MULTI-POLARIZED MULTI-USER MASSIVE MIMO: PRECODER DESIGN AND PERFORMANCE ANALYSIS

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ABSTRACT

Space limitation and multi-antenna channel acquisition prevent Massive multiple-input multiple-output (MIMO) from being easily deployed. The use of multi-polarized antennas can be one solution to alleviate the first obstacle. Furthermore, the dual structured precoding, in which a preprocessing based on the spatial correlation and a subsequent linear precoding based on the short-term channel state information at the transmitter (CSIT) are concatenated, can reduce the feedback overhead efficiently. To reduce the feedback overhead further, we propose a dual structured multi-user linear precoding, in which the subgrouping method based on co-polarization is additionally applied to the spatially grouped mobile stations (MSs) in the preprocessing stage. By investigating the behavior of the asymptotic performance as a function of the cross-polar discrimination (XPD) parameter, we also propose a new dual structured precoding, in which the XPD, spatial correlation, and CSIT quality are jointly utilized in the precoding/feedback for the multi-polarized multi-user massive MIMO system.

Index Terms**—** Multi-polarized Massive MIMO, Dual structured precoding with long-term/short-term CSIT

1. INTRODUCTION

By deploying a large number of antennas at the base station (BS), high spectral gain can be achieved [1]. However, the channel acquisition of a large number of antennas prevents us from taking the benefit of Massive multiple-input multiple-output (MIMO) system. To resolve the channel acquisition burden at BS, the dual structured precoding, in which a preprocessing based on the long-term channel state information at the transmitter (CSIT) (mainly, spatial correlation) and a subsequent linear precoding based on the short-term CSIT (that generally has lower dimension than the number of transmit antennas) are concatenated, can be exploited and reduce the feedback overhead efficiently [2, 3]. Another challenge of the massive MIMO system is the antenna space limitation. The multi-polarized antennas can be one solution to alleviate the space limit due to the large number of antennas [2, p.602].

In this paper, we first model the massive multi-user MIMO system, where BS is equipped with a large number of multi-polarized antennas and mobile stations (MSs) are equipped with a single single-polarized antenna and present the dual structured precoding based on long-term/short-term CSIT. By grouping the spatially correlated MSs and multiplying the same preprocessing matrix based on spatial correlation, the dimension of the precoding signal space after preprocessing can be reduced, resulting in the feedback overhead reduction [3]. To reduce the feedback overhead further, we propose a dual structured linear precoding, in which the subgrouping method based on the polarization is applied to the spatially grouped MSs in the preprocessing stage. By subgrouping co-polarized MSs in each group, we let the MS report the CSI from the transmit antennas having the same polarization as its polarization and further reduce the short-term CSI feedback overhead. Under the imperfect CSIT, two different dual structured precodings with preprocessing of i) grouping based only on the spatial correlation and ii) subgrouping based on both the spatial correlation and polarization can be asymptotically analyzed based on random matrix theory [4]. From the asymptotic results, we can find that even though the dual precoding using the subgrouping can reduce the feedback overhead, its performance can be affected by the the cross-polar discrimination (XPD) parameter (the long-term statistics of the antennas and channel depolarization that measures the ability to distinguish the orthogonal polarization). Under the same feedback overhead, the dual precoding with subgrouping can utilize more accurate CSIT, but undergoes a performance degradation according to the XPD compared to that with spatial grouping only. Accordingly, we identify the region that the dual precoding with subgrouping outperforms that with spatial grouping. The region depends on the XPD, the spatial correlation, and the short-term CSIT quality. Based on the result, we propose a new dual structured precoding in which the XPD, the spatial correlation, and the short-term CSIT quality are jointly utilized in the precoding/feedback for the multi-polarized multi-user massive MIMO system.

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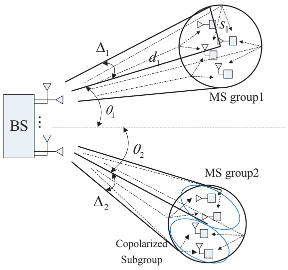


Fig. 1. An example of a dual-polarized multi-user downlink.

