A LOW COMPLEXITY COHERENT CPM RECEIVER WITH MODULATION INDEX ESTIMATION

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ABSTRACT

In this paper we address the problem of low-complexity coherent detection of continuous phase modulation (CPM) signals. We exploit the per-survivor-process technique to build a reduced-state trellis and apply a Viterbi algorithm with modified metrics. In the case where the modulation index can vary, we propose a maximum-likelihood (ML) estimation of the modulation index and compare the performance of the resulting structure with a non-coherent receiver structure of the state of the art. Simulations on an additive white Gaussian noise (AWGN) channel both for binary and M-ary CPM show the efficiency of the proposed receiver.

Index Terms— CPM, modulation index estimation, per-survivor processing, reduced-complexity, Viterbi decoding, modulation index mismatch.

1. INTRODUCTION

In applications with low-cost transmitters, the RF front end may suffer from non linearities. In this case, continuous phase modulations (CPM) [1], which have a constant amplitude, are a good candidate to limit the transmitted signal distorsion. A CPM is perfectly defined by the symbol alphabet, the frequency pulse and the modulation index. When the modulation index varies, either randomly in a continuous interval as for the AIS analog transmitter [2] or, intentionally as for the Bluetooth standard [3], the priority is given to non coherent detection due to its robustness. Discriminator detectors [4] are the most used for their low computation cost, at the expense of severely degraded error rate performance. Efficient non-coherent receiver structures such as the one proposed in [5] by Lampe *et al.* achieve better error rate performance with a reasonable increase of complexity. The best error rate performance is obtained through coherent maximum likelihood sequence estimation (MLSE). Based on the trellis state description of the CPM, the Viterbi algorithm performs optimally, but suffers from a high complexity. Complexity reduction has been studied in [6], [7], [8], and [9]. The per survivor processing (PSP) has been applied in [10] in a specific case (binary full-response CPFSK) to reduce the trellis state number and hence the computation cost. However, like the Viterbi algorithm, even a slight modulation index variation yields a severe error rate degradation. In [11], the PSP has also been applied to soft-in soft-out (SISO) CPFSK detection with an irrational modulation index. Furthermore the modulation index mismatch between the transmitter and the receiver is taken into account in the metric computation, enhancing the robustness of the resulting modified SISO BCJR algorithm. However a perfect knowledge of the modulation index is assumed. In this paper, we generalize the principle of the PSP approach for state number reduction and metric modification for modulation index mismatch correction to any kind of CPM. We introduce a maximum-likelihood estimation of the modulation index in the resulting coherent receiver. We compare the proposed coherent receiver with the non-coherent structure of [5] taking into account the spectral efficiency loss involved by the ML modulation index estimation. Simulations carried out on an AWGN channel, for both binary (Bluetooth and AIS standards parameters) and non-binary CPM (Wireless M-Bus standard parameters), show the efficiency of the proposed receiver. The paper is organized as follows: we first briefly introduce the system model in Section 2. Then we define the generalized receivers for CPM signals for any modulation index as well as the modulation index estimation in Section 3. The applications and simulations are given in Section 4, followed by a conclusion and some perspectives in Section 5.

2. SYSTEM MODEL

2.1. CPM signal Model

The complex baseband CPM signal is defined as:

$$s(t,\boldsymbol{\alpha}) = \sqrt{\frac{E}{T}} e^{j\phi(t,\boldsymbol{\alpha})}, \tag{1}$$

where E is the average symbol energy, T is the symbol duration and $\phi(t,\boldsymbol{\alpha})$ the information-carrying phase given by:

$$\phi(t,\boldsymbol{\alpha}) = 2\pi h_{tx} \sum_{i=0}^{\infty} \alpha_i q(t-iT). \tag{2}$$

