ADAPTIVE STABILIZATION OF ELECTRO-DYNAMICAL TRANSDUCERS

Wolfgang Klippel

KLIPPEL GmbH, Dresden, Germany

ABSTRACT

A new control technique for electro-dynamical transducer is presented which stabilizes the voice coil position, compensates for nonlinear distortion and generates a desired transfer response by preprocessing the electrical input signal. The control law is derived from transducer modeling using lumped elements and identifies all free parameters of the model by monitoring the electrical signals at the transducer terminals. The control system stays operative for any stimulus including music and other audio signals. The active stabilization is important for small loudspeakers generating the acoustical output at maximum efficiency.

***Index Terms*—** loudspeaker, nonlinear adaptive control, stability, bifurcation, DC displacement

1. INTRODUCTION

Electro-dynamical transducers generating the required acoustical output at high efficiency, low cost, small size and minimum weight are strongly nonlinear systems causing not only harmonic and intermodulation distortion but also generating a DC-displacement which drives the coil away from the rest position [1]. The DC-component may exceed the RMS-value of the AC-component as shown in Fig. 1.

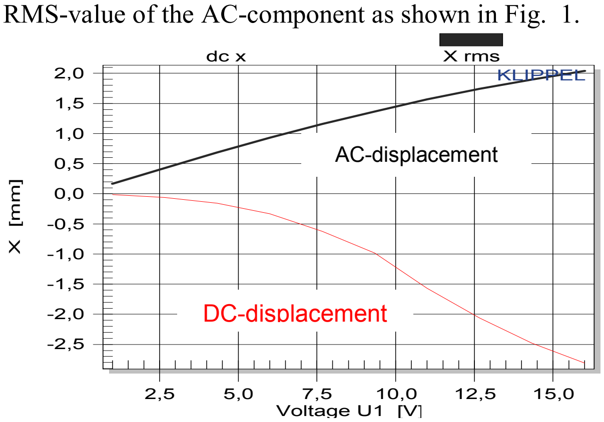

Fig. 1. Amplitude of the AC and DC components of the voice coil displacement for a sinusoidal stimulus above resonance

A well-made transducer having a high efficient motor structure may bifurcate into two states with a positive or negative DC displacement if the stiffness of suspension system becomes softer due to aging of the material over time or climate impact. A significant DC displacement is caused by an instability inherent in the electro-dynamical transduction principle and a major concern in the development of loudspeaker systems. The transducer engineer can only cope with the potential instability by adjusting the voice coil carefully in the gap to ensure a symmetrical shape of the nonlinear force factor characteristic $Bl(x)$ versus displacement x and by using a sufficient progressive stiffness $K_{ms}(x)$ of the mechanical suspension (spider, surround) to keep the coil in the gap.

This paper develops a new concept for stabilizing such transducers actively by digital signal processing.

2. CONTROL WITHOUT STABILIZATION

Before presenting the new approach the basic principle of the transducer-related control technique [3] will be discussed. The basic principle is illustrated as a signal flow chart in Fig. 2.

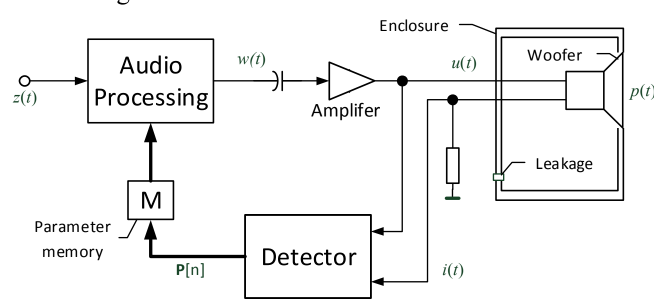

Fig. 2. Adaptive control system without stabilization

A digital signal processor generates from an audio input signal $z(t)$ a control output signal $w(t)$ which is supplied via a power amplifier to the terminals of a woofer, tweeter, micro-speaker or any other electro-dynamical transducer. The power amplifier has usually a low output impedance and uses a high-pass to avoid driving the transducer with a DC component. Based on the voltage $u(t)$ and the measured input current $i(t)$ a detector identifies the free parameters of a transducer model summarized in vector **P**. The parameter are assumed as time-invariant over a limited period (T < 1 min) but depend on the ambient climate condition, aging of the transducer materials and other changes of the mechanical or acoustical load driven by the transducer [2]. To cope with these parameter variations the detector generates updates of vector **P**[n] which are fed back to the audio signal processing. Contrary to the negative feed-back of state signals the parameter feed-back can cope with any latency caused by DAC, ADC and other digital signal processing. It is also useful to store the vector **P**[n] in a memory M and to use old values as initial parameters after restarting the adaptive control system.

