BLIND SEPARATION OF SOURCES WITH FINITE RATE OF INNOVATION

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ABSTRACT

We propose a method for recovering the parameters of periodic signals with finite rate of innovation sampled using a raised cosine pulse. We show that the proposed method exhibits the same numerical stability as existing methods of its type, and we investigate the effect of oversampling on the performance of our method in the presence of noise. Our method can also be applied to non-periodic signals and we assess the efficacy of signal recovery in this case. Finally, we show that the problem of cochannel QPSK signal separation can be converted into a general finite rate of innovation framework, and we test the effectiveness of this approach.

Index Terms— Raised Cosine, QPSK, Finite Rate of Innovation, Signal Separation

1. INTRODUCTION

Due to its practical importance, the problem of separating cochannel digital communications signals has been extensively studied by the communications community. For example, separation of cochannel signals can be used to improve the demodulation performance in the presence of cochannel interference.In the literature, a number of methods have been proposed for this problem. Most of them are based on traditional statistical techniques such as PCA [1] and ICA [2][3], as well as taking advantage of such signals transmitting from a finite set of symbols [4][5].

In this paper we study Finite Rate of Innovation (FRI) methods for signal reconstruction. FRI methods have previously been applied to medical imaging [6], ECG and EEG [7][8], image processing [9], and compression [10]. They have not however been applied very much to digital communications, and in particular not to the problem of source separation. We adapt an existing finite rate of innovation method to a raised cosine sampling kernel, chosen for its existing use as a filter in digital communications, and show how this can be used to perform cochannel demodulation of digital communications signals in the single sensor, multiple transmitters case. We consider a linear instantaneous mixing model, which is characteristic of signals with dominant line of sight components, e.g. satellite signals.

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In section 2 we first adapt the method proposed in [11] for a periodic stream of Dirac pulses sampled with a sinc sampling kernel, to a periodic stream of Dirac pulses sampled with a raised cosine kernel. Subsequently, we test the practical efficiency of this approach. In section 3, we describe the problem of separating mixed QPSK signals and propose a practical solution. Section 4 is concerned with the evaluation of our method. Finally, Section 5 concludes the paper.

2. FRI FRAMEWORK

2.1. Signal Sampling and Reconstruction

The signal we consider is periodic with period ρ and K Dirac pulses in each period. Hence, the signal admits the representation:

$$x(t) = \sum_{k\in\mathbb{Z}} c_k \delta(t - t_k) \tag{1}$$

The periodicity condition signifies that $c_{k+K} = c_k$ and $t_{k+K} = t_k + \rho$.

As shown in [11], (1) can be rewritten as

$$x(t) = \sum_{m\in\mathbb{Z}} \left(\frac{1}{\rho} \sum_{k=0}^{K-1} c_k e^{-\frac{2\pi i m t_k}{\rho}} \right) e^{\frac{2\pi i m t}{\rho}} \tag{2}$$

Using (2), we get the Fourier transform of $x(t)$:

$$X[m] = \frac{1}{\rho} \sum_{k=0}^{K-1} c_k e^{-\frac{2\pi i m t_k}{\rho}} \tag{3}$$

As in [11], an annihilating filter is used in order to find the locations of the Dirac pulses. The z-transform of the annihilating filter is given by (4)

$$A(z) = \prod_{k=0}^{K-1} (1 - e^{-\frac{2\pi i t_k}{\rho}} z^{-1}) \tag{4}$$

As each exponential in (3) is annihilated by a root of $A(z)$, we get:

$$A[m] * X[m] = 0 \quad (5)$$

(Here, $A[m]$ is the Fourier transform of the annihilating filter)

In this paper we sample $x(t)$ with a raised cosine kernel, $rc_{T,\alpha}$. The kernel is given by:

$$\mathrm{rc}_{T,\alpha} = \frac{\sin(\frac{\pi t}{T})\cos(\frac{\pi\alpha t}{T})}{\frac{\pi t}{T}(1-(\frac{2\alpha t}{T})^2)} \quad (6)$$

The parameters T, α, should be chosen such that the bandwidth of the kernel exceeds the rate of innovation, i.e.

