

# SEGMENTATION OF 3D DYNAMIC MESHES BASED ON REEB GRAPH APPROACH

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## ABSTRACT

This paper presents a new segmentation approach, for 3D dynamic meshes, based upon ideas from Morse theory and Reeb graphs. The segmentation process is performed using topological analysis of smooth functions defined on 3D mesh surface. The main idea is to detect critical nodes located on the mobile and immobile parts. Particularly, we define a new continuous scalar function, used for Reeb graph construction. This function is based on the heat diffusion properties. Clusters are obtained according to the values of scalar function while adding a refinement step. The latter is based on curvature information in order to adjust segmentation boundaries. Experimental results performed on 3D dynamic articulated meshes demonstrate the high accuracy and stability under topology changes and various perturbations through time.

**Index Terms**— 3D dynamic meshes, segmentation, Reeb graph, heat diffusion, curvature information.

## 1. INTRODUCTION

3D dynamic shapes are becoming a media of increasing importance. They constitute a fundamental and time consuming task in 3D animation systems. This dynamic shapes are usually represented by a sequence of 3D meshes with temporal information provided by time-varying geometry. A dynamic articulated model is subject to a wide variety of processing operations such as segmentation, compression and indexation.

Mesh surface segmentation has been studied in computer vision, especially for compression and simplification. It consists in partitioning mesh elements (vertices, edges and faces) into disjoint sets according to certain criterion (curvature, patch area,...). The issue of 3D static meshes segmentation has rapidly gained the interest of the scientific community in recent years. However few existing works in the literature studies motion based segmentation for 3D dynamic meshes [1, 2, 3].

The pioneer method developed by Lengyel [1] proposes to partition the mesh into components, whose motion can be accurately described by a 3D affine transform. This heuristic approach select randomly 10% of triangles in mesh. The

remaining vertices are classified according to the motion of these triangles. The drawback of this approach resides in the large number of clusters which is independent of the motion. Amjoun *et al.* [2] extended this approach, to develop region growing segmentation algorithm. The authors propose to search the  $N$  farthest triangles in geodetic sense. The choice of triangles germs is based only on the geometry of the first frame. This may justify the lack of efficiency of the segmentation approach. The authors of [3] propose another alternative of [1]. The main idea consists on calculating, for each vertex  $v$ , the affine transform that optimally describes the motion of his neighborhood. All the vertices whose motion can be described by the same affine transform with respect to a minimum error motion compensation are grouped in the same cluster. The sub-optimal selection of vertices leads to a misclassification of the ones located on the borders between clusters.

More Recently Rosman *et al.* [4, 5] developed a motion-based segmentation technique to partition an articulated 3D shape into rigid parts. The proposed approaches rebuild on the Ambrosio-Tortorelli scheme for Mumford-Shah segmentation [6]. The latter establishes an optimality criterion to segment the 3D shape into sub-regions. The technique presented in [5] consists on performing a segmentation by resolving a piecewise-smooth regularization problem.

Skeletonisation and segmentation tasks are closely related, one being the driver to the other. Mesh segmentation is often formulated as graph clustering. Few existing works in the literature use Reeb-graphs or skeletons to accomplish a structural based segmentation of static meshes [7, 8]. Up to date no work based on Reeb-graphs has been investigated for 3D dynamic meshes segmentation.

In this paper, we propose an implicit segmentation method which exploits the temporal information. The main contribution consists in partitioning dynamic meshes based on Kinematic Reeb graph (KRG) extraction method. It is no worthy that KRG technique allows the extraction of topological features which are preserved through the time. In this work, we first define a new scalar function based on the eccentricity in term of diffusion distance to construct the KRG. Then we present our implicit segmentation algorithm based on curva-

ture and boundary information.

The rest of this paper is organized as follows. In Section 2 we describe the KRG construction technique which is the core of the proposed approach. In Section 3, we describe our segmentation method in detail. In Section 4, we investigate the performance of our system in terms of accuracy and robustness. Finally, in Section 5, we conclude with final remarks and present some future work.