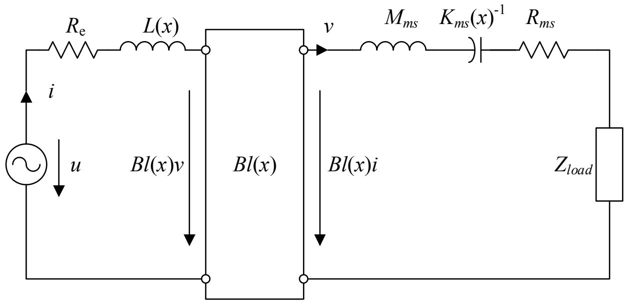

Fig. 3. Equivalent circuit of the electro-dynamical transducer

2.1. Lumped parameter modeling

The parameter identification in the detector and the processing of the audio signal are based on the lumped parameter model [4] shown in Fig. 3. This model corresponds to the integro-differential equations

$$u = R_e i + \frac{d\left(L(x)i\right)}{dt} + Bl(x)\frac{dx}{dt} \qquad (1)$$

$$Bl(x)i = \left(K_{ms}(x) - K_{ms}(0)\right)x + \mathrm{L}^{-1}\left\{sZ_m(s)\right\} * x$$

with the force factor

$$Bl(x) = \sum_{i=0}^{N} b_i x^i, \qquad (2)$$

the stiffness of the mechanical suspension

$$K_{ms}(x) = \sum_{i=0}^{N} k_i x^i \qquad (3)$$

and the voice coil inductance

$$L(x) = \sum_{i=0}^{N} l_i x^i, \qquad (4)$$

which are nonlinear functions of the voice coil displacement $x(t)$. This representation uses the inverse Laplace transform $\mathrm{L}^{-1}\{\ \}$ and the convolution denoted by *. The linear parameters are the voice coil resistance R_e and the total mechanical impedance

$$Z_m(s) = \frac{\sum_{i=0}^{M} a_i s^i}{\sum_{i=0}^{M} c_i s^i} \qquad (5)$$

describing the effect of the mechanical stiffness $K_{ms}(x=0)$ at the rest position, the mechanical resistance R_{ms}, the moving mass M_{ms} and the load impedance $Z_{load}(s)$ of the coupled acoustical and mechanical system. The order M describes the number of poles and zeroes in the rational transfer function $Z_m(s)$. A transducer mounted in a sealed enclosure can be modeled by a second-order function $Z_m(s)$ while a vented box system, panel or in a horn requires a higher-order system, which makes the identification of the linear parameters a_i and c_i more difficult [5].

2.2. Parameter identification

The free parameters of the electrical equivalent model summarized in the parameter vector

$$\mathbf{P} = [P_1 \ldots \quad P_j \quad \ldots \quad P_J]^T \qquad (6)$$

can be identified from the voltage and current monitored at the terminals reproducing an audio signal (e.g. music) by the transducer. The system identification is based on minimizing an error signal such as the difference

$$e(t) = u'(t) - u(t) \qquad (7)$$

between voltage $u'(t)$ predicted by the model in (**1**) and the measured voltage $u(t)$. The optimal parameter vector **P** can be determined by searching for the minimum of the mean squared error

$$\mathbf{P} = \arg\min_{\mathbf{P}}\left(E\{e(t)^2\}\right) \qquad (8)$$

corresponding to the requirement

$$\frac{\partial E\left\{e(t)^2\right\}}{\partial P_j} = 2e(t)\frac{\partial e}{\partial P_j} = 2e(t)\frac{\partial u'(t)}{\partial P_j} \overset{!}{=} 0 \qquad j = 1,\ldots,J \qquad (9)$$

giving the Wiener-Hopf-equation

$$\mathbf{P} = \mathbf{R}^{-1}\mathbf{Y} = \left(E\left(\mathbf{G}(t)\mathbf{G}^H(t)\right)\right)^{-1} E\left(u(t)\mathbf{G}(t)\right) \qquad (10)$$

with the expectation value $E\{\ \ldots\}$ and the gradient vector **G***(t)*

$$\mathbf{G}(t) = \left[\frac{\partial u'(t)}{\partial P_1} \quad \ldots \quad \frac{\partial u'(t)}{\partial P_j} \quad \ldots \quad \frac{\partial u'(t)}{\partial P_J}\right]^T \qquad (11)$$

