CLOSED-FORM APPROXIMATIONS OF THE PAPR DISTRIBUTION FOR MULTI-CARRIER MODULATION SYSTEMS

Marwa Chafii, Jacques Palicot

SUPELEC/IETR
{marwa.chafii, jacques.palicot} @supelec.fr
35576 Cesson - Sévigné Cedex, France

Rémi Gribonval

Inria Rennes - Bretagne Atlantique
remi.gribonval@inria.fr
35042 Rennes Cedex, France

ABSTRACT

The theoretical analysis of the Peak-to-Average Power Ratio (PAPR) distribution for an Orthogonal Frequency Division Multiplexing (OFDM) system, depends on the particular waveform considered in the modulation system. In this paper, we generalize this analysis by considering the Generalized Waveforms for Multi-Carrier (GWMC) modulation system based on any family of modulation functions, and we derive a general approximate expression for the Cumulative Distribution Function (CDF) of its continuous and discrete time PAPR. These equations allow us to directly find the expressions of the PAPR distribution for any particular family of modulation functions, and they can be applied to control the PAPR performance by choosing the appropriate functions.

Index Terms**—** Distribution, Peak-to-Average Power Ratio (PAPR), Orthogonal Frequency Division Multiplexing (OFDM), Generalized Waveforms for Multi-Carrier (GWMC), Multi-Carrier Modulation (MCM).

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a technique used to send information over several orthogonal carriers in a parallel manner. Compared to single-carrier modulation, this system shows a better behaviour against frequency selective channels and gives a better interference reduction. However, the OFDM signal presents large amplitude variations. Based on this fact, non-linear distortion occurs during the introduction of the signal into a non linear device, as High Power Amplifier (HPA). In order to study these high amplitude fluctuations, the Peak-to-Average Power Ratio (PAPR) has been defined. The PAPR is a random variable, as the symbols arrive randomly at the modulation input. To study this measure, several researchers have analysed the distribution law for a particular modulation waveform, such as the Fourier basis [1] [2], the Discrete Cosine Transform (DCT) [3], and the Wavelet basis [4]. Others have studied the PAPR based on the Fourier modulation basis by using different waveforms, such as the Square Root of Raised Cosine (SRRC) [5], the Isotropic Orthogonal Transform Algorithm (IOTA) [6] and other prototype filters in the PHYDYAS project [7] for the Filter Bank Multi-Carrier (FBMC) systems. For the Wavelet modulation basis, the PAPR can be studied by using several types of wavelets, such as the Daubechies wavelets, the Haar wavelet, and the Biorthogonal wavelet [8]. To the best of our knowledge, this is the first work that generalizes the previous studies by considering any family of modulation functions to derive the CDF of the PAPR. The equations are applied to check the expressions of the PAPR distribution, which have been derived by other authors for the conventional OFDM and FBMC. In addition, the PAPR distribution, which is based on the general expression is validated by means of a simulation for the Nonorthogonal FDM (NOFDM) [9].
We first propose a general definition of the continuous-time PAPR and derive its Cumulative Distribution Function (CDF) in Section 2. In Section 3, we perform the same analysis for the discrete time context. Applications of the theoretical results are illustrated in Section 4. Finally, the conclusions are presented in Section 5.

2. A CONTINUOUS-TIME ANALYSIS OF THE PAPR

We consider in this section the most general theoretical case: transmitting infinite data in a continuous way over time. We first model the Generalized Waveforms for Multi-Carrier (GWMC) signal at the modulation output in section 2.1. In section 2.2, we define the PAPR in this case and find an approximation of its distribution law in section 2.3.

2.1. System description

Information-bearing symbols, resulting from modulation performed by any type of constellation, are decomposed into several blocks. Each block of symbols is inserted in parallel into a modulation system. At the output of the modulator, the GWMC signal can be expressed as:

Definition 1. (GWMC continuous signal)

$$X(t) = \sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} C_{m,n} \underbrace{g_m(t-nT)}_{g_{m,n}(t)}, \quad (1)$$

- *T: duration of a block of input symbols,*
- *M: number of carriers assumed greater than 8 [1],*
- *$C_{m,n} = C^R_{m,n} + jC^I_{m,n}$: input symbols from a mapping technique, that take complex values,*
- *$(g_m)_{m\in[\![0,M-1]\!]}$: $\in$ $L^2(\mathbb{R})$ (the space of square integrable functions), family of functions representing the modulation system with $g_{m,n}(t) = g^R_{m,n}(t) + jg^I_{m,n}(t)$.*