$\boldsymbol{\alpha} = \{\alpha_i\}$ denotes the information sequence. The information symbols α_i are assumed to be independent and identically distributed and to take values in the M-ary alphabet $\mathcal{M} = \{\pm 1, \pm 3, ..., \pm(M-1)\}$. h_{tx} is the modulation index used at the transmitter. The phase response $q(t)$ is defined on $[0, \ +\infty)$ and satisfies :

$$q(t) = \frac{1}{2}, \qquad t \geq LT. \tag{3}$$

Using the properties of q given in (3), the information-carrying phase during the n-th time interval, $[nT, \ (n+1)T]$, $n \in \mathbb{N}$, can be written as:

$$\begin{aligned} \phi(t, \boldsymbol{\alpha}) &= 2\pi h_{tx} \sum_{i=0}^{\infty} \alpha_i q(t - iT), \\ &= \pi h_{tx} \sum_{i=0}^{n-L} \alpha_i + 2\pi h_{tx} \sum_{i=n-L+1}^{n} \alpha_i q(t - iT), \\ &= \theta_{tx,n} + \phi_{tx,n}(t). \end{aligned} \tag{4}$$

2.2. Coherent receiver

From (4) we observe that the modulated signal over the n-th time interval depends both on the phase state denoted by $\theta_{tx,n}$ and on the L most recent symbols, i.e $(\alpha_n, \alpha_{n-1}, ..., \alpha_{n-L+1})$. For a rational modulation index $h_{tx} = \frac{k_{tx}}{p_{tx}}$, the phase state $\theta_{tx,n}$ modulo 2π can take only p_{tx} or $2p_{tx}$ different values for even and odd k_{tx}, respectively. Therefore, the phase evolution can be described by a finite-state machine, where each state is represented by an L-dimensional vector $(\theta_{tx,n}, \alpha_{n-1}, \alpha_{n-2}, ..., \alpha_{n-L+1})$ and where the number of such states is $p_{tx}M^{L-1}$. We assume that the signal is transmitted over a Gaussian channel. The equivalent baseband received signal, denoted by $r(t)$, is defined as:

$$r(t) = s(t, \boldsymbol{\alpha}) + n(t), \tag{5}$$

where $n(t)$ is a realization of a zero-mean wide sense stationary complex circularly symmetric Gaussian noise, independent of the signal, and with double-sided power spectral density $2N_0$ over the bandwidth of $s(t, \boldsymbol{\alpha})$. The MLSE-detector aims at maximizing the scalar product between $r(t)$ and all the realizations of $s(t, \boldsymbol{\alpha})$. Assuming N_d transmitted symbols, the MLSE estimation of the information symbols $\alpha_0, \alpha_1, ..., \alpha_{N_d-1}$ is given by:

$$(\tilde{\alpha}_0, \tilde{\alpha}_1, , ..., \tilde{\alpha}_{N_d-1}) = \underset{\tilde{\boldsymbol{\alpha}} \in \mathcal{M}^{N_d}}{\arg\max} \Re \left[\int_0^{N_d T} r(t) s^*(t, \tilde{\boldsymbol{\alpha}}) dt \right], \tag{6}$$

where $\Re(x)$ denotes the real part of x.
As $\int_0^{N_d T} r(t) s^*(t, \tilde{\boldsymbol{\alpha}}) dt = \sum_{n=0}^{N_d-1} \int_{nT}^{(n+1)T} r(t) e^{-j\phi(t, \tilde{\boldsymbol{\alpha}})} dt$, the computation of all scalar products over $[0, \ N_d T]$ is equivalent to the sum of all scalar products over a symbol duration given the previous phase state $\theta_{tx,n}$. We introduce and denote by $BM_{i,j}^{(n)}$ the branch metric corresponding to the scalar product associated to the transition from state $\sigma^i = \left(\theta_{tx,n}^i, \alpha_{n-1}, \alpha_{n-2}, ..., \alpha_{n-L+1}\right)$ to state $\sigma^j = \left(\theta_{tx,n+1}^j, \alpha_n, \alpha_{n-1}, ..., \alpha_{n-L+2}\right)$, where α_n is the symbol related to this transition during the n-th time interval. It is calculated as follows:

$$BM_{i,j}^{(n)} = \Re\{ \int_{nT}^{(n+1)T} r(t) e^{-j(\theta_{tx,n}^i + \phi_{tx,n}(t))} dt \}. \tag{7}$$