The gradient vector **G***(t)* is generated in the gradient calculator GC in Fig. 4 based on the state vector $\mathbf{S}_1$ generated by a transducer model in accordance with (1).

Alternatively, the optimal parameter vector

$$\mathbf{P}[n] = \mathbf{P}[n-1] + \boldsymbol{\mu}(t)e(t)\mathbf{G}(t) \qquad (12)$$

can be estimated iteratively by using the stochastic gradient method (LMS-algorithm) with the learning speed matrix **μ**(t) as shown in Fig. 4.

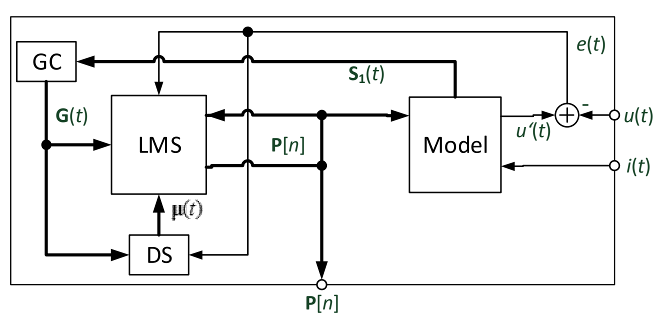

Fig. 4. Detector estimating the parameter vector **P** based on monitored voltage *u* and current *i*

If the stimulus has a sparse spectrum and comprises for examples only a few tones and provides no persistent excitation [6] of the loudspeaker, the matrix **R** in (10) becomes positive semi-definite and has no inverse. Under those conditions there exist no unique solution of the optimization problem and the LMS-algorithm might diverge from the optimal values of the transducer parameters [7]. Furthermore, a badly conditioned matrix **R** reduces the learning speed and the accuracy of the parameter measurement process.

Therefore, a supervisory system DS in Fig. 4 sets at least one learning speed parameter in $\boldsymbol{\mu}(t)$ to zero if the rank rk(**R**) of the matrix **R** is smaller than the number J of free parameters in vector **P**.

2.3. Audio signal processing

The identified parameter vector **P** is used for the processing of the audio signal as shown in Fig. 5. A state estimator generates a state vector $\mathbf{S}_p$ which comprises the voice coil displacement

$$x(t) = \mathrm{L}^{-1}\{H_x(s)\}y(t) \qquad (13)$$

corresponding to the target transfer function $H_x(s)$ of the linearized overall system and the nonlinear input current i predicted by (**1**).

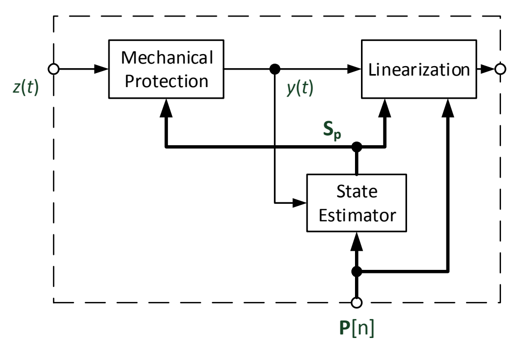

Fig. 5. Processing of the audio signal with mechanical protection and feed-forward linearization (mirror filter).

A mechanical protection system attenuates the input signal $z(t)$ if the predicted absolute value of the voice coil displacement $|x|$ exceeds a permissible limit value x_{max}. The parameter and state information are also used in the control law

$$w(t) = \alpha(\mathbf{P},\mathbf{S}_P)\left[y(t) + \beta(\mathbf{P},\mathbf{S}_P)\right] \qquad (14)$$

to compensate the nonlinear distortion and synthesize the desired overall transfer behavior [4-6]. The prediction of the state vector $\mathbf{S}_p$ and the generation of the both control gain $\alpha(\mathbf{P},\mathbf{S}_P)$ and the control additive $\beta(\mathbf{P},\mathbf{S}_P)$ exploits the nonlinear transducer model in (**1**) as described in [11].