## 2. REEB GRAPH CONSTRUCTION

Reeb graph is a topological structure determined using a continuous scalar function defined on an object of arbitrary dimension. More specifically, a Reeb-graph is a schematic way to encode the behavior of a given continuous function  $f$  on a surface. It is a structure that represents the evolutions of the level lines of a  $f$ , defined over objects of any dimension ( $k$ -manifolds) [9]. The proposed approach for Reeb graph construction is performed in two steps. First, we extract the feature points based on the diffusion distance. Then, the set of these feature points is taken as the initial data to compute the scalar function  $\mu$ .

### 2.1. Feature points extraction

In the literature, many approaches were proposed to detect the local extremum for rigid and non rigid models. To extract the set of feature points, we adopted a strategy which is inspired from Tierny *et al.* [10] method. The extraction process starts by looking for the two farthest points,  $v_1$  and  $v_2$ , in geodesic sense. To extract two local properties groups, the authors of [10] defined two geodesic functions associated with each points  $v_1$  and  $v_2$ . The intersection of the resulting groups provides the set of feature points. This approach produces a set of well-localized points. But the feature points are very sensitive against topological changes. To overcome this problem and ensure stability under eventual perturbations over time, we propose using the diffusion distance  $d_t^2$  instead of geodesic one while keeping the same process. It is worthwhile to note that global properties of the shape are detected through the behavior of heat diffusion over longer time. On the overhand local properties are detected through the behavior of heat diffusion over short time. Indeed, the heat diffusion is fully described by the heat kernel associated with the Laplace-Beltrami operator. For a given surface  $S$ ,  $d_t^2$  measures the connectivity distance between two points  $x, y \in S$  at a given time  $t$  according to the following equation:

$$\begin{aligned} d_t^2(x, y) &= k_t(x, x) + k_t(y, y) - 2k_t(x, y) \\ &= \sum_i^\infty e^{(\lambda_i t)} (\phi_i(x) - \phi_i(y))^2. \end{aligned} \quad (1)$$

The heat kernel  $k_t(x, x)$  does not admit an explicit function; it can be computed based on the Laplace-Beltrami operator, with  $\lambda_i$  and  $\phi_i$  are respectively the  $i^{th}$  eigenvalues and eigenfunctions of the Laplace-Beltrami. Considering discrete

meshes, many cotangent schemes have been proposed to estimate the Laplace-Beltrami operator [11, 12, 13]. In this work, we suggest to use the solution proposed in [13] to approximate the Laplace-Beltrami operator and calculate the set of eigenvalues and eigenfunctions. Note for small  $t$ , the variation of the heat kernel function is large but decays as  $t$  increases. Therefore, to ensure an accurate detection of feature points, we scale the temporal domain logarithmically. This gives a more faithful approximation of local shape properties at the choosing time range  $[t_1, t_2]$ . Let  $v_i$  be one of the two farthest points ( $v_i \in \{v_1, v_2\}$ ). The set of feature points  $f_{v_i}$  corresponding to  $v_i$  is given by:

$$f_{v_i} = d_t^2(v, v_i) = \int_{t_1}^{t_2} k_t(v, v) + k_t(v_i, v_i) - 2k_t(v, v_i) d \log t.$$

Elements belonging to both  $f_{v_1}$  and  $f_{v_2}$  groups, constitute the set of feature points  $F$ . Thus,  $F = f_{v_1} \cap f_{v_2}$  will be used as origin to compute the scalar function defined in Section 2.2.

### 2.2. Proposed scalar function

According to the Morse theory, a continuous function defined on a closed surface characterizes the topology of the surface on its critical points. Thus, a Reeb graph can be obtained assuming a continuous function  $\mu$  calculated over the 3D object surface [14]. In the following we only consider objects which are closed 2-manifold meshes with vertices located in a Cartesian frame  $\mathbb{R}(x; y; z)$ .