The Viterbi algorithm is based on the recursive computation of the accumulated metric at state σ^i and time instant nT denoted by $CM_i^{(n)}$ and defined as: $CM_i^{(n)} = \max_j \left(CM_j^{(n-1)} + BM_{i,j}^{(n)} \right)$. At the end, the MLSE based-decision rule is given by the sequence $\tilde{\boldsymbol{\alpha}}$ yielding the maximum over i of $CM_i^{(N)}$.
We assume that the modulation index h_{tx} is not perfectly known at the receiver. The coherent detection requires an estimate of h_{tx}. In that case, N_e pilot symbols are inserted at the beginning of the frame. Denoting by $\mathcal{H}$ the set of possible values for h_{tx}, the ML estimate of h_{tx} denoted by $\hat{h}_{tx}$, is computed as:

$$\hat{h}_{tx} = \underset{h_i \in \mathcal{H}}{\arg\max} \Re \left[\int_0^{N_e T} r_{pilots}(t) {S_{pilots}^i}^*(t) dt \right], \tag{8}$$

where $r_{pilots}(t)$ is the received signal and S_{pilots}^i is the modulated training sequence with h_i.

3. PROPOSED RECEIVERS

3.1. Generalized Non-coherent CPM receiver

In this section we generalize to M-ary CPM the non-coherent sequence detection receiver for Bluetooth systems referred to as adaptive NSD (Non-coherent sequence detection) and developed in [5] . Non-coherent receivers are designed to solve the problem of robustness to modulation index variation when a perfect estimation cannot be guaranteed. In [5] a trellis is defined with a reduced number of states. The states corresponding to $[nT, (n+1)T]$ depend on $(\alpha_n, \ldots, \alpha_{n-L+1})$ only. The phase state $\theta_{tx,n}$ is determined by employing the per-survivor processing approach (PSP) [12]. In order to be robust to phase variations, an adaptive phase reference recursively updated $q_{ref}(n)$ is added. So, the new expression of the branch metric becomes:

$$\begin{aligned} BM_{i,j}^{(n)} = &\Re\{ \left[q_{ref}^{(i)}(n) \right]^* \int_{nT}^{(n+1)T} r(t). \\ & e^{-j(\theta_{tx,n}^i + 2\pi h_{tx} \sum_{k=n-L+1}^{n} \alpha_k q(t-kT))} dt \}, \end{aligned} \tag{9}$$

where the phase reference is updated as follows:

$$\begin{aligned} q_{ref}^{(j)}(n+1) = \beta q_{ref}^{(i)}(n) + (1-\beta) \int_{nT}^{(n+1)T} r(t). \\ e^{-j(\theta_{tx,n}^i + 2\pi h_{tx} \sum_{k=n-L+1}^{n} \alpha_k q(t-kT))} dt, \end{aligned} \tag{10}$$

where $q_{ref}^{(k)}$ is the phase reference related to the kth state and i is the previous state of the state j. The parameter β, $0 \leq \beta < 1$ acts as a forgetting factor.
However, the estimation of modulation index h_{tx} in the NSD receiver is done with the help of the first N_e received symbols to adaptively choose the modulation index yielding the maximum cumulative metric (9):

$$\Big(\hat{\alpha}_1, \hat{\alpha}_2, ..., \hat{\alpha}_{N_e}, \hat{h}_{tx}\Big) = \underset{\hat{\alpha}_1, \hat{\alpha}_2, ..., \hat{\alpha}_{N_e}, h_i \in \mathcal{H}}{\arg\max} \Bigg\{ \Re\Big\{ \int_0^{N_e T} r(t) \big(S_{seq}^i(t)\big)^* q_{ref}^*(t) dt \Big\} |h_i \Bigg\} \tag{11}$$

where $S^i(t)$ corresponds to the modulation of $(\hat{\alpha}_1, \ldots, \hat{\alpha}_{N_e})$ with $h_{tx} = h_i$. After the estimation period $\hat{h}_{tx}$ is used for detecting the remaining sequence. This receiver does not require the insertion of pilots.