2. SYSTEM MODEL

Fig 1 shows an example of dual-polarized multi-user downlink system, where one BS has $\frac{M}{2}$ pairs of co-located vertically/horizontally polarized antenna elements and N MSs have a single antenna with either vertical or horizontal polarization[1]. Since human activity is usually confined in small clustered regions such as buildings, locations of MSs tend to be spatially clustered, e.g., G groups. Then, the received signal $\mathbf{y}_g \in \mathbb{C}^{N_g}$ of the gth group is given by

$$\mathbf{y}_g = \begin{bmatrix} \mathbf{y}_g^v \\ \mathbf{y}_g^h \end{bmatrix} = \mathbf{H}_g^H \mathbf{x} + \mathbf{n}_g, \quad (1)$$

where $\mathbf{y}_g^v$ and $\mathbf{y}_g^h$ are the received signal for MSs with vertical and horizontal polarization, respectively, and $\mathbf{n}_g$ is a zero-mean complex Gaussian noise vector having a covariance matrix $\mathbf{I}_{N_g}$. Here, N_g denotes the number of MSs in the gth group. For simplicity, it is assumed that $N_1 = ... = N_G = \bar{N}$, where $\bar{N}$ is even, and both $\mathbf{y}_g^v$ and $\mathbf{y}_g^h$ are $\frac{\bar{N}}{2} \times 1$ vectors. The channel of the kth MS in the gth group is then given as $\mathbf{h}_{gk} = [\mathbf{H}_g]_k$. The $M \times 1$ vector, $\mathbf{x}$, is the linearly precoded transmit signal expressed as $\mathbf{x} = \sum_{g=1}^{G} \mathbf{V}_g \mathbf{d}_g$, where $\mathbf{V}_g \in \mathbb{C}^{M \times \bar{N}}$ and $\mathbf{d}_g = \begin{bmatrix} \mathbf{d}_g^v \\ \mathbf{d}_g^h \end{bmatrix}$ are the linear precoding matrix and the data symbol vector for the MSs in the gth group, respectively. The precoded signal $\mathbf{x}$ should satisfy the power constraint $E[\|\mathbf{x}\|^2] \leq P$.

By using the Karhunen-Loeve transform and the polarized MIMO channel modeling with infinitesimally small antenna elements described in [2,p.76], the downlink channel to the gth group, $\mathbf{H}_g$, can be represented as

$$\mathbf{H}_g = \left(\mathbf{I}_2 \otimes (\mathbf{U}_g \mathbf{\Lambda}_g^{\frac{1}{2}}) \right) \left(\mathbf{G}_g \odot (\mathbf{X} \otimes \mathbf{1}_{r \times \frac{\bar{N}}{2}}) \right) \quad (2)$$

where $\mathbf{\Lambda}_g$ is an $r \times r$ diagonal matrix with the non-zero eigenvalues of the spatial covariance matrix $\mathbf{R}_g^s$ for the gth group[2] (generally, $r \ll M$), and $\mathbf{U}_g \in \mathbb{C}^{\frac{M}{2} \times r}$ has the associated eigenvectors of $\mathbf{R}_g^s$ as columns. The matrix $\mathbf{G}_g$ is defined as $\mathbf{G}_g = \begin{bmatrix} \mathbf{G}_g^{vv} & \mathbf{G}_g^{hv} \\ \mathbf{G}_g^{vh} & \mathbf{G}_g^{hh} \end{bmatrix}$, and the elements of $\mathbf{G}_g^{pq} \in \mathbb{C}^{r \times \frac{\bar{N}}{2}}$, $p, q \in \{h, v\}$ are complex Gaussian distributed with zero mean and unit variance. The matrix $\mathbf{X}$ describes the power imbalance between the orthogonal polarizations and is given as $\mathbf{X} = \begin{bmatrix} 1 & \sqrt{\chi} \\ \sqrt{\chi} & 1 \end{bmatrix}$, where the parameter $0 \leq \chi \leq 1$ is the inverse of the XPD, where $1 \leq \text{XPD} \leq \infty$. Accordingly, (2) can be rewritten as

$$\mathbf{H}_g = \begin{bmatrix} \mathbf{U}_g \mathbf{\Lambda}_g^{\frac{1}{2}} \mathbf{G}_g^{vv} & \sqrt{\chi} \mathbf{U}_g \mathbf{\Lambda}_g^{\frac{1}{2}} \mathbf{G}_g^{hv} \\ \sqrt{\chi} \mathbf{U}_g \mathbf{\Lambda}_g^{\frac{1}{2}} \mathbf{G}_g^{vh} & \mathbf{U}_g \mathbf{\Lambda}_g^{\frac{1}{2}} \mathbf{G}_g^{hh} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_g^{vv} & \mathbf{H}_g^{hv} \\ \mathbf{H}_g^{vh} & \mathbf{H}_g^{hh} \end{bmatrix}, \quad (3)$$

