black vertical lines.

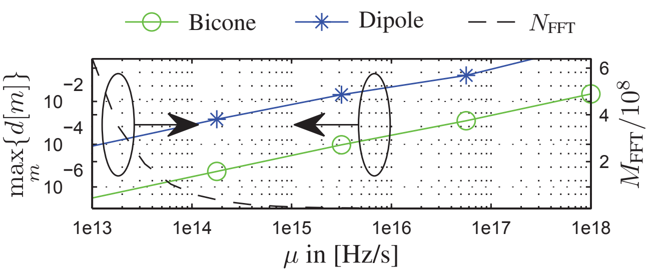

Fig. 6. Error maximum $\max_m\{d[m]\}$ and FFT length M_{FFT} vs. sweeprate for both, Bicone and Dipole. For both systems, the error linearly scales down with the sweeprate.

versus the sweeprate obtained from both antennas in receive mode. Again, an approximately linear dependence of error magnitude and sweeprate is obvious. In addition, as expected from the simplified example shown above, the error using a wideband biconical antenna is again around 100 times smaller than the error obtained from the dipole antenna. Note that, although the radar system from [1] is designed for a sweeprate of $\mu = 3 \cdot 10^{12}[\mathrm{Hz/s}]$, the lowest simulated sweeprate is $\mu = 1 \cdot 10^{13}[\mathrm{Hz/s}]$. This is due to the high computational complexity of the FFT-based frequency response computation, compare the M_{FFT} curve from figure 6 and (19).

8. CONCLUSIONS & OUTLOOK

We numerically investigated the QS response of antennas to wideband LFMCW excitation and showed that, depending on the resonant behavior of the system and with an increasing sweeprate, the QS approximation becomes less accurate. Hence, in fast-sweep wideband LFMCW systems of future radar sensor systems, one should carefully check whether the QS response and thus the related parameters describing the LFMCW ramp distortion (group delay, phase response) can be applied. The proposed method is indeed suitable to investigate those effects, but results in very high computational complexity for numerically evaluating practically relevant radar systems. Hence future work may be based on using analytical formulas [7] to reduce the computational complexity.

REFERENCES

[1] Markus Gardill, Georg Fischer, Robert Weigel, and Alexander Koelpin, "Design of an Ultra-Wideband Monocone Circular Antenna Array for Vehicle-Integrated Industrial Local Positioning Applications," in *Proc. 7th European Conference on Antennas and Propagation (EuCAP)*, 2013.

[2] K.B. Cooper, R.J. Dengler, G. Chattopadhyay, E. Schlecht, J. Gill, A. Skalare, I. Mehdi, and P.H. Siegel, "Submillimeter-wave active radar imager," *Infrared and Millimeter Waves*, pp. 922–923, 2007.

[3] P. Bezousek, M. Hajek, and M. Pola, "Effects of signal distortion in a FMCW radar on range resolution," in *Proc. 15th Int Microwave Techniques (COMITE) Conf*, 2010, pp. 113–116.

[4] F. Perez-Martinez, M. Burgos-Garcia, and A. Asensio-Lopez, "Group delay effects on the performance of wideband CW-LFM radars," *IEE Proc. -Radar, Sonar and Navigation*, vol. 148, no. 2, pp. 95–100, 2001.

[5] C. Bares, C. Brousseau, L. Le Coq, and A. Bourdillon, "Effect of antenna phase centre displacement on FM-CW measurements: application to radar system," *Electronics Letters*, vol. 39, no. 11, pp. 867–868, 2003.

[6] E. Baghdady, "Theory of Low-Distortion Reproduction of FM Signals in Linear Systems," *IRE Transactions on Circuit Theory*, vol. 5, no. 3, pp. 202–214, 1958.

[7] D. D. Weiner and B. J. Leon, "The quasi-stationary response of linear systems to modulated waveforms," *Proc. of the IEEE*, vol. 53, no. 6, pp. 564–575, 1965.

[8] S. Licul and W. A. Davis, "Unified frequency and time-domain antenna modeling and characterization," *IEEE Transactions on Antennas and Propagation*, vol. 53, no. 9, pp. 2882–2888, 2005.

[9] Werner Soergel and Werner Wiesbeck, "Influence of the antennas on the ultra-wideband transmission," *EURASIP J. Appl. Signal Process.*, vol. 2005, pp. 296–305, Jan. 2005.

[10] M. D. E. Turley, "Bandwidth formula for Linear FMCW radar waveforms," in *International Radar Conference (RADAR)*, Oct 2009, pp. 1–6.