ADAPTIVE WAVEFORM SELECTION AND TARGET TRACKING BY WIDEBAND MULTISTATIC RADAR/SONAR SYSTEMS

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ABSTRACT

An adaptive waveform selection algorithm for target tracking by multistatic radar/sonar systems in wideband environments is presented to minimize the tracking mean squared error. The proposed selection algorithm is developed based on the minimization of the trace of error covariance matrix for the target state estimates (i.e. the target position and target velocity). This covariance matrix can be computed using the Cramér-Rao lower bounds of the wideband radar/sonar measurements. The performance advantage of the proposed adaptive waveform selection algorithm over the conventional fixed waveforms with minimum and maximum time-bandwidth products is demonstrated by simulation examples using various FM waveform classes.

Index Terms— adaptive waveform selection, wideband, multistatic, radar/sonar, target tracking.

1. INTRODUCTION

Adaptive waveform selection has been shown to significantly improve the tracking performance of radar/sonar systems [1–5], where the track information obtained from the past measurements is utilized for selecting the waveform of the next signal transmission in order to achieve optimal tracking performance. The early work in adaptive waveform selection considered target tracking in a one-dimensional clutter-free environment [1]. The authors then extended their work to a cluttered scenario [2]. The problem of adaptive waveform selection was further investigated for two-dimensional target tracking in different scenarios (single or multiple targets, narrowband or wideband environments, and clutter-free or cluttered environments) [3–5].

All of these works were conducted for monostatic systems, in which the transmitter and receiver are collocated. In many applications, it has become apparent that multistatic radar/sonar systems can provide superior performance over monostatic radar/sonar systems [6]. Adaptive waveform selection, which has been addressed for the monostatic case, is a promising approach to improve the tracking performance of multistatic radar/sonar systems. However, there are significant differences between monostatic and multistatic radar, particularly in terms of performance. The multistatic radar performance depends not only on transmitted waveform but also on radar geometry [7, 8]. Thus it is not readily obvious how to apply adaptive waveform selection to the multistatic case. In our previous work [9, 10], we have successfully demonstrated the superior performance of adaptive waveform selection over conventional fixed waveforms in a multistatic setting. However, the work in [9, 10] is restricted to narrowband signals.

The narrowband condition $TB \ll c/\dot{r}$ can be easily satisfied in radar applications because the speed of light c is very large compared to the speed of typical targets. Here TB is the time-bandwidth product, and r is the total bistatic target range. However, this narrowband condition is generally very hard to satisfy in sonar applications, where the speed of sound ($c_s \approx 1500$ m/s) is comparable to the speed of underwater targets (on the order of 10 m/s) and the time-bandwidth product of the sonar signal is usually large ($TB > 100$) [5, 11]. In this scenario, wideband signal models are commonly employed whereby the received signal is not simply a time-shifted and frequency-shifted copy of the transmitted signal with no time scaling as in the narrowband case. The Cramér-Rao lower bounds (CRLB) of the wideband radar/sonar measurements are also different to those of the narrowband case [12].

In this paper, we consider the problem of adaptive waveform selection for target tracking by wideband multistatic radar/sonar systems. We first derive a new wideband bistatic signal model. Note that a bistatic signal model was first introduced by Tsao *et al.* [13], but for narrowband signals only. A bistatic signal model for wideband radar signals was then derived in [14]. However, this model does not work well in sonar applications, where the speed of sound is comparable to the speed of typical underwater targets. By following the derivation in [13], we form a wideband bistatic signal model which can be employed for both radar and sonar applications. The target tracking problem by wideband multistatic radar/sonar systems is then defined utilizing the new derived signal model. The adaptive waveform selection algorithm is developed to minimize the tracking mean squared error of the target state estimate, i.e. target position and velocity, by equivalently minimizing the trace of the tracking error covariance matrix. The superior performance of the proposed waveform selection algorithm vis-à-vis conventional fixed waveforms is demonstrated by simulation examples.