and its covariance matrix is given as

$$\mathbf{R}_g = \begin{bmatrix} \mathbf{R}_g^s & \mathbf{0} \\ \mathbf{0} & \chi \mathbf{R}_g^s \end{bmatrix} + \begin{bmatrix} \chi \mathbf{R}_g^s & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_g^s \end{bmatrix} = \mathbf{R}_{gv} + \mathbf{R}_{gh}, \quad (4)$$

where $\mathbf{R}_{gv}$ and $\mathbf{R}_{gh}$ are the covariance matrices of the vertically and the horizontally co-polarized MS subgroups, respectively.

The short-term CSI parameter $\mathbf{G}_g$ is varying independently over the short-term coherence time. The feedback of instantaneous channel incurs a significant feedback overhead and imperfect CSIT at the BS. Accordingly, the imperfect CSI $\hat{\mathbf{G}}_g$ available at the transmitter is modeled as $\hat{\mathbf{G}}_g = \sqrt{1-\tau} \mathbf{G}_g + \tau \mathbf{Z}_g$, where the elements of $\mathbf{Z}_g$ are complex Gaussian distributed with zero mean and unit variance and $\tau \in [0, 1]$ indicates the accuracy of available CSIT. That is, the case of $\tau = 0$ implies the perfect CSIT. From (3), $\hat{\mathbf{H}}_g$ and $\hat{\mathbf{H}}_g^{pp}$ can be defined as the imperfect CSIT of $\mathbf{H}_g$ and $\mathbf{H}_g^{pp}$, respectively, by using $\hat{\mathbf{G}}_g$. Note that the kth MS in the gth group can have much smaller feedback overhead by sending the essential channel information of $\mathbf{g}_{gk} \in \mathbb{C}^{2r \times 1}$ rather than $\mathbf{h}_{gk}$.

3. DUAL STRUCTURED PRECODING BASED ON LONG-TERM/SHORT-TERM CSIT

The dual structured precoding matrix for the gth group is given as $\mathbf{V}_g = \mathbf{B}_g \mathbf{P}_g$, where $\mathbf{B}_g \in \mathbb{C}^{M \times \bar{B}}$ is the preprocessing matrix based on the long-term channel statistics with $\bar{N} \leq \bar{B} \leq 2r \ll M$ and $\mathbf{P}_g \in \mathbb{C}^{\bar{B} \times \bar{N}}$ is the precoding matrix for the effective (instantaneous) channel $\mathbf{H}_g^H \mathbf{B}_g$. Here, $\bar{B}$ is a design parameter that determines the dimension of the transformed channel using the long-term CSIT. The system equation (1) can then be rewritten as

$$\mathbf{y}_g = \mathbf{H}_g^H \mathbf{B}_g \mathbf{P}_g \mathbf{d}_g + \sum_{l=1, l \neq g}^{G} \mathbf{H}_g^H \mathbf{B}_l \mathbf{P}_l \mathbf{d}_l + \mathbf{n}_g. \quad (5)$$

Preprocessing using block diagonalization (BD) based on spatial correlation: To null out the leakage to other groups,

[1] Throughout the paper, we consider the dual-polarized antennas at the BS for ease of explanation, but our approach can be extended to the multi-polarized case without difficulty.

[2] For simplicity, it is assumed that the spatial covariance matrix is the same for both polarizations.

it is desirable that the preprocessing matrix $\mathbf{B}_g$ based on the spatial correlation is designed as

$$\mathbf{H}_l^H \mathbf{B}_g \approx \mathbf{0}, \text{ for } l \neq g. \quad (6)$$

Then, $\mathbf{P}_g$ in (5) can be computed with the decoupled system model $\mathbf{y}_g \approx \mathbf{H}_g^H \mathbf{B}_g \mathbf{P}_g \mathbf{d}_g + \mathbf{n}_g$ where the inter-group interferences have been eliminated.