Given a surface  $S$  of a 3D object and a real continuous function  $\mu : S \mapsto \mathbb{R}$ , the Reeb graph is the quotient space of the graph of  $\mu$  in  $S \times \mathbb{R}$  with the equivalence relation “ $\sim$ ” between  $X \in S$  and  $Y \in S$  is given by:

$$X \sim Y \iff \begin{cases} \mu(X) = \mu(Y) \\ Y \in \mu^{-1}(\mu(X)). \end{cases} \quad (2)$$

More specifically, for two nodes  $v_i, v_j$ :

$$(v_i, \mu(v_i)) \sim (v_j, \mu(v_j)),$$

if and only if  $\mu(v_i) = \mu(v_j)$ , where  $v_i, v_j$  belong to the same connected component of  $\mu^{-1}(\mu(v_j))$ .

Several work have been developed to propose a Reeb graph construction for rigid models [15, 16]. The proposed approaches have been later extended to non-rigid models [17, 18]. In particular, the pioneer method developed by Hilağa *et al.* [15] proposes to calculate a scalar function based on geodesic distance. Gal *et al.* [18] extended this context for non-rigid models. Nevertheless, scalar function based on geodesic distance is penalized by its sensitivity to topology changes.

In our work we propose to define an appropriate continuous function  $\mu$  in order to guarantee invariance and stability of the graph structure. The proposed scalar function, which exploits the temporal information, is based on the eccentricity

$ecc$  in terms of the diffusion distance. The latter is defined as the mean square of the diffusion distance on the whole surface of  $S$ :

$$ecc_t(x) = \frac{1}{area(S)} \int_S d_t^2(x, y) dy. \quad (3)$$

By using the discrete case, the  $\mu$  function is computed, as the sum of the eccentricity from  $v$  to each one of the feature points. Assuming that the surface  $S$  is approximated through a discrete triangular mesh  $M$ , for each vertex  $v \in M$ ,  $\mu(v)$  is established by:

$$\mu(v) = \frac{1}{area(M)} \sum_{p \in F} d_t^2(v, p) area(p), \quad (4)$$

where  $area(M)$  being the surface area of  $M$ ,  $F$  represents the set of feature points, which are extracted in the first step, and  $area(p)$  being the area that  $p$  occupies.

As result, the triangular mesh  $M$  is divided into regions depending on the values of  $\mu_n$ . Each connected component associated with each level set identifies a set of equivalent surface points and is represented with a node in the graph. The graph structure is obtained by linking the nodes of the connected regions.

### 3. PROPOSED SEGMENTATION APPROACH

A scalar function value  $\mu(v)$  is associated with each vertex  $v \in 1, \dots, V$ . We obtained a vector that contains all the values of  $\mu$  function for a given Frame. The vector size is the number of vertices in the consider mesh. Depending on the value of  $\mu$ , the vector is divided into intervals according to the number of connected components associated with each interval. Contiguous intervals with the same number of connected components are merged into a single interval. The process is repeated iteratively in order to reduce the number of regions (clusters). At each step, the system performs a fusion operation on the intervals groups that have the same number of connected components. The iterative loops stops when all the resulting intervals admit a different number of connected components.

Using a scalar function  $\mu$  based on the heat diffusion leads to a set of critical points well located on articulations at both immobile and mobile parts. However, computing the value of  $\mu$  in the discrete case may prevents the detection of region boundaries. Therefore, in order to adjust segmentation boundaries with respect to deep surface concavities, a refinement step is required. The additional treatment may leads to an accurate detection of region boundaries.