$$\frac{1+\alpha}{T}\pi \geq \frac{2K}{\rho} \quad (7)$$

Set $M = \left\lfloor \frac{(1+\alpha)\rho}{2T} \right\rfloor$ and take N samples at equally spaced intervals, $t = nT_s$, $n = 0, \ldots, N-1$, $N \geq 2M+1$

$$y_n = \int_{-\infty}^{\infty} \mathrm{rc}_{T,\alpha}(t - nT_s - t')x(t')dt' \quad (8)$$

The samples y_n obtained through (8) are sufficient to reconstruct $x(t)$. This is based on the following arguments: To recover the signal parameters from the samples y_n we use the Fourier series of $x(t)$ to rewrite y_n as

$$y_n = \sum_m X[m] \int_{-\infty}^{\infty} \mathrm{rc}_{T,\alpha}(t - nT_s - t')e^{\frac{2\pi i m t'}{\rho}} dt' \quad (9)$$

$$= \sum_{m=-M}^{M} X[m]\hat{rc}_{T,\alpha}(\frac{2\pi m}{\tau})e^{\frac{2\pi i m n T_s}{\rho}} \quad (10)$$

Note that (10) is a system of linear equations in $X[m]$, where $\hat{rc}_{T,\alpha}$ is the Fourier transform of $rc_{T,\alpha}$.

This system can be rewritten as (11), where $X'[m] = X[m]\hat{rc}_{T,\alpha}(\frac{2\pi m}{\tau})$

$$y_n = \sum_{m=-M}^{M} X'[m]e^{\frac{2\pi i m n T_s}{\rho}} \quad (11)$$

Thus, following the arguments in [11], we conclude that (11) is not invertible if $\frac{\rho}{T_s} = \frac{p}{q}$, for some arbitrary fraction $\frac{p}{q}$ with $p < N$, and in all other cases is of maximal rank. Once $X[m]$ is computed (from (11)), the annihilating filter $A[m]$ can be obtained from the system (12).

$$X[m] = -\sum_{k=0}^{K-1} A[k]X[m-k] \quad (12)$$

Relying on $A[m]$, we determine the z-transform $A(z)$ and its roots $u_k = e^{\frac{-2\pi u t_k}{\rho}}$. Using the roots we get the locations of Dirac pulses t_k. Finally, to find the pulse weights, we solve the system (13).

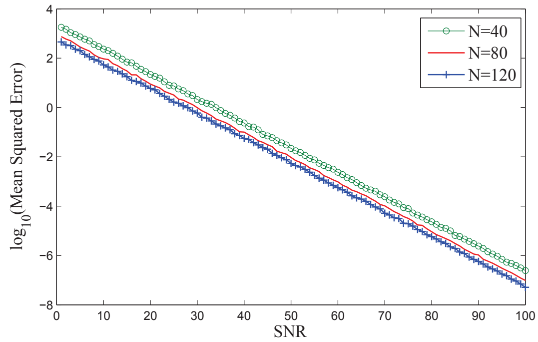

Fig. 2. Logarithm of the mean squared error of the recovered signal compared to the original signal

$$X[m] = \frac{1}{\rho}\sum_{k=0}^{K-1} c_k e^{-\frac{2\pi i m t_k}{\rho}} = \frac{1}{\rho}\sum_{k=0}^{K-1} c_k u_k^m \quad (13)$$

The derivation for a root raised cosine filter proceeds exactly as above.

2.2. Numerical Simulations

In order to test the practical efficiency of the proposed method, we apply it to a signal with the following parameters: $T = \frac{\sqrt{3}}{2}$, $\alpha = 0.3$, $t_k = k+1$, $c_k = k-6$, $k = 0, \ldots, 9$, $\rho = 15$, $T_s = \frac{\sqrt{2}}{3}$, $N = 40$.

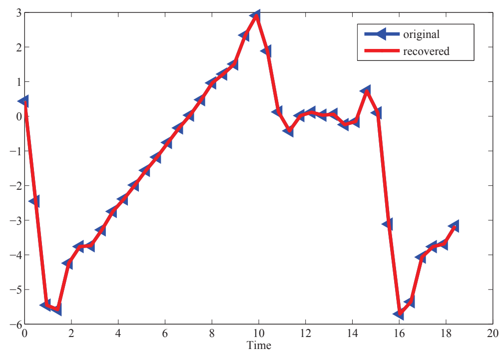

Fig. 1. Original and recovered signals

Figure 1 shows that with these parameters the recovered signal is indistinguishable from the recovered signal, while Figure 2 clearly indicates the improvement achieved by oversampling when noise is added to the signal. More sophisticated methods of dealing with noise are discussed in are discussed in [12] .