2. WIDEBAND BISTATIC SIGNAL MODEL

Consider a bistatic channel between the transmitter Tx, the receiver Rx, and the target Tgt. Let $s(t)$ denote the transmitted bandpass signal. Then the received signal is given by

$$r(t) = As[t - \tau(t)] + n(t) \tag{1}$$

where $\tau(t)$ is the total travel time of the signal from Tx to Tgt and to Rx, A is the signal attenuation, and $n(t)$ is the noise at the receiver. Denote the target-transmitter distance and the target-receiver distance by R_T and R_R, respectively, with $R = R_T + R_R$ giving the total distance. Using the results derived in [13] without narrowband approximation, we have

$$\tau(t) = \tau_a + \frac{\dot{R}(t_*)}{c + \dot{R}_R(t_*)}(t - \tau_a) \tag{2}$$

where t_* is the time instant when the transmitted signal reaches Tgt, and τ_a is the actual total time delay given by

$$\tau_a = \frac{R(t_*)}{c} = \frac{R_T(t_*) + R_R(t_*)}{c}. \tag{3}$$

Using (2) and (3), the received signal $r(t)$ in (1) becomes

$$r(t) = As[\beta(t - \tau_a)] + n(t) \tag{4}$$

where β is the Doppler stretch factor given by

$$\beta = 1 - \frac{\dot{R}(t_*)}{(c + \dot{R}_R(t_*))}. \tag{5}$$

Note that, since $\dot{R}_R \ll c$ in radar applications, we have $\beta \approx 1 - \dot{R}(t_*)/c$ [14]. In sonar applications, where $\dot{R}_R$ is comparable to the speed of propagation of sound, this approximation does not hold. Note that a wideband environment is characterized by $\beta \neq 1$ compared to $\beta = 1$ for narrowband environments.

3. TARGET TRACKING BY MULTISTATIC RADAR/SONAR SYSTEM

We consider an active multistatic radar/sonar system with a single dedicated transmitter at the origin $[0, 0]$ and N receivers located separately at $[x^i_{\text{Rx}}, y^i_{\text{Rx}}]$, $i = 1, 2, \ldots, N$ for tracking a single target in a two-dimensional wideband environment as shown in Fig. 1. The considered tracking scenario is assumed to be clutter-free with moderately high signal-to-noise ratio (SNR) at the receivers. We also assume that communication links are available between the transmitter and receivers with negligible time-synchronization errors.

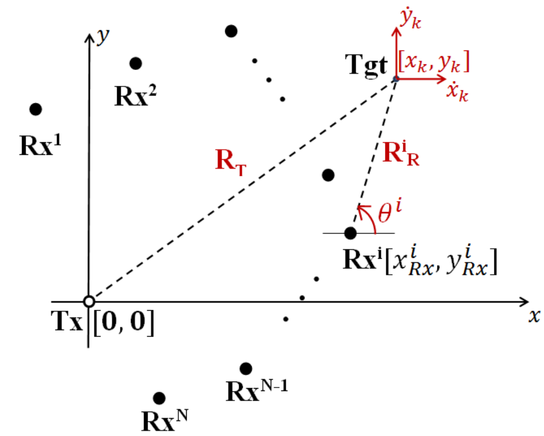

Fig. 1. Illustration of multistatic radar/sonar geometry.

At each receiver, the measurements of time delay, Doppler stretch factor, and arrival angle are available. These measurements from all receivers are sent to a central processor located at the transmitter site and used for target tracking and waveform selection. To deal with the nonlinearity between the target state vector $\mathbf{x}_k$ and the measurement vector $\mathbf{z}_k$, an extended Kalman filter (EKF) is employed for tracking the target. Note that in this section we utilize the wideband signal model in Section 2 to formulate the measurement model of the target tracking problem.

Let $\mathbf{x}_k = [x_k, y_k, \dot{x}_k, \dot{y}_k]^T$ denote the target state vector at time $k = 0, 1, \ldots$, with x_k, y_k corresponding to the target position and $\dot{x}_k, \dot{y}_k$ corresponding to the target velocity in Cartesian coordinates. The target dynamics are modelled by a nearly constant velocity model given by

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \mathbf{w}_k \tag{6}$$

where $\mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ is the process noise. The matrices $\mathbf{F}$ and $\mathbf{Q}$ are given in [15].