To obtain $\mathbf{B}_g$ satisfying the condition (6), the BD can be utilized. That is, due to the block diagonal structure in (4), we first define

$$\mathbf{U}_{-g} = [\mathbf{U}_1^a, ..., \mathbf{U}_{g-1}^a, \mathbf{U}_{g+1}^a, ..., \mathbf{U}_G^a] \in \mathbb{C}^{\frac{M}{2} \times (G-1)r^a}, \quad (7)$$

where $\mathbf{U}_g^a = [\mathbf{U}_g]_{1:r^a}$ and $r^a (\leq r)$ is the number of *dominant* eigenvalues of $\mathbf{R}_g^s$, a design parameter. Note that the perfect orthogonality is guaranteed when $r^a = r$. The matrix $\mathbf{U}_{-g}$ in (7) then has a singular value decomposition (SVD) as $\mathbf{U}_{-g} = [\mathbf{E}_{-g}^{(1)}, \mathbf{E}_{-g}^{(0)}] \begin{bmatrix} \mathbf{\Lambda}_{-g}^{(1)} & \\ & \mathbf{\Lambda}_{-g}^{(0)} \end{bmatrix} \mathbf{V}_{-g}^H$, where $\mathbf{E}_{-g}^{(1)}$ (resp. $\mathbf{E}_{-g}^{(0)}$) is the left singular vectors associated with the $(G-1)r^a$ dominant (resp. $\frac{M}{2} - (G-1)r^a$ non-dominant) singular values $\mathbf{\Lambda}_{-g}^{(1)}$ (resp. $\mathbf{\Lambda}_{-g}^{(0)}$). Then, because $(\mathbf{E}_{-g}^{(0)})^H \mathbf{U}_{-g} = 0$, by defining $\tilde{\mathbf{H}}_g = (\mathbf{I}_2 \otimes \mathbf{E}_{-g}^{(0)})^H \mathbf{H}_g$, $\tilde{\mathbf{H}}_g$ is orthogonal to the dominant eigen-space spanned by other groups' channel. Note that the covariance matrix of $\tilde{\mathbf{H}}_g$ is then given by

$$\tilde{\mathbf{R}}_g = (\mathbf{I}_2 \otimes \mathbf{E}_{-g}^{(0)})^H \mathbf{R}_g (\mathbf{I}_2 \otimes \mathbf{E}_{-g}^{(0)}), \quad (8)$$

and by defining $\tilde{\mathbf{R}}_g^s = (\mathbf{E}_{-g}^{(0)})^H \mathbf{R}_g^s \mathbf{E}_{-g}^{(0)}$, we have its eigenvalue decomposition (EVD) as $\tilde{\mathbf{R}}_g^s = \mathbf{F}_g \tilde{\mathbf{\Lambda}}_g \mathbf{F}_g^H$, where $\mathbf{F}_g$ is the eigenvectors of $\tilde{\mathbf{R}}_g^s$. Then by letting $\mathbf{F}_g^{(1)} = [\mathbf{F}_g]_{1:\frac{\bar{B}}{2}}$, the preprocessing matrix can be given as

$$\mathbf{B}_g = \mathbf{I}_2 \otimes \mathbf{B}_g^s, \quad \mathbf{B}_g^s = \mathbf{E}_{-g}^{(0)} \mathbf{F}_g^{(1)}. \quad (9)$$

With $\mathbf{B}_g$, we can transform the transmit signal for the gth group into the $\bar{B}$ dimensional dominant eigen-space that is orthogonal to the subspace spanned by other groups' channel.

Multi-user precoding for each decoupled group: Because the effective channel for the gth group is $\bar{\mathbf{H}}_g = \mathbf{B}_g^H \mathbf{H}_g$, the corresponding covariance matrix is given by

$$\bar{\mathbf{R}}_g = \mathbf{B}_g^H \mathbf{R}_g \mathbf{B}_g = \mathbf{B}_g^H (\mathbf{R}_{gv} + \mathbf{R}_{gh}) \mathbf{B}_g = \bar{\mathbf{R}}_{gv} + \bar{\mathbf{R}}_{gh}. \quad (10)$$

Furthermore, the precoding matrix $\mathbf{P}_g$ is designed such that the intra-group interferences are nulled out based on the short-term CSIT of the gth group. That is, the regularized ZF precoding matrix [3] can be utilized as $\mathbf{P}_g = \xi_g \hat{\bar{\mathbf{K}}}_g \hat{\bar{\mathbf{H}}}_g$, where $\hat{\bar{\mathbf{K}}}_g = \left(\hat{\bar{\mathbf{H}}}_g \hat{\bar{\mathbf{H}}}_g^H + \bar{B} \alpha \mathbf{I}_{\bar{B}} \right)^{-1}$, $\hat{\bar{\mathbf{H}}}_g = \mathbf{B}_g^H \hat{\mathbf{H}}_g$. Here, $\alpha = \frac{\bar{N}}{\bar{B}P}$, which is equivalent with the MMSE linear filter [2,p.241]. The normalization factor ξ_g is then given as