In order to apply the Generalized Central Limit Theorem (G-CLT) in section 2.3, our system must satisfy the following two assumptions:

Assumption 1. (*) Independence of input symbols

- $(C^R_{m,n})_{(n\in\mathbb{Z},\, m\in[\![0,M-1]\!])}$ are independent, each with zero mean and a variance of $\frac{\sigma_c^2}{2}$,
- $(C^I_{m,n})_{(n\in\mathbb{Z},\, m\in[\![0,M-1]\!])}$ are also independent, each with zero mean and a variance of $\frac{\sigma_c^2}{2}$,
- $(\forall m,p \in [\![0,M-1]\!]), (\forall n,q \in \mathbb{Z})$ $C^R_{m,n}$ and $C^I_{p,q}$ are independent.

The symbols from constellation diagrams of the usual digital modulation schemes satisfy these conditions.

Assumption 2. (**) Lyapunov's condition:

$$\frac{\sqrt[3]{\sum_{m=0}^{M-1} E|\sum_{n\in\mathbb{Z}} C^R_{m,n}g^R_{m,n}(t) - C^I_{m,n}g^I_{m,n}(t)|^3}}{\sqrt{\frac{\sigma_c^2}{2}\sum_{m=0}^{M-1}\sum_{n\in\mathbb{Z}} |g_m(t-nT)|^2}} < \epsilon_M \ll 1,$$

$$\frac{\sqrt[3]{\sum_{m=0}^{M-1} E|\sum_{n\in\mathbb{Z}} C^R_{m,n}g^I_{m,n}(t) + C^I_{m,n}g^R_{m,n}(t)|^3}}{\sqrt{\frac{\sigma_c^2}{2}\sum_{m=0}^{M-1}\sum_{n\in\mathbb{Z}} |g_m(t-nT)|^2}} < \epsilon_M^{'} \ll 1.$$

2.2. General definition of the continuous-time PAPR

For a finite observation duration of NT, we can define the PAPR for the continuous GWMC signal expressed in Eq.(1) as follows:

Definition 2. (Continuous-time PAPR of GWMC signal for a finite observation duration)

$$PAPR_c^N = \frac{\max_{t\in[0,NT]} |X(t)|^2}{P_{c,mean}}, \quad (2)$$

$$\textit{with} \quad P^c_{mean} = \frac{\sigma_c^2}{T}\sum_{m=0}^{M-1} \|g_m\|^2, \quad (3)$$

$$\textit{and} \quad \|g_m\|^2 = \int_{-\infty}^{+\infty} |g_m(t)|^2 \quad (4)$$

[1]This is an assumption made for the validity of Central Limit Theorem (CLT) using Berry-Essen theorem

The subscript c corresponds to the continuous-time context and the exponent N is the number of GWMC symbols considered in our observation. The average power can be calculated as follows:

$$P^c_{mean} = \lim_{t_0\to+\infty} \frac{1}{2t_0}\int_{-t_0}^{t_0} E(|X(t)|^2)\, dt$$

$$\overset{(*)}{=} \lim_{t_0\to+\infty} \frac{1}{2t_0}\int_{-t_0}^{t_0} \sum_{m=0}^{M-1}\sum_{n\in\mathbb{Z}} \sigma_c^2 |g_{m,n}(t)|^2\, dt,$$

Let us put $t_0 = \frac{KT}{2}, K \in \mathbb{N}$

$$= \frac{\sigma_c^2}{T}\lim_{K\to+\infty}\frac{1}{K}\sum_{m=0}^{M-1}\int_{-\frac{KT}{2}}^{\frac{KT}{2}}\sum_{n\in\mathbb{Z}} |g_{m,n}(t)|^2\, dt$$

$$\overset{\text{(by periodicity)}}{=} \frac{\sigma_c^2}{T}\sum_{m=0}^{M-1}\int_{-\frac{T}{2}}^{\frac{T}{2}}\sum_{n\in\mathbb{Z}} |g_m(t-nT)|^2\, dt$$

$$\overset{\text{(by periodicity)}}{=} \frac{\sigma_c^2}{T}\sum_{m=0}^{M-1}\sum_{n\in\mathbb{Z}}\int_{nT-\frac{T}{2}}^{nT+\frac{T}{2}} |g_m(t-nT)|^2\, dt$$