The measurement equation for centralized target tracking is given by

$$\begin{aligned}\mathbf{z}_k =& [\tau^1_k, \beta^1_k, \theta^1_k, ..., \tau^N_k, \beta^N_k, \theta^N_k]^T \\ =& \mathbf{h}(\mathbf{x}_k) + \mathbf{n}_k \\ =& [\mathrm{h}^1_\tau(\mathbf{x}_k), \mathrm{h}^1_\beta(\mathbf{x}_k), \mathrm{h}^1_\theta(\mathbf{x}_k), .., \mathrm{h}^N_\tau(\mathbf{x}_k), \mathrm{h}^N_\beta(\mathbf{x}_k), \mathrm{h}^N_\theta(\mathbf{x}_k)]^T \\ &+ [n^1_{\tau k}, n^1_{\beta k}, n^1_{\theta k}, ..., n^N_{\tau k}, n^N_{\beta k}, n^N_{\theta k}]^T \end{aligned} \tag{7}$$

where $\tau^i_k, \beta^i_k, \theta^i_k$ are the measurements of time delay, Doppler stretch factor, and arrival angle at the i-th receiver, respectively, and $\mathbf{n}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{N}_k)$ is the measurement error at time k. Using the wideband bistatic signal model in Section 2, i.e. equations (3) and (5), we have

$$\mathrm{h}^i_\tau(\mathbf{x}_k) = \frac{\|[x_k, y_k]\| + \|[x_k, y_k] - [x^i_{\text{Rx}}, y^i_{\text{Rx}}]\|}{c} \tag{8a}$$

$$\mathrm{h}^i_\beta(\mathbf{x}_k) = 1 - \frac{\dot{R}^i}{c + \dot{R}^i_R} = 1 - \frac{\dot{R}_T + \dot{R}^i_R}{c + \dot{R}^i_R} \tag{8b}$$

where

$$\dot{R}_T = \frac{\dot{x}_k x_k + \dot{y}_k y_k}{\|[x_k, y_k]\|} \tag{9a}$$

$$\dot{R}_R^i = \frac{\dot{x}_k(x_k - x_{\mathrm{Rx}}^i) + \dot{y}_k(y_k - y_{\mathrm{Rx}}^i)}{\|[x_k, y_k] - [x_{\mathrm{Rx}}^i, y_{\mathrm{Rx}}^i]\|}. \tag{9b}$$

Furthermore, $\mathrm{h}_\theta^i(\mathbf{x}_k)$ in (7) is given by

$$\mathrm{h}_\theta^i(\mathbf{x}_k) = \mathrm{atan2}\left(\frac{y_k - y_{\mathrm{Rx}}^i}{x_k - x_{\mathrm{Rx}}^i}\right) \tag{10}$$

where atan2 denotes the four quadrant arctangent.

In this paper we assume, as commonly done, that the measurement errors $\mathbf{n}_k$ can achieve their CRLB. Thus $\mathbf{N}_k$ equals to the CRLB $\mathbf{C}_k$ of the measurement vector $\mathbf{z}_k$ that consists of all measurements in the system. As the measurement errors at different receivers are statistically independent, we have

$$\mathbf{N}_k = \mathbf{C}_k = \mathrm{diag}(\mathbf{C}_k^1, \mathbf{C}_k^2, \ldots, \mathbf{C}_k^N) \tag{11}$$

where $\mathbf{C}_k^i$ is the CRLB of the measurements $(\tau_k^i, \beta_k^i, \theta_k^i)$ at the i-th receiver. Since the CRLB $C_{\theta_k}^i$ corresponding to θ_k^i is independent from the CRLB $\mathbf{C}_{(\tau_k,\beta_k)}^i$ corresponding to (τ_k^i, β_k^i) [16], we have $\mathbf{C}_k^i = \mathrm{diag}(\mathbf{C}_{(\tau_k,\beta_k)}^i, C_{\theta_k}^i)$. Moreover, $C_{\theta_k}^i$ only depends on SNR at receiver [16], thus can be modelled by $C_{\theta_k}^i = \sigma_\theta^2/\mathrm{SNR}_k^i$ where σ_θ is a constant. $\mathbf{C}_{(\tau_k,\beta_k)}^i$, on the other hand, can be evaluated from the wideband ambiguity function of the transmitted waveform [12]. In fact $\mathbf{C}_{(\tau_k,\beta_k)}^i$ is the inverse of the Fisher information matrix $\mathbf{J}_{(\tau_k,\beta_k)}^i$. For a general form of the transmitted bandpass signal given as

$$s(t) = a(t)\exp j(\psi(t) + 2\pi f_c t) \tag{12}$$

where $a(t)$ and $\psi(t)$ are the amplitude and phase modulation functions, respectively, and f_c is the carrier frequency, we have $\mathbf{J}_{(\tau_k,\beta_k)}^i = 2\mathrm{SNR}_k^i \times \mathbf{I}_{(\tau,\beta)}$ with the elements of $\mathbf{I}_{(\tau,\beta)}$ given by [12]