3.2. Generalized coherent CPM receiver

We first assume a perfect knowledge of the modulation index h_{tx}. The proposed receiver relies on the decomposition of h_{tx} in the form $h_{tx} = h_{rx} + \Delta h$ with h_{rx} being a rational number. The key idea is to use the Viterbi algorithm with modified branch and state metrics on a trellis designed from h_{rx}. It takes into account a phase difference proportional to Δh and computed on a per survivor processing basis [12].
The PSP application for state number reduction purpose was introduced in [10], [11] in the case of binary full response CPFSK. In [11], the branch metric computation takes into account Δh, and is applied in a SISO algorithm namely the BCJR [13]. In this section we generalize the PSP branch metric computation of [13] to any kind of CPM modulation, whatever the modulation indices h_{tx} and h_{rx}, the value of L or the frequency pulse $g(t)$ are. Going back to the information-carrying phase expression given in (4), we express it as a function of h_{rx} and Δh :

$$\begin{aligned} \phi(t, \boldsymbol{\alpha}) &= \pi h_{tx} \sum_{i=0}^{n-L} \alpha_i + 2\pi h_{tx} \sum_{i=n-L+1}^{n} \alpha_i q(t - iT) \\ &= \pi h_{rx} \sum_{i=0}^{n-L} \alpha_i + \pi \Delta h \sum_{i=0}^{n-L} \alpha_i \\ &+ 2\pi h_{rx} \sum_{i=n-L+1}^{n} \alpha_i q(t - iT) \\ &+ 2\pi \Delta h \sum_{i=n-L+1}^{n} \alpha_i q(t - iT) \\ &= \theta_{rx,n} + \phi_{rx,n}(t) + \lambda_n^i \\ &+ 2\pi \Delta h \sum_{i=n-L+1}^{n} \alpha_i q(t - iT) \end{aligned} \tag{12}$$

with $\theta_{rx,n} = \pi h_{rx} \sum_{i=0}^{n-L} \alpha_i$,
$\phi_{rx,n}(t) = 2\pi h_{rx} \sum_{i=n-L+1}^{n} \alpha_i q(t - iT)$
and $\lambda_n^i = \pi \Delta h \sum_{i=0}^{n-L} \alpha_i$. The first two terms in (12) are tracked by the Viterbi algorithm and the third term is the resulting phase difference which is built up at every symbol. This accumulation is calculated using the PSP technique by associating to each state an additional parameter λ_n^i. The last term of (12) is calculated at the output of the matched filter. We can write the following relation between the corrected and the reference phase states.

$$\begin{aligned} \theta_{tx}^i &= \lambda_n^i + i \frac{k_{rx}}{p_{rx}} \pi, \qquad i = \{0, 1, 2, ..., 2p_{rx} - 1\} \\ &= \lambda_n^i + \theta_{rx}^i. \end{aligned} \tag{13}$$

The only additional task that needs to be performed is the PSP update of the phase difference for each state, λ_n^i. The update equation is given by:

$$\lambda_{n+1}^i = \lambda_n^{j^*} + \alpha_{(j^*, i)} \pi \Delta h \tag{14}$$

where j^* is selected from the M previous states of σ_i, as the index of the maximum cumulative metric and $\alpha_{(j^*,i)}$ is the corresponding data symbol involved in the transition from state σ^{j^*} to σ^i in the receiver trellis structure. We define a new expression of the branch metric $\sigma^j \to \sigma^i$ used in the proposed algorithm: $\big(\theta_{rx}^j(\lambda_n^j), \alpha_{n-1}, \alpha_{n-2}, ..., \alpha_{n-L+1}\big)$ $\xrightarrow{\alpha_n} \big(\theta_{rx}^i(\lambda_{n+1}^i), \alpha_n, \alpha_{n-1}, \alpha_{n-2}, \ldots, \alpha_{n-L+2}\big)$.
The branch metric is then calculated as follows:

$$\begin{aligned} BM_{j,i}^{(n)} =& \Re\{e^{-j\lambda_n^j} \int_{nT}^{(n+1)T} r(t). \\ & e^{-j(\theta_{rx}^j + 2\pi h_{rx} \sum_{k=n-L+1}^{n} \alpha_k q(t-iT))} \\ & e^{-j2\pi \Delta h \sum_{k=n-L+1}^{n} \alpha_k q(t-kT)} dt\} \end{aligned} \tag{15}$$

In the simulations, h_{tx} is estimated from N_e pilot symbols by applying Equation (8). The branch metrics are computed by replacing Δh by $\widehat{\Delta h}$ in (15).