2.4. Adaptive control limitations

The control and protection system is based on a transducer model which neglects creep and other viscoelastic behavior of the suspension. The creep effect reduces the stiffness at low frequencies and generates a higher value of the DC displacement than predicted by (1). A similar problem occurs when the transducer is operated in an enclosure which is sufficiently sealed for audio frequencies. A small leakage in the enclosure required to compensate for static air pressure variations will reduce the stiffness of the enclosed air volume at very low frequencies. Thus the DC displacement is significantly higher than predicted by the total stiffness found at the resonance frequency. Extended models developed for viscoelastic behavior of the mechanical suspension [12] and [13] and for leaky enclosures are less useful for adaptive control because the accurate identification of the model parameters requires a mechanical sensor which measures displacement at low frequencies. This information cannot be derived from electro-motive force (EMF) found in electrical signals at the transducer terminals.

A further disadvantage of the known control technique is that the adaptive parameter estimation requires a persistent excitation of the transducer. For example the parameters of a transducer cannot be identified with a single tone stimulus while generating a high DC displacement as shown in Fig. 1.

3. ACTIVE TRANSDUCER STABILIZATION

To cope with the unknown voice coil position in the adaptive control system an additional variable $x_{\mathrm{off}}(t)$ is introduced in the nonlinear parameters

$$Bl(x) = \sum_{i=0}^{N} b_i\left(x + x_{off}(t)\right)^i \qquad (15)$$

$$K_{ms}(x) - K_{ms}(0) = \sum_{i=1}^{N} k_i\left(x + x_{off}(t)\right)^i$$

$$L(x) = \sum_{i=0}^{N} l_i\left(x + x_{off}(t)\right)^i .$$

The offset $x_{\mathrm{off}}(t)$ may be considered as time-variant parameter depending on the interactions between transducer nonlinearities and stimulus, viscoelastic behavior of the suspension, gravity and other external influences. The offset $x_{\mathrm{off}}(t)$ may be also interpreted as a state variable which comprises only low frequency components far below the audio band. By introducing the offset $x_{\mathrm{off}}(t)$ the time variance of the coefficients b_i, k_i and l_i in (15) can be reduced.

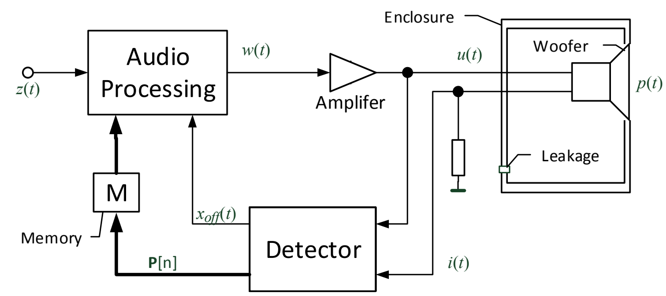

Fig. 6. Control System with stabilization of voice coil position

3.1. Voice coil offset identification

The detector has to identify the coil offset $x_{\mathrm{off}}(t)$ in the back-EMF at the transducer terminals by exploiting the nonlinear distortion generated in the audio band. The instantaneous value of $x_{\mathrm{off}}(t)$ is permanently supplied to the audio processing as illustrated in Fig. 6.

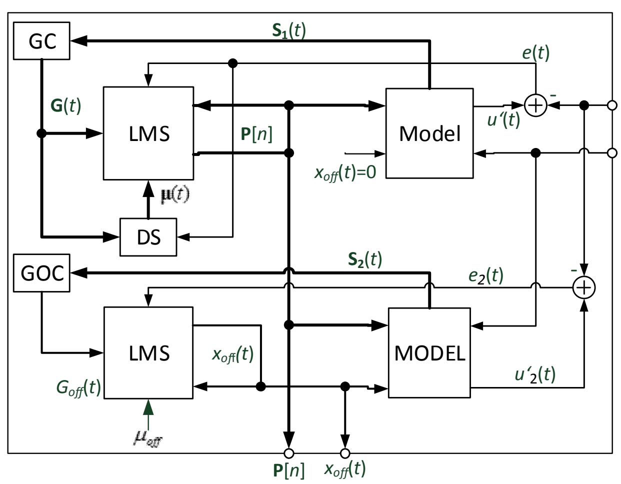

Fig. 7. Detector estimating the parameter vector **P** and the voice coil offset x_{off} based on monitored voltage u and current i