$$\xi_g^2 = \frac{\bar{N} \frac{P}{N}}{\frac{P}{N} tr(\hat{\bar{\mathbf{H}}}_g^H \hat{\bar{\mathbf{K}}}_g^H \mathbf{B}_g^H \mathbf{B}_g \hat{\bar{\mathbf{K}}}_g \hat{\bar{\mathbf{H}}}_g)} = \frac{\bar{N}}{tr(\hat{\bar{\mathbf{H}}}_g^H \hat{\bar{\mathbf{K}}}_g^H \hat{\bar{\mathbf{K}}}_g \hat{\bar{\mathbf{H}}}_g)}. \quad (11)$$

Denoting $\hat{\bar{\mathbf{h}}}_{gk} = [\hat{\bar{\mathbf{H}}}_g]_k$ as the effective channel estimate of the kth MS in the gth group, The SINR of the kth MS in the gth group with p polarization is then given by

$$\gamma_{gpk}^{BD} = \frac{\frac{P}{N} \xi_g^2 |\mathbf{h}_{gk}^H \mathbf{B}_g \hat{\bar{\mathbf{K}}}_g \hat{\bar{\mathbf{h}}}_{gk}|^2}{\frac{P}{N} \sum_{j \neq k} \xi_g^2 |\mathbf{h}_{gk}^H \mathbf{B}_g \hat{\bar{\mathbf{K}}}_g \hat{\bar{\mathbf{h}}}_{gj}|^2 + \frac{P}{N} \sum_{l \neq g} \sum_j \xi_l^2 |\mathbf{h}_{gk}^H \mathbf{B}_l \hat{\bar{\mathbf{K}}}_l \hat{\bar{\mathbf{h}}}_{lj}|^2 + 1}.$$

Dual precoding using block diagonalization and sub-grouping (BDS) based on both spatial correlation and polarization: Note that, when χ becomes small (i.e., the antennas can favorably discriminate the orthogonally polarized signals), the interference signals through the cross-polarized channels can be naturally nulled out. This suggests that we can make the subgroups of co-polarized MSs in each group and let the BS precode the signal for the co-polarized subgroup by using the short-term CSIT of transmit antennas having the same polarization with the associated subgroup. That is, from (1) and (5), the received signal for the co-polarized subgroup with p polarization, for $p \in \{h, v\}$, in the gth group can be written as

$$\mathbf{y}_g^p = \mathbf{H}_{gp}^H \mathbf{B}_{gp} \mathbf{P}_{gp} \mathbf{d}_g^p + \sum_{\substack{q \in \{h,v\} \\ q \neq p}} \mathbf{H}_{gp}^H \mathbf{B}_{gq} \mathbf{P}_{gp} \mathbf{d}_g^q + \sum_{l=1, l \neq g}^{G} \sum_{q \in \{h,v\}} \mathbf{H}_{gp}^H \mathbf{B}_{lq} \mathbf{P}_{lq} \mathbf{d}_l^q + \mathbf{n}_g^p,$$

where $\mathbf{H}_{gv} = \begin{bmatrix} \mathbf{H}_g^{vv} \\ \mathbf{H}_g^{vh} \end{bmatrix}$ and $\mathbf{H}_{gh} = \begin{bmatrix} \mathbf{H}_g^{hv} \\ \mathbf{H}_g^{hh} \end{bmatrix}$ from (3). Here, $\mathbf{B}_{gp}$ for $p \in \{h, v\}$ are given as $\mathbf{B}_{gv} = \begin{bmatrix} \mathbf{B}_g^s \\ \mathbf{0} \end{bmatrix}$, $\mathbf{B}_{gh} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_g^s \end{bmatrix}$, where $\mathbf{B}_g^s$ is given in (9). Note that, when $\chi \approx 0$, we can easily find that $\mathbf{H}_{lp}^H \mathbf{B}_{gq} \approx \mathbf{0}$, for $p \neq q$. Furthermore, because $\mathbf{H}_{gq}$, $q \neq p$ has no influence on $\mathbf{P}_{gp}$, the MSs do not need to feed back the instantaneous CSI from cross polarized transmit antennas at BS.