2.3. Numerical Stability

We consider the proposed methods for the parameters, $N = M = \lfloor \frac{1.3}{2\tau} \rfloor, \tau = 100$, which are chosen to ensure the invertibility of the system (11).

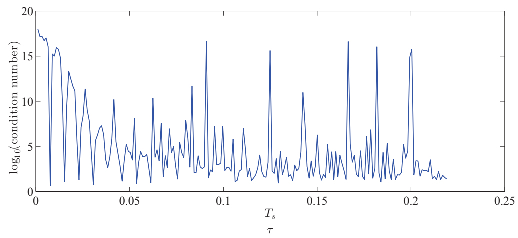

Fig. 3. Logarithm of the condition number of the linear system for the ratio $\frac{T_s}{\tau}$

According to Figure 3, as $\frac{T_s}{\rho} \to 0$, the condition number of the linear system grows very fast. Indeed, although the ratios were taken to be of the form $\frac{\sqrt{1.2}}{1000}k$, there are some spikes which are probably caused by this ratio being very close to some arbitrary fraction $\frac{p}{q}$, $p < N$, which would mean the system is very close to being uninvertible, as discussed in [11].

3. QPSK SIGNAL SEPARATION

3.1. QPSK Signals

In this section, the proposed method is applied to the separation of QPSK signals. These signals can be described as follows. We transmit a series of binary digits. To do so, we encode each block of n bits as a constellation point drawn from a set of size 2^n in $\mathbb{C}$. In the particular case of the QPSK signals we consider, the constellation set is $\{e^{\frac{\pi i}{4}}, e^{\frac{\pi i}{4}+\frac{2\pi i}{4}}, e^{\frac{\pi i}{4}+\pi i}, e^{\frac{\pi i}{4}+\frac{3\pi i}{2}}\}$, or some rotation of these symbols. The data is encoded only by the phase of the symbol, and not by the amplitude. As a result of encoding, a series of symbols c_k is produced. We would like to transmit c_k as a series of weighted Dirac pulses.

$$x(t) = \sum_{k=1}^{K} c_k \delta(t - kT - \tau) \tag{14}$$

In this case, T is the symbol period and τ is the delay. Hence the symbol c_k is transmitted at time $kT + \tau$. To restrict the transmission to a small frequency band, we filter the signal. Ideally, we would like to use a filter with frequency response 1 in the band we wish to transmit in and 0 elsewhere. In such an approach, $\delta(t)$ is replaced by $\text{sinc}(t)$. As $\text{sinc}(t)$ decays slowly, significant intersymbol interference occurs. Instead, we use a root raised cosine filter. Its impulse response is:

$$\text{rrc}_{T,\alpha}(t) = \frac{1}{\sqrt{T}} \frac{\sin(\frac{\pi(1-\alpha)t}{T}) + \frac{4\alpha t}{T}\cos(\frac{\pi\alpha t}{T})}{\frac{\pi t}{T}(1 - (\frac{4\alpha t}{T})^2)} \tag{15}$$

Where $\alpha \in [0,1]$ is the roll-off factor. Note that for $\alpha = 0$ (15) is just a scaled sinc function. This has much faster convergence in the time domain (for $\alpha > 0$), at the cost of using more bandwidth in the frequency domain, by a factor of $1 + \alpha$. Note that convolving an rrc filter with itself gives a raised cosine (rc) filter, with impulse response as given in (6). (6) satisfies the Nyquist property, because it vanishes at non-zero multiples of the symbol period, $t = kT$, $k \neq 0$. Consequently, applying a matched filter at the receiver eliminates intersymbol interferences.
In this case, the filtered signal can be calculated from formulae:

$$\begin{aligned} B(t) &= \int_{-\infty}^{\infty} x(t')\text{rrc}_{T,\alpha}(t-t')\mathrm{d}t' \\ &= \int_{-\infty}^{\infty} \sum_k c_k \delta(t' - kT - \tau)\text{rrc}_{T,\alpha}(t-t')\mathrm{d}t' \\ &= \sum_k c_k \text{rrc}_{T,\alpha}(t - kT - \tau) \end{aligned} \tag{16}$$

Note that in practice this pulse-shaping convolution sum is only computed to a small number of terms at the transmitter.