The detector as shown in Fig. 7 uses two transducer models according to (1) and (15). The first model describes the long-term behavior of the loudspeaker where the expectation value $E\{x_{\mathrm{off}}(t)\}=0$. Thus, the identification of parameter vector **P** corresponds to the conventional approach shown in Fig. 4. The second model considers the instantaneous coil offset and generates a second error signal $e_2(t)$ which is used in a second LMS algorithm

$$x_{off}(t) = x_{off}(t-\Delta t) + \mu_{off}\, e_2(t) G_{off}(t) \qquad (16)$$

with the gradient

$$G_{off}(t) = \frac{\partial e_2(t)}{\partial x_{off}} \qquad (17)$$

$$= \frac{\partial u_2'(t)}{\partial Bl(x)} \frac{\partial Bl(x)}{\partial x_{off}} + \frac{\partial u_2'(t)}{\partial K_{ms}(x)} \frac{\partial K_{ms}(x)}{\partial x_{off}} + \frac{\partial u_2'(t)}{\partial L(x)} \frac{\partial L(x)}{\partial x_{off}}$$

generated by the gradient calculator GOC from the state vector $\mathbf{S}_2$ provided by the second model. The constant learning parameter μ_{off} permanently enables learning the offset variable $x_{\mathrm{off}}(t)$ for any stimulus (including a single tone !) supplied to the loudspeaker. The adaptive estimation of the offset variable $x_{\mathrm{off}}(t)$ only stagnates if the stimulus generates no displacement of the voice coil.

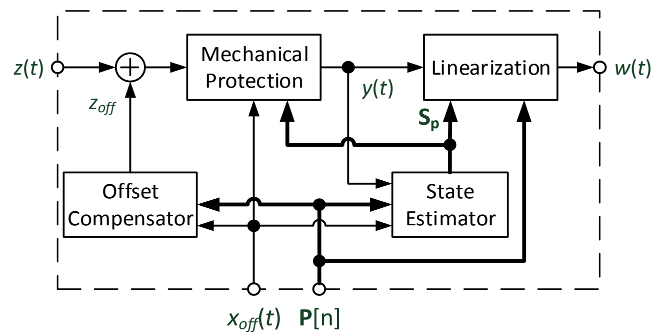

Fig. 8. Processing of the audio signal with mechanical protection, linearization and stabilization.

If the stimulus $z(t)$ provides persistent excitation of the transducer to identify all parameters b_i, k_i and l_i in (15), the parameter vector **P** will represent the voice coil position, and the offset variable $x_{\mathrm{off}}(t)$ becomes zero eventually. Therefore only the parameter vector **P** will be stored in the memory M and used with $x_{\mathrm{off}}(t)=0$ as initial values after restarting the control system.

3.2. Audio processing with stabilization

The state estimator as shown in Fig. 8 and the control law

$$w(t) = \alpha(\mathbf{P}, \mathbf{S}_P, x_{off}) \left[y(t) + \beta(\mathbf{P}, \mathbf{S}_P, x_{off}) \right] \qquad (18)$$

consider the instantaneous voice coil offset $x_{\mathrm{off}}(t)$ in the nonlinear parameter definition (15). The mechanical protection system compares the total displacement $|x(t)+x_{\mathrm{off}}(t)|$ with the permissible limit value $x_{\max}$ to detect a mechanical overload of the transducer and to attenuate the input signal in time.