The precoding matrix $\mathbf{P}_{gp}$ is then designed such that the intra-subgroup interferences are nulled out using the co-polarized short-term CSIT. That is, letting $\hat{\mathbf{H}}_{gp}$ denotes the imperfect CSIT of $\mathbf{H}_{gp}$, $p \in \{h, v\}$, the regularized ZF precoding matrix can be computed as $\mathbf{P}_{gp} = \xi_{gp} \hat{\bar{\mathbf{K}}}_{gp} \hat{\bar{\mathbf{H}}}_{gp}$, where $\hat{\bar{\mathbf{K}}}_{gp} = \left(\hat{\bar{\mathbf{H}}}_{gp} \hat{\bar{\mathbf{H}}}_{gp}^H + \frac{\bar{B}}{2} \alpha \mathbf{I}_{\frac{\bar{B}}{2}} \right)^{-1}$ and $\hat{\bar{\mathbf{H}}}_{gp} = \mathbf{B}_{gp}^H \hat{\mathbf{H}}_{gp} = (\mathbf{B}_g^s)^H \hat{\mathbf{H}}_g^{pp}$, the effective channel estimate that is available at the BS. The normalization factor ξ_{gp} is then given as

$$\xi_{gp}^2 = \frac{\bar{N}/2}{tr(\hat{\bar{\mathbf{H}}}_{gp}^H \hat{\bar{\mathbf{K}}}_{gp}^H \hat{\bar{\mathbf{K}}}_{gp} \hat{\bar{\mathbf{H}}}_{gp})}. \quad (12)$$

Assuming equal power allocation, the SINR of the kth MS in the subgroup with p polarization of the gth group is then given by

$$\gamma_{gpk}^{BDS} = \frac{\frac{P}{N} \xi_{gp}^2 |\mathbf{h}_{gpk}^H \mathbf{B}_{gp} \hat{\bar{\mathbf{K}}}_{gp} \mathbf{B}_{gp}^H \hat{\mathbf{h}}_{gpk}|^2}{IN_{gpk}}, \quad (13)$$

where $IN_{pgk} = \frac{P}{N}\sum_{j\neq k}\xi_{gp}^2|\mathbf{h}_{gpk}^H\mathbf{B}_{gp}\hat{\hat{\mathbf{K}}}_{gp}\mathbf{B}_{gp}^H\hat{\mathbf{h}}_{gpj}|^2 + \frac{P}{N}\sum_{q\neq p}\sum_j\xi_{gq}^2|\mathbf{h}_{gpk}^H\mathbf{B}_{gq}\hat{\hat{\mathbf{K}}}_{gq}\mathbf{B}_{gq}^H\hat{\mathbf{h}}_{gqj}|^2 + \frac{P}{N}\sum_{l\neq g}\sum_q\sum_j\xi_{lq}^2|\mathbf{h}_{gpk}^H\mathbf{B}_{lq}\hat{\hat{\mathbf{K}}}_{lq}\mathbf{B}_{lq}^H\hat{\mathbf{h}}_{lqj}|^2 + 1$. Here, $\mathbf{h}_{gpk} = [\mathbf{H}_{gp}]_k$, and $\hat{\mathbf{h}}_{gpk} = [\hat{\mathbf{H}}_{gp}]_k$, respectively.

4. ASYMPTOTIC PERFORMANCE ANALYSIS FOR A LARGE M

Based on the analytic results using random matrix theory [4, 5], we can derive the asymptotic SINR for two different dual precoding schemes – dual precoding with i) BD and ii) BDS. However, due to the limit of space, we omit the detailed results (see the expanded version [6]), but we introduce two propositions about the asymptotic SINR and its behavior as f function of the XPD parameter χ.

Proposition 1 *For a large M, the asymptotic SINR of the dual precoding with BD is written as the function $\gamma_{gpk}^{BD,o}(\chi)$ of χ given by*

$$\gamma_{gpk}^{BD,o}(\chi) \approx \frac{A_0(1-\tau_{BD}^2)}{B_0(1+D_0\tau_{BD}^2)+(1+E_0)(D_0+1)}, \quad (14)$$

where A_0, B_0, D_0, and $E_0 \ll 1$ depends only on $\bar{\mathbf{R}}_{gv}$ and $\bar{\mathbf{R}}_{gh}$ in (10) and τ_{BD} is the accuracy of available CSIT for the dual precoding with BD. For detailed expressions of the parameters, please see [6]. Note that $\gamma_{gpk}^{BD,o}(\chi)$ is approximately independent of χ (i.e., $\gamma_{gpk}^{BD,o}(\chi) \approx \gamma_{gpk}^{BD,o}(0)$).