$$\mathbf{I}_{1,1} = \int_{-\lambda/2}^{\lambda/2} (\dot{a}^2(t) + a^2(t)\Psi^2(t))dt - \left[\int_{-\lambda/2}^{\lambda/2} a^2(t)\Psi(t)dt\right]^2 \tag{13a}$$

$$\mathbf{I}_{1,2} = \mathbf{I}_{2,1} = \int_{-\lambda/2}^{\lambda/2} t(\dot{a}^2(t) + a^2(t)\Psi^2(t))dt - \int_{-\lambda/2}^{\lambda/2} a^2(t)\Psi(t)dt \int_{-\lambda/2}^{\lambda/2} ta^2(t)\Psi(t)dt \tag{13b}$$

$$\mathbf{I}_{2,2} = \int_{-\lambda/2}^{\lambda/2} t^2(\dot{a}^2(t) + a^2(t)\Psi^2(t))dt - \left[\int_{-\lambda/2}^{\lambda/2} ta^2(t)\Psi(t)dt\right]^2 - \frac{1}{4} \tag{13c}$$

where $\Psi(t) = \dot{\psi}(t) + 2\pi f_c$.

Note that the ambiguity function and CRLB with respect to the target-to-receiver range and bisector velocity is often considered in multistatic radars [8, 13, 17]. This CRLB can also be utilized for the considered tracking problem. However, it is to be emphasized that, in the context of target tracking, the ultimate objective is to minimize the mean squared error of the target state estimate, i.e. target position and velocity (see Section 4). We can show that the tracking error covariance can be equivalently computed using either the CRLB of time delay and Doppler or the CRLB of receiver-to-target range and bisector velocity. Moreover, using the CRLB of receiver-to-target range and bisector velocity requires more computations as this CRLB is needed to compute from the CRLB of time delay and Doppler using the derivative chain rule [8]. Therefore, based on these considerations, the CRLB of time delay and Doppler stretch is employed here.

4. ADAPTIVE WAVEFORM SELECTION

As can be seen in Section 3, the CRLB for the measurements of time delay and Doppler stretch are dependent on the transmitted waveform; where the elements of $\mathbf{I}_{(\tau,\beta)}$ are computed from $a(t)$ and $\psi(t)$. As a result, the measurement error covariance $\mathbf{N}_{k+1}$, is also dependent on the transmitted waveform. In other words, $\mathbf{N}_{k+1}(\mathbf{\Omega}_{k+1})$ can be explicitly shown to be a function of the transmitted waveform parameters $\mathbf{\Omega}_{k+1}$. Note that the waveform parameters might be different depending on waveform classes. By using the EKF's update equations, the error covariance matrix $\mathbf{P}_{k+1|k+1}$ of the target state estimate at time $k+1$ can be computed prior to the signal transmission at time $k+1$ as follows [15]:

$$\begin{aligned}
\mathbf{P}_{k+1|k} &= \mathbf{F}\mathbf{P}_{k|k}\mathbf{F}^T + \mathbf{Q} \\
\mathbf{S}_{k+1}(\mathbf{\Omega}_{k+1}) &= \mathbf{H}_{k+1}\mathbf{P}_{k+1|k}\mathbf{H}_{k+1}^T + \mathbf{N}_{k+1}(\mathbf{\Omega}_{k+1}) \\
\mathbf{K}_{k+1}(\mathbf{\Omega}_{k+1}) &= \mathbf{P}_{k+1|k}\mathbf{H}_{k+1}^T\mathbf{S}_{k+1}(\mathbf{\Omega}_{k+1})^{-1} \\
\mathbf{P}_{k+1|k+1}(\mathbf{\Omega}_{k+1}) &= [\mathbf{I} - \mathbf{K}_{k+1}(\mathbf{\Omega}_{k+1})\mathbf{H}_{k+1}]\mathbf{P}_{k+1|k}
\end{aligned}$$

where $\mathbf{H}_{k+1}$ is the Jacobian matrix of $\mathbf{h}(\mathbf{x}_{k+1})$ [15], which is straightforward to derive from (8)–(10). $\mathbf{P}_{k+1|k+1}$ now becomes a function of the parameters $\mathbf{\Omega}_{k+1}$ of the waveform to be transmitted at time $k+1$: $\mathbf{P}_{k+1|k+1}(\mathbf{\Omega}_{k+1})$.