4. APPLICATIONS AND SIMULATIONS

The proposed algorithms can be used in all cases where the original Viterbi algorithm fails to achieve the best trade-off between performance and complexity. In all simulations we take an oversampling of 8 samples per symbol interval and we denote by N_d the data length. For the coherent detection, we take into account the spectral efficiency loss in the computation of $\frac{E_b}{N_0}$.

4.1. Bluetooth

The first example corresponds to the Bluetooth standard which employs the binary Gaussian frequency-shift keying (GFSK) modulation with a frequency pulse length $L = 3$ and a frame length $N_d = 2748$. The modulation index is assumed to vary in a finite alphabet $\mathcal{H}$ known at the receiver [5], [14]. For the modulation index estimation, the best training sequence has all symbols equal to 1, since it allows to have the maximum decorrelation between the reference signals S^i_{seq}. In Figure 1 we compare the coherent receiver to the adaptive NSD. We fix $h_{tx} = 0.35$ and $h_{rx} = \frac{2}{3}$. $h_{rx} = \frac{2}{3}$ ensures the best state number reduction. We fix the forgetting factor $\beta = 0.9$ and $N_e = 50$ in the case of NSD and let N_e vary in the case of the coherent receiver. We observe that the coherent receiver requires at least $N_e = 15$ to outperform the NSD. For $N_e = 20$ its performance coincides with the optimum MLSE one and exhibits a gain of 1 dB compared to NSD for a $BER = 10^{-4}$.

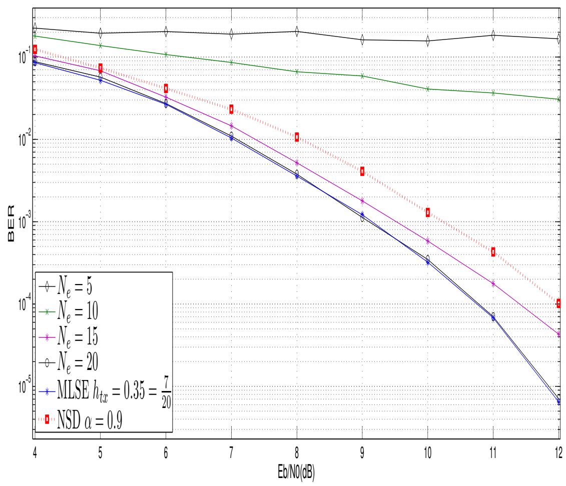

Fig. 1. Bluetooth detection for a transmission with $h_{tx} = 0.35$ and a coherent reception with $h_{rx} = \frac{2}{3}$ with $N_d = 2748$

4.2. Automatic Identification System (AIS)

The second application case is the random variation of the modulation index around its nominal value as in AIS systems due to an analog low-cost transmitter [2]. The AIS standard uses a binary GMSK modulation with $BT = 0.4$. At the transmitter, we generate randomly the transmission modulation index h_{tx} in the interval $[0.5 - 0.035 \quad 0.5 + 0.035]$. At the reception we discretize this interval with a step μ_h to estimate h_{tx} in the same manner as in (8). For the NSD receiver we fix the forgetting factor $\beta = 0.9$, $\mu_h = 0.001$ and $N_e = 50$ and let μ_h and N_e vary for the coherent receiver. The MLSE performance is plotted as a lower bound. The coherent receiver is built with $h_{rx} = \frac{2}{3}$. From Figure 2, we see that for $\mu_h = 0.001$ and $N_e = 40$ the coherent receiver outperforms the NSD receiver with a gain of 0.3 dB and a loss of only 0.2 dB compared to the lower bound for a $BER = 10^{-4}$.