Each region boundary is a level set and is thus associated with a value of  $\mu$  that corresponds to a critical point. In our case, the proposed  $\mu$  function is stable against perturbations and invariant under isometric transformations. Thus we have overcome the locality problem. A perturbation of the  $\mu$  value

does not affect the locality boundaries. As result, the latter remains stable and does not shift to a new level set. In the refinement step, to identify a perceptually salient decomposition, we only consider the concavity problem. The optimal value of  $\mu_{opt}$  should determine a boundary that matches a deep concavity profile on the object surface. It should be close to  $\mu_c$  that correspond to the closest critical point. The

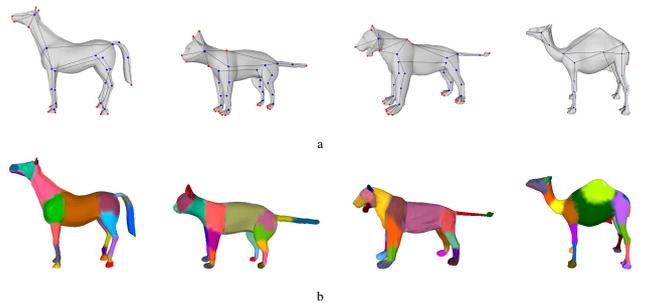
**Table 1.** Properties of the tested dynamic meshes.

	Vertices	Frames	$N_b$ regions	time (s)
Horse	8431	47	26	181
Cat	7207	10	20	34
Lion	5000	11	12	49
Camel	21887	11	38	217

objective aimed at is to found the the optimal value  $\mu_{opt}$  that determines the boundary profile. The issue maybe considered as an optimization problem, which consists of minimizing the concavity function  $E_{concave}(\mu')$  of each region boundary associated with a value of  $\mu'$ .  $E_{concave}(\mu')$  is defined by:

$$E_{concave}(\mu') = \min_{\mu'} (K_{min}(c_{(\mu')R}(t)) \otimes G_{\sigma}(t)), \quad (5)$$

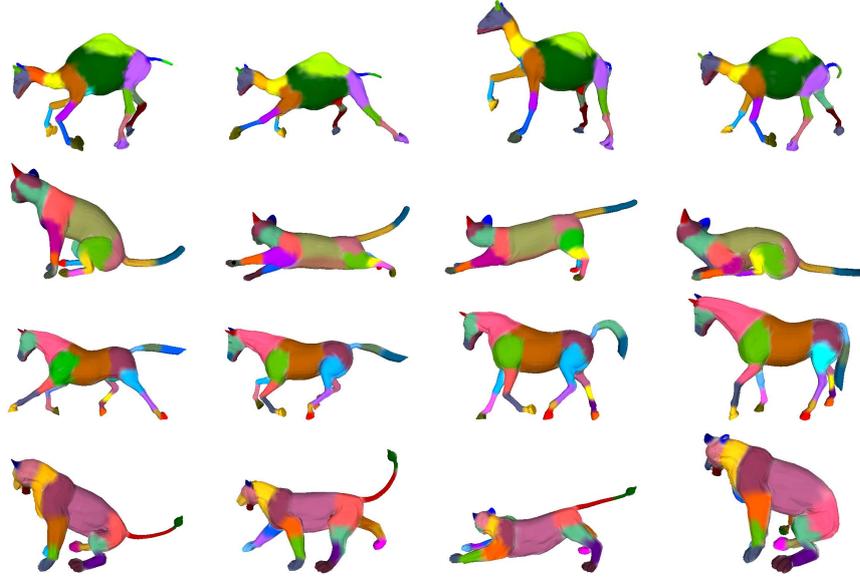
being  $K_{min}(\cdot)$  a function returning the sequence of  $K_{min}$  curvature values, computed according to [19], along the boundary profile,  $c_{(\mu')R}(t)$  the curve-parameterized with respect to the normalized arc-length  $t$ . In fact,  $c_{(\mu')R}(t)$  represents the portion of the  $\mu$  values set corresponding to the boundary of region  $R$ . Convolution  $\otimes$  with a gaussian smoothing kernel  $G_{\sigma}(t)$  leads to smoothing values of the  $K_{min}$ . Consequently, the minimum identification will be more efficient and stable. Curvature information is exploited to refine the segmentation and adjust region boundaries so as to match deep surface concavities.