3.2. Transmitted and received signals

The baseband signal is multiplied by a carrier wave:

$$C(t) = e^{2\pi i(ft+\phi)} \tag{17}$$

with carrier frequency f and phase offset ϕ. The signal $S(t) = C(t)B(t)$ is transmitted through a channel, and the received signal is obtained:

$$R(t) = G(t) * S(t) + n(t) \tag{18}$$

Here, $G(t)$ is the channel response, and $n(t)$ is additive white Gaussian noise. We assume that $G(t) = g\delta(t)$, where $g \in \mathbb{R}^+$ is a constant. (Note that any phase change caused by the channel can be absorbed into the parameter ϕ)

Hence, the received signal can be written out as follows:

$$R(t) = ge^{2\pi i(ft+\phi)} \sum_k c_k \text{rrc}_{T,\alpha}(t - kT - \tau) + n(t) \tag{19}$$

3.3. Signal mixture

In our study, we assume that there are two sources, the a-side source and the b-side source, transmitting independent streams of symbols a_k, and b_k. These sources are modelled by a linear mixture model, hence the signal admits the representation:

$$R(t) = g_1 e^{2\pi i(f_1 t+\phi_1)} \sum_k a_k \mathrm{rrc}_{T,\alpha_1}(t-kT-\tau_1) + g_2 e^{2\pi i(f_2 t+\phi_2)} \sum_k b_k \mathrm{rrc}_{T,\alpha_2}(t-kT-\tau_2) \quad (20)$$

Here, we adopt the convention that the stronger of the two signals is referred to as the a-side signal, i.e. $g_1 \geq g_2$

In our study, we assume that $\alpha_1 = \alpha_2$ and that the receiver is able to exactly filter the signal it receives with a matched root raised cosine filter. We also assume that $|f_2 - f_1| \ll \frac{1}{T}$.Then, we have:

$$R(t) = g_1 e^{2\pi i(f_1 t+\phi_1)} \sum_k a_k rc(t-kT-\tau_1) + g_2 e^{2\pi i(f_2 t+\phi_2)} \sum_k b_k rc(t-kT-\tau_2) \quad (21)$$

Consequently,

$$\frac{R(t)}{g_1 e^{2\pi i(f_1 t+\phi_1)}} = \sum_k a_k rc(t-kT-\tau_1) + \frac{g_2}{g_1} e^{2\pi i((f_2-f_1)t+\phi_2-\phi_1)} \sum_k b_k rc(t-kT-\tau_2) \quad (22)$$

Assuming that $\frac{1}{T}$ is large in comparison to $|f_2 - f_1|$, we get:

$$\frac{R(t)}{g_1 e^{2\pi i(f_1 t+\phi_1)}} \approx \sum_k a_k rc(t-kT-\tau_1) + \sum_k b_k' rc(t-kT-\tau_2) \quad (23)$$

Where $b_k' = \frac{g_2}{g_1} e^{2\pi i((f_2-f_1)(kT+\tau_2)+\phi_1-\phi_2)} b_k$. Note that this approximation is exact if the two carrier frequencies are the same, i.e. $f_1 = f_2$.

Signal (23) is a stream of Dirac pulses filtered with a raised cosine filter, and after recovering them, it is just a matter of sorting successive samples into the right signal. For example, if $\tau_1 < \tau_2$, the stream of recovered symbols is c_k, $a_k = c_{2k-1}$, $b_k = c_{2k}$, $k = 1, ..., K$.