The objective of the proposed adaptive waveform selection scheme is to minimize the total tracking mean squared error of the target state estimate in both target position and velocity. This can be achieved by minimizing the trace of $\mathbf{P}_{k+1|k+1}(\mathbf{\Omega}_{k+1})$. Therefore, the optimal transmitted waveform is obtained by

$$\mathbf{\Omega}_{k+1}^* = \underset{\mathbf{\Omega}_{k+1} \in \mathrm{Library}}{\arg\min} \ \mathrm{Tr}(\mathbf{P}_{k+1|k+1}(\mathbf{\Omega}_{k+1})). \tag{15}$$

Note that $\mathbf{\Omega}_{k+1}^*$ is found using a grid search over a finite search space (i.e. a waveform library). The waveform library

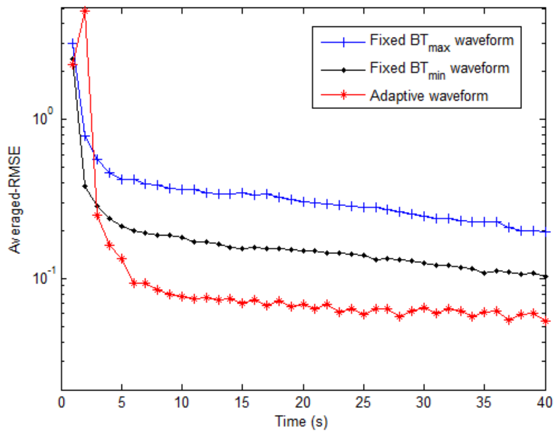

Fig. 2. Tracking performance for LFM waveform.

may contain a single waveform class with varying parameters or a number of different waveform classes. This method of discrete grid search is computationally cheaper than gradient-based methods and perform well in practice [3]. However, the computation of discrete grid search is quite expensive if the waveform library is large, thus a closed-form or approximation solution is needed. This is out of the scope of the present paper and will be considered in our future work.

5. SIMULATION EXAMPLES

In this section, we consider a multistatic sonar system with a dedicated transmitter and four receivers tracking an underwater target in a wideband environment, where the transmitter is located at the origin $[0, 0 \text{ m}]$, and the receivers are located at $[500, 0]$ m, $[250, 500]$ m, $[250, 200]$ m, and $[0, 250]$ m. The carrier frequency of the transmitted waveform is $f_c = 25$ kHz. The speed of sound under water is assumed to be constant at 1500 m/s. The initial position and velocity of the target are $[600, 400]$ m and $[-4, -2]$ m/s, respectively. The sampling interval is $T = 1$s, and the constant associated with the maneuver level of the target in the process noise covariance matrix $\mathbf{Q}$ is $q = 0.01$. The SNR at the i-th receiver is modelled by $\text{SNR}_k^i = R_0^4/(R_{Tk}R_{Rk}^i)^2$, where $R_0 = 1000$ m. The constant σ_θ associated with C_θ^i is set to 0.01 rad. We consider three different waveform classes, viz. the linear FM (LFM), hyperbolic FM (HFM), and exponential FM (EFM) [4]. Their phase functions $\psi(t)$ are given in Table 1

Waveform	Phase function $\psi(t)$	Δ_F
LFM	$2\pi b[t/\gamma + (t+\lambda/2)^2/2)]$	$b\lambda$
HFM	$2\pi b[\ln(t+\gamma+\lambda/2)]$	$b\frac{\lambda}{\gamma(\gamma+\lambda)}$
EFM	$2\pi b[\exp(-(t+\lambda/2)/\gamma)]$	$\frac{b}{\gamma}(\mathrm{e}^{-\frac{\lambda}{\gamma}}-1)$

Table 1. FM waveforms considered in the simulation.

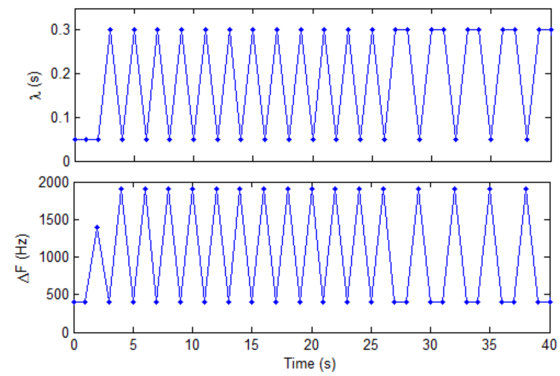

Fig. 3. A pattern of selected waveform parameters for LFM.