**Fig. 1.** Reeb graphs (Part a) and segmentation (Part b) of different 3D meshes.

### 4. EXPERIMENTAL RESULTS

In order to evaluated the proposed segmentation approach, we consider some 3D dynamic meshes named: “Horse”, “Cat”,



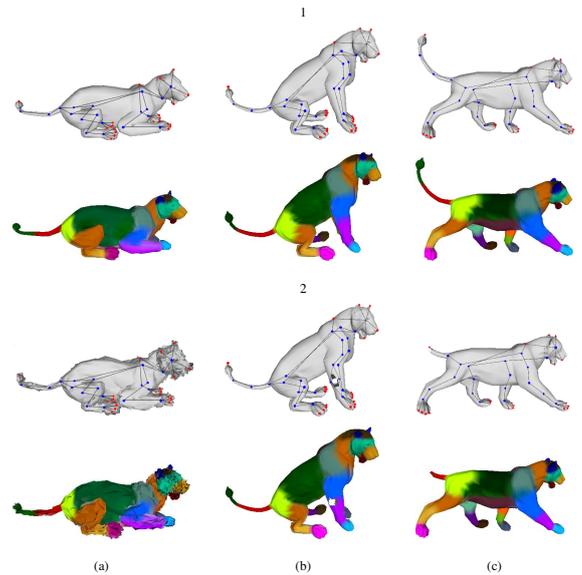
**Fig. 2.** Segmentation of different 3D mesh sequences.

“Lion” and “Camel”. These models are characterized by their various motions and complexities. Moreover, they offer a good variability in terms of spatial and temporal sizes. Table 1 summarizes their properties, expressed in terms of numbers of vertices, number of frames, number of obtained clusters and the average running time including the Reeb Graph extraction process.

#### 4.1. Accuracy assessment

To evaluate the execution-time, the tests were conducted on a laptop with an Intel Core 3 CPU M350 at 2.23 GHz, and operating system Windows 7 SP 1. From Table. 1 one can notice that the running time is smaller when the number of vertices decreases. This is due to the computation of laplace operator eigenvalue and eigenvector during the Reeb graph extraction. Fig. 1.a illustrates the test 3D models, represented by their reference frame, and their extracted Reeb graph plotted on the same figure. We can clearly see that the proposed approach allows detecting a set of extremum local points (red nodes) which are located on the object boundary. Fig. 1.b shows the segmentation results using the extracted Reeb graphs. For each test mesh we show regions related to each node in the Reeb graph, with a different palette of colors. Fig. 2 depicts some segmentation results for the test sequences. The number of clusters varies from one model to another with characteristics reported in Table. 1. We note that the segmentation process allows partitioning the mesh into clusters consisting of topologically connected vertices.

To assess the robustness of the proposed segmentation approach, we applied some transformations like noise addition (Fig. 3.2.a), holes (Fig. 3.2.b) and missing parts (Fig. 3.2.c). These transformations were performed to a set of interme-



**Fig. 3.** Robustness of the proposed approach against various transformations, Part 1: original sequence, Part 2: modified sequence applying noise addition in (a), holes in (b) and missing parts in (c).

diate frames of the “Lion” model. In Fig. 3, we compare the extracted Reeb graphs corresponding to the modified sequence and the original one. We can observe that our algorithm produces well-localized feature points, which are stable against the tested transformations. From the same figure, we can notice that the segmentation is invariant under almost all applied transformations. This proves the high stability of the

proposed approach.

## 5. CONCLUSION

In this paper, we presented a novel segmentation approach combined with a Reeb graph construction method for 3D dynamic meshes. By using an efficient and stable continuous function, 3D dynamic meshes are partitioned on set of level lines depending on their curvature and boundary information. Preliminary experimental results have shown that the constructed kinematic Reeb graphs preserve the topology of the test models. Consequently, we obtain a more faithful segmentation despite the perturbations occurred over time. The accuracy of the vertices distribution, on the border between clusters, has been enhanced by exploiting the curvature information.

In our future work, we plan to investigate the proposed approach to develop a 3D dynamic compression scheme.

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