The new concept of active stabilization of electro-dynamical transducers exploits information from transducer modeling. All free parameters in this model and uncertainties due to viscoelastic behavior and external influences can be identified by monitoring electrical signals at the terminals only. There is no need to measure the absolute position of the voice coil by using a mechanical sensor such as a laser triangulation sensor. There are no additional hardware requirements to ensure reliable protection and linearization of the transducer. If a DC-coupled power amplifier is used in the application, an offset compensator as shown in Fig. 8 detects the symmetry point x_{sym} in the nonlinear force factor characteristic $Bl(x)$ and synthesizes a DC signal z_{off} which moves the coil's rest position to the symmetry point x_{sym}, ensuring highest efficiency and minimum distortion.

4. CONCLUSION

Active voice coil position stabilization is a fundamental requirement for all control objectives such as mechanical protection, linearization and equalization of the transducer. For example, a significant DC component as shown in Fig. 1 reduces the maximum amplitude of the AC component by 6 dB where bottoming of the voice coil at the back plate occurs.

The moving coil transducer's tendency to generate DC and unstable rest position increases in smaller loudspeakers, which generate sound with less hardware resources and energy. A nonlinear motor structure optimized for highest force factor $Bl(x=0)$ at the rest position at the rest position $x=0$ provides maximum overall efficiency for common audio signals having bell-shaped probability density function $pdf(x)$ of the voice coil displacement. Such a nonlinear but efficient motor structure can be combined with a soft mechanical suspension resulting in a lower resonance frequency and more low frequency displacement. While beneficial for most applications, such a transducer requires active stabilization to generate the desired transfer behavior over the entire life time of the product.

The adaptive control algorithm presented in this paper has been illustrated on a moving coil transducer but the same approach can also be applied to other transduction principles such as the balance armature transducer used in hearing aids and in-ear phones.

REFERENCES

[1] Klippel, Tutorial: "Loudspeaker Nonlinearities - Causes, Parameters, Symptoms" *J. Audio Eng. Society* 54, No. 10 pp. 907-939 (Oct. 2006).

[2] W. Klippel, "Modeling load-induced aging of loudspeaker suspension," Proceedings of the Acoustics 2012 Nantes Conference, 23-27 April, 2012, Nantes, France.

[3] W. Klippel, „Active Compensation of Transducer Nonlinearities," preprint in 23rd International Conference of the *Audio Eng. Society* on "Signal Processing in Audio Recording and Reproduction", Copenhagen, Denmark, 23-25 May 2003.

[4] A. J. Kaiser, "Modeling of the Nonlinear Response of an Electrodynamic Loudspeaker by a Volterra Series Expansion," J. Audio Eng. Soc. 35, p. 421, (1987 June).

[5] R. H. Small, "Vented-Box Loudspeaker Systems Part I: Small-Signal Analysis, J. Audio Eng. Soc., Vol. 21, No. 5, pp. 363- 372.

[6] L. Ljung, T. Glad, "On global identifiability for arbitrary model parametrizations," Automatica, Vol. 30, Issue 2, Feb. 2, 1994, p. 265-276.

[7] W. Klippel, "Adaptive Nonlinear Control of Loudspeaker Systems," *J. Audio Eng. Society* 46, pp. 939 - 954 (1998).

[8] H. Nijmeijer and A.J. van der Schaft, Nonlinear Dynamical Control Systems (Springer, New York, 1990).

[9] J. Suykens, J. Vandewalle and J. van Gindeuren, "Feedback Linearization of Nonlinear Distortion in Electrodynamic Loudspeakers," J. Audio Eng. Soc., Vol. 43, No. 9, pp. 690-694 (1995).

[10] W. Klippel, "Direct Feedback Linearization of Nonlinear Loudspeaker Systems," *J. Audio Eng. Society* 46, pp. 499-507, (1998).

[11] W. Klippel, "The Mirror Filter - a New Basis for Reducing Nonlinear Distortion Reduction and Equalizing Response in Woofer Systems", *J. Audio Eng. Society* 32, Heft 9, S. 675-691, (1992).

[12] M. H. Knudsen and J. G. Jensen, "Low-Frequency Loudspeaker Models that Include Suspension Creep," *J. Audio Eng. Soc.,* vol. 41, pp. 3–18 (1993 Jan./Feb.).

[13] F. Agerkvist and T. Ritter, "Modeling Viscoelasticity of Loudspeaker Suspensions using Retardation Spectra", presented at the 129th Convention of the Audio Eng. Soc. in San Francisco, Nov. 4-7, 2010, preprint 8217.