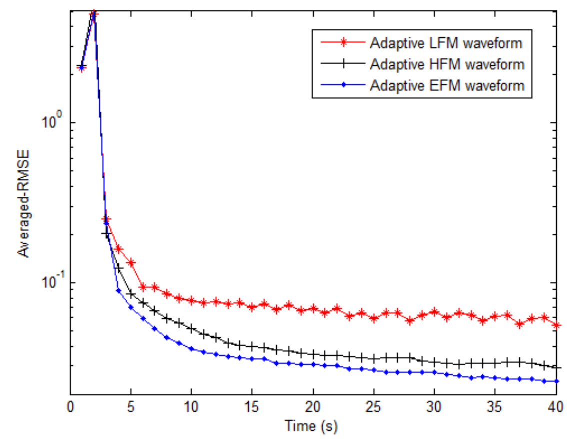

Fig. 4. Tracking performance of different waveform classes.

with reference time $t_r = 1$. A trapezoidal envelope is used for the amplitude modulation of the transmitted waveform given by [4]

$$a(t) = \begin{cases} \frac{\alpha}{t_f}(t - \frac{T_s}{2} - t_f), & -T_s/2 - t_f \le t < -T_s/2 \\ \alpha, & -T_s/2 \le t < T_s/2 \\ \frac{\alpha}{t_f}(\frac{T_s}{2} + t_f - t), & T_s/2 \le t < T_s/2 + t_f \end{cases}$$

where α is chosen so that $s(t)$ in (12) has unit energy, and $t_f \ll T_s/2$ is the rise/fall time. We choose $t_f = 0.001\lambda$, where $\lambda = T_s + 2t_f$ is the chirp duration. The chirp duration λ and the frequency sweep Δ_F are the waveform parameters with $\lambda \in \{0.05, 0.1, 0.15, 0.2, 0.25, 0.3\}$ s and $\Delta_F \in \{400, 900, 1400, 1900\}$ Hz. The averaged root mean squared error (averaged-RMSE) obtained from 500 Monte Carlo simulation runs is used for evaluating tracking performance. It should be noted that this averaged-RMSE includes tracking errors in both target position and velocity.

Fig. 2 compares the tracking performance of the proposed adaptive waveform selection scheme with those of the conventional fixed waveform schemes with the maximum and minimum time-bandwidth product (BT_{max} or BT_{min}) using the LFM waveform class. It is apparent in Fig. 2 that the proposed adaptive waveform significantly outperforms the fixed BT_{min} and BT_{max} waveforms. Fig. 3 shows a pattern of the selected waveforms for a single Monte Carlo simulation run.

Fig. 4 compares the tracking performance of the adaptive waveform selection algorithm corresponding to three different waveform classes (LFM, HFM, EFM). Note that the same parameter ranges of the chirp duration λ and frequency sweep Δ_F are used for these three waveform classes. It can be seen that the tracking performance of the adaptive HFM and EFM waveforms are significantly better than the adaptive LFM waveform's performance while the adaptive EFM waveform performs the best among the three waveforms for the simulated tracking scenario.

We observed that the proposed algorithm works well in high SNR conditions where CRLB can be used to approximate the radar measurement errors. However, CRLB is not a tight bound in low SNR conditions [18]. Therefore, our assumption on the measurement error achieving their CRLB is not valid for the case of low SNR, hence other lower bounds may need to be derived. Moreover, EKF may diverge if SNR is low.

6. CONCLUSION

We presented an adaptive waveform selection algorithm for target tracking by wideband multistatic radar/sonar systems to minimize the tracking mean squared error. A wideband bistatic signal model for both radar and sonar application was derived and utilized to formulate the wideband target tracking problem. The proposed waveform selection algorithm is developed by selecting the waveform which yields the smallest trace of the error covariance matrix of the target state estimate. This error covariance is computed using the CRLB of the wideband measurements of time delay, Doppler stretch, and arrival angle. Our simulation examples demonstrate that the tracking performance of wideband multistatic radar/sonar systems can be significantly improved by the proposed adaptive waveform selection algorithm. The considered problem can be further extended to multiple target tracking and/or with the presence of clutter. This extension will be considered in our future work.

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