3.4. Recovery by approximation to a periodic signal

In our study, we consider a signal $x(t)$ consisting of the first K Dirac pulses of an infinite series of Dirac pulses. We assume that ρ is proportional to K, $\rho = aK$. We further consider that all the Dirac pulses are contained in a subinterval of the period, $[0, b\rho]$, $b < 1$. As $K \to \infty$, the contributions from each Dirac outside the interval $[0,\rho]$ to $y(t) =< \hat{rc}_{\alpha,T}(t'-t), x(t') >$ (that would exist if the signal was periodic) are $\mathcal{O}(\frac{1}{t^3})$, where t is the distance from the sampling point. Hence their overall contribution is $\mathcal{O}(\frac{1}{\rho^2})$. In other words on the intervals $[0, baK]$, the filtered periodic signals converge to the filtered non-periodic signals in the ℓ_∞ sense. This justifies using the method for periodic signals on non-periodic signals. However, such an approach would not work well for a sinc sampling kernel because of its slow decay. Note that since the original signal is not periodic, the parameter ρ is used only in the recovery algorithm, and so we can view it as a tunable parameter in the recovery algorithm.

4. APPLICATION TO QPSK SIGNALS

4.1. Signal Separation

In this section, the proposed method is applied to the signal separation in the framework described in section 3. Here, we assume that $f_1 = f_2$. We consider the signal with the parameters $T = 1$, $\alpha = 0.3$, $g_1 = g_2 = 1$, $\tau_1 = -0.1$, $\tau_2 = 0.3$, $N = 25$ and when recovering, $\rho = 30$

We assume the a-side signal transmits $[e^{\frac{\pi i}{4}+\pi i}, e^{\frac{\pi i}{4}+\frac{\pi i}{2}}, e^{\frac{\pi i}{4}}, e^{\frac{\pi i}{4}+\pi i}, e^{\frac{\pi i}{4}}]$, while the b-side signal transmits $[e^{\frac{\pi i}{4}}, e^{\frac{\pi i}{4}}, e^{\frac{\pi i}{4}+\frac{3\pi i}{2}}, e^{\frac{\pi i}{4}+\frac{\pi i}{2}}, e^{\frac{\pi i}{4}}]$. The results obtained using the proposed method are presented in Figures 4 and 5.

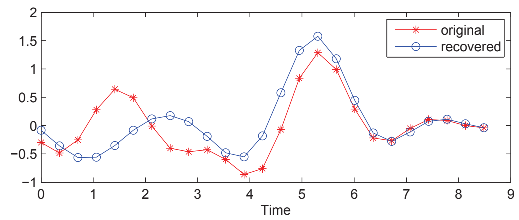

Fig. 4. Real parts of original and recovered signals

As suggested by Figure 4 the recovery is not perfect. However if we know the times of the Dirac pulses, we achieve much better recovery, (as seen in Figure 5).

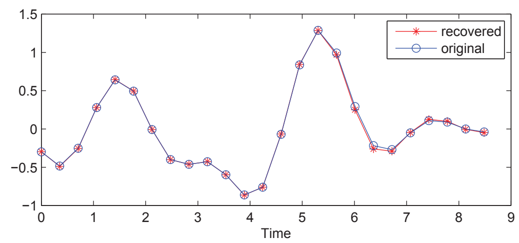

Fig. 5. Real parts of original signal and signal recovered using known times

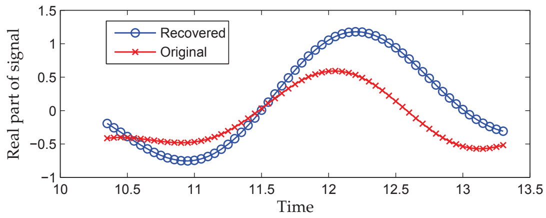

Fig. 6. Real parts of original and recovered signals, using windowing

4.2. Windowed signal recovery

Due to the complexity of the algorithm, and to the problems with numerical stability, applying the proposed method to a long signal is impractical. Instead, we look at sampling and recovering just a small windowed part of a larger signal. We take the same signal parameters as previously with longer randomly generated signals. For recovery, we take $T_s = 0.05T$, $N = 90$, $\rho = 7\sqrt{2}$, and recover two symbols from each side.

Note that ρ had to be selected with some care to achieve the level of success shown in Figure 6. The best recovery is achieved in the centre of the period being considered, and this is unsurprising, because this is where the contributions from outside the period being considered are weakest.

5. CONCLUSIONS

We have shown that a stream of Dirac pulses sampled using a raised cosine filtered can be reconstructed exactly from samples taken at the rate of innovation. We have also shown how to transform mixed QPSK signals into a form where this can be used to separate them. Further work could focus on improving the robustness of the parameter recovery and increasing the length of the signals that can be effectively recovered.

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