Proposition 2 *For a large M, the asymptotic SINR of the dual precoding with BDS is written as the function $\gamma_{gpk}^{BDS,o}(\chi)$ of χ given by*

$$\gamma_{gpk}^{BDS,o}(\chi) \approx \frac{A_0(1-\tau_{BDS}^2)}{(B_0(1+D_0\tau_{BDS}^2)+(1+E_0)(D_0+1))(1+c_0\chi)}. \quad (15)$$

where $c_0 \approx \frac{B_0(D_0+1)}{B_0D_0\tau_{BDS}^4+B_0+D_0+1}$ [6] and τ_{BDS} is the accuracy of available CSIT for the dual precoding with BDS. Then, $\gamma_{gpk}^{BDS,o}(\chi) \approx \frac{\gamma_{gpk}^{BDS,o}(0)}{1+c_0\chi}$.

Note that, from Proposition 2, the asymptotic SINR of the dual precoding with BDS decreases when χ increases. This is because the subgroups are formed with the assumption that the interferences through the cross-polarized channels are perfectly nulled out and $\mathbf{P}_{gp}$ is determined based only on co-polarized CSIT. Therefore, the interference power increases proportionally to χ. In contrast, because the dual precoding with BD nulls out the intra-group interferences based on both co/cross polarized CSIT, it exhibits performances somehow robust to the variation of the polarization parameter χ.

5. A NEW DUAL STRUCTURED PRECODING

Even though the performance of the dual precoding with BDS is degraded according to the XPD parameter, it can utilize more accurate short-term CSIT compared to the dual precoding with BD under the same number of feedback bits. When random vector quantization (RVQ) with N_B bits is utilized [7], the quantization error for the short-term CSIT in the dual precoding with BD (i.e., the columns of $\mathbf{G}_g$) is upper bounded as $\tau_{BD}^2 < 2^{-\frac{N_B}{2r-1}}$. Here, the codebook is fixed given r and there is no adaptive codebook that would adapt as a function of r and XPD. For the dual precoding with BDS, the quantization error is upper bounded as $\tau_{BDS}^2 < 2^{-\frac{N_B}{r-1}}$. Because the bound is tight for a large N_B [7], by assuming $\tau_{BD}^2 = 2^{-\frac{N_B}{2r-1}} (\approx \tau_{BDS})$, we have the following proposition.

Proposition 3 *For a given χ and a large M, when*

$$N_B \lesssim (2r-1)\left(\log_2\left(1+\frac{D_0}{B_0(D_0+1)}\right) - \log_2\chi\right), \quad (16)$$

the dual precoding with BDS outperforms that with BD.

Proof: The dual precoding with BDS outperforms that with BD when $\gamma_{gpk}^{BDS,o}(\chi) \geq \gamma_{gpk}^{BD,o}(\chi)$. By letting $\tau_{BD}^2 \triangleq \tau^2 = 2^{-\frac{N_B}{2r-1}} (\approx \tau_{BDS})$, after a simple calculation with (14) and (15), the inequality becomes

$$\chi \leq \frac{(B_0D_0 + B_0 + (1+E_0)(D_0+1))\tau^2}{c_0(B_0D_0\tau^4 + B_0 + (1+E_0)(D_0+1))}. \quad (17)$$

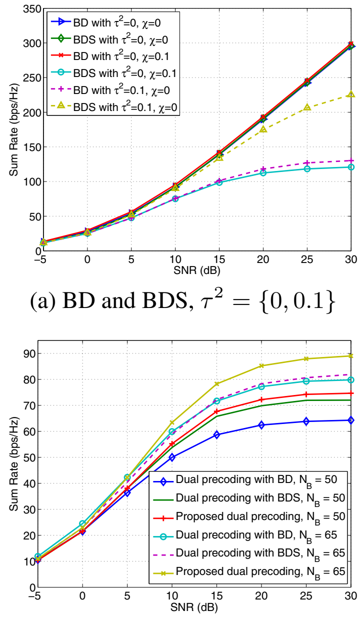
Because $E_0 \ll 1$, by substituting c_0 in (15) into (17), we have $\chi \lesssim (1 + \frac{D_0}{B_0(D_0+1)})\tau^2$, which induces (16). □

From Proposition 3, when the feedback bits are not enough to describe the short-term CSIT accurately, the dual precoding with BDS exhibits a better performance than that with BD. That is, it is preferable that by forming the co-polarized subgroup, each MS feeds back the short-term CSI from the co-polarized transmit antennas. In addition, from (16), when $\chi \to 0$, the dual precoding with BDS always exhibits better performance than that with BD. Therefore, based on Proposition 3, a new dual precoding/feedback scheme can be developed in which, depending on the long-term CSI (spatial correlation, polarization) and the number of feedback bits (or, the short-term CSIT accuracy τ), the precoding mode is switched between BD and BDS.