4.3. Wireless M-Bus

The third application case is the demodulation of quaternary GFSK signals. This application is known as "Wireless M-

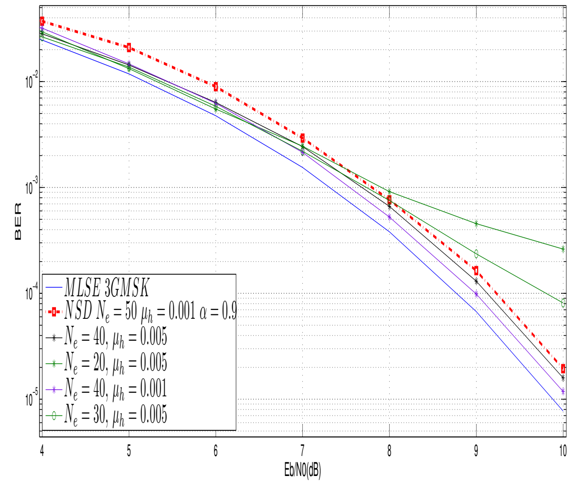

Fig. 2. AIS detection for a transmission with random modulation index h_{tx} and a coherent reception with $h_{rx} = \frac{2}{3}$ with $N_d = 256$

Bus" which is a European Standard for wireless Metering BUS [15]. To lower the cost and the power consumption, an analog transmitter can be implemented on the meters, yielding index modulation variation as for the AIS case. The best training sequence has all symbols equal to 3. In the quaternary modulation case, we found from several simulations that the value of $h_{rx} = \frac{2}{5}$ is the best choice for a compromise between performance and complexity. The performance of Figure 3 are obtained by averaging over several values of h_{tx} taken randomly from the interval $[0.5 - 0.035 \quad 0.5 + 0.035]$. For $\mu = 0.001$ and $N_e = 50$ we obtain near optimal performance with a negligible loss of 0.3 dB compared to the optimum MLSE and a gain of 0.2 dB compared to the NSD receiver one for a $BER = 10^{-4}$.

5. CONCLUSIONS AND PERSPECTIVES

In this paper, we have extended the PSP application and branch metric modification of trellis-based coherent receivers, initially proposed for binary CPFSK, to detect M-ary CPM signals with any modulation index and phase response. We have applied this generalization to cases where the modulation index is unknown at the receiver and has to be estimated. We have simulated the error rate performance in three different applications, namely Bluetooth, Automatic Identification System (AIS) and the Wireless Metering Bus European standard. The error rate performances have been compared to the performance of an adaptive non-coherent receiver proposed for Bluetooth detection, which is, to the best of the authors' knowledge, one of the most promising receiver for the Bluetooth standard, where robustness against unknown modulation index is a considerable advantage. For this comparison, we have extended the adaptive non-coherent receiver to non-binary CPM. The comparison shows that, despite the unknown modulation index, the coherent receiver outperforms the non-coherent receiver.

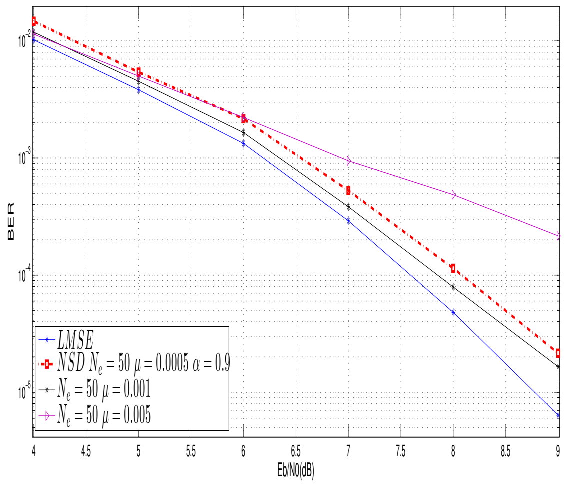

Fig. 3. Wireless M-Bus detection for a transmission with random modulation index h_{tx} and a coherent reception with $h_{rx} = \frac{2}{5}$ with N_d=1000 quaternary symbols

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