6. SIMULATION RESULTS

In the simulation, it is assumed that BS has a dual polarized linear array antenna with $M = 120$ and all the antenna elements are perfectly aligned with either vertical or horizontal polarization. For consideration about the polarization mismatch due to the random variation of MS orientation, please see [6]. It is also assumed that $N = 32$, $G = 4$, $\bar{N} = 8$. For preprocessing, we set as $\bar{B} = \min(2\bar{N}, 2r)$, where r is the rank of $\mathbf{R}_g^s$. In addition, we consider the one-ring model for the spatially correlated channel [8]. That is, the correlation between the channel coefficients of antennas $1 \le m, n \le M$ is given by $[\mathbf{R}_g^s]_{m,n} = \frac{1}{2\Delta_g}\int_{-\Delta_g}^{\Delta_g} e^{-j\pi\lambda_0^{-1}\mathbf{\Omega}(\alpha+\theta_g)(\mathbf{r}_m-\mathbf{r}_n)}d\alpha$, where θ_g and Δ_g are, respectively, the azimuth angle at which the gth group is located and the angular spread of the departure waves to the gth group which is determined as $\Delta_g \approx tan^{-1}(s_g/d_g)$. Here, s_g and d_g are, respectively, the radius of the ring of scatterers for the gth group and the distance between the BS and the gth group. (see Fig. 1.) The parameter λ_0 is the wavelength of signal, $\mathbf{r}_m = [x_m, y_m]^T$ is the position vector of the mth antenna, and $\mathbf{\Omega}(\alpha)$ indicates the wave vector with the angle-of-departure (AoD), α, given by $\mathbf{\Omega}(\alpha) = (\cos(\alpha), \sin(\alpha))$. Here, $d_s = \frac{\lambda_0}{2}$, $\Delta = \frac{\pi}{12}$, and $\theta_g = -\frac{\pi}{4} + \frac{\pi}{6}(g-1)$ for $g = 1, ..., 4$. Fig. 2(a) shows the sum rates of the dual precodings with BD and BDS (solid lines) when the perfect CSIT is assumed (i.e., $\tau^2 = 0$). Note that when $\chi = 0$, the dual precoding with BD and BDS exhibit the same sum rate performance. However, when $\chi = 0.1$, the performance of the dual precoding with BDS is degraded, while that of the dual precoding with BD does not change significantly compared to the case of $\chi = 0$. In Fig. 2(a), we additionally plot the sum rates when $\chi = 0$ for the imperfect CSIT with $\tau^2 = 0.1$ (dashed lines). We can see that the BDS exhibits better performance than the BD because the BDS can exploit more accurate short-term CSIT due to the feedback of the smaller dimensional channel instance.

In Fig. 2(b), we have compared the sum rates of the new dual structured precoding in Section 5 with those of dual precodings with BD and BDS. We set $N_B = \{50, 65\}$ and χ is uniformly distributed on $[0, 0.5]$. The sum rates for $N_B = 65$ is higher than that for $N_B = 50$ and the sum rates of all schemes are saturated at high SNR due to the imperfect CSIT. Note that the proposed scheme exhibits higher performance than the other two schemes.



(a) BD and BDS, $\tau^2 = \{0, 0.1\}$

(b) BD, BDS, and a proposed dual precoding, $N_B = \{50, 65\}$

Fig. 2. Sum rates of dual precodings.

7. CONCLUSION

In this paper, we have investigated the dual structured linear precoding in the multi-polarized MU massive MIMO system. We have found that the dual precoding with BD exhibits substantially robust performance against the XPD, while the performance of that with BDS is affected by the XPD parameter. Because the dual precoding with BDS can have more accurate CSIT than that with BD under the same number of feedback bits, the region where the BDS exhibits better performance than the BD is analytically derived. Finally, we have proposed a new dual structured precoding/feedback in which the precoding mode is switched between BD and BDS depending on the XPD, spatial correlation, and the number of short-term feedback bits (short-term CSIT quality).

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