$$= \frac{\sigma_c^2}{T}\sum_{m=0}^{M-1}\int_{-\infty}^{+\infty} |g_m(t)|^2\, dt$$

$$P^c_{mean} = \frac{\sigma_c^2}{T}\sum_{m=0}^{M-1} \|g_m\|^2.$$

2.3. Approximation of the CDF for the continuous-time PAPR

The CDF or its complementary function is usually used in the literature as a performance criterion of the PAPR. The CDF is the probability that a real-valued random variable (the PAPR here) with a given probability distribution will be found at a value less than or equal to γ, which can be expressed in our case as:

$$\Pr(\text{PAPR}_c^N \le \gamma) = \Pr[\frac{\max_{t\in[0,NT]} |X(t)|^2}{P_{c,mean}} \le \gamma]. \quad (5)$$

In what follows, we give an approximation of this expression. Let us consider a partition $(A_i)_{i\in[\![0,Q_N-1]\!]}$ of $[0,NT]$ defined as:

$$\begin{cases} [0,NT] = \bigcup_{i\in[\![0,Q_N-1]\!]} A_i \\ A_i \cap A_j = \varnothing \quad i \neq j \\ \forall i \in [\![0,Q_N-2]\!],\ A_i = [t_i - \frac{\epsilon_i}{2}, t_i + \frac{\epsilon_i}{2}[\\ A_{Q_N-1} = [t_{Q_N-1} - \frac{\epsilon_i}{2}, t_{Q_N-1} + \frac{\epsilon_i}{2}], \end{cases}$$

and let us take these following simplifying assumptions for this partition:

- ϵ_i is chosen with respect to X. For each interval of length ϵ_i, X is a quasi-stationary signal.
- $\forall i \neq j$ $\{|X(t)|^2, t\in A_i\}$ and $\{|X(s)|^2, s\in A_j\}$ are independent.

Q_N is the number of intervals A_i that we should consider for N GWMC symbols. By the density of staircase functions in L^2, we can approach the signal X by a staircase signal

X'. The latter is constant over small intervals A_i. Therefore the $\max |X(t)|^2$ over each A_i is approximately equal to the $\max |X'(t)|^2$ over the same interval. The maximum of X' is reached at all the points of A_i, in particular t_i, since it is constant over each A_i, then:

$$\begin{aligned}\Pr(\text{PAPR}_c^N \leq \gamma) &= \Pr(\max_{t\in[0,NT]} |X(t)|^2 \leq \gamma P_{c,mean}) \\ &\approx \Pr(\forall i \in [\![0, Q_N-1]\!] \max_{t\in A_i} |X^{'}(t)|^2 \leq \gamma P_{c,mean}) \\ &\approx \Pr(\forall i \in [\![0, Q_N-1]\!] |X(t_i)|^2 \leq \gamma P_{c,mean}),\end{aligned}$$

Given the simplifying assumption that samples are independent, which is true when t_i corresponds to the sampling instant at the symbol frequency, we have:

$$\Pr(\text{PAPR}_c^N \leq \gamma) \approx \prod_{i\in[\![0,Q_N-1]\!]} \Pr(|X(t_i)|^2 \leq \gamma P_{c,mean}). \quad (6)$$

We should now look for the distribution law of $|X(t_i)|^2$.
We first find the distribution of the real part $X^R(t)$ of $X(t)$ and after that, do the same for the imaginary part $X^I(t)$.
We have the random variables $X_0^R(t)$, $X_1^R(t)$, $X_2^R(t)$,. . ., $X_{M-1}^R(t)$ that are independent with zero mean and satisfy Lyapunov's condition. Thus, we can apply the G-CLT to get for large M:

$$\sum_{m=0}^{M-1} X_m^R(t) \sim \mathcal{N}(0, \underbrace{\frac{\sigma_c^2}{2} \sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} |g_{m,n}(t)|^2}_{\frac{\sigma_X^2(t)}{2}}) \quad (7)$$

$$X^R(t) \sim \mathcal{N}(0, \frac{\sigma_X^2(t)}{2}). \quad (8)$$

following the same steps we get:

$$X^I(t) \sim \mathcal{N}(0, \frac{\sigma_X^2(t)}{2}). \quad (9)$$

Thus $X(t)$ follows a complex Gaussian process with zero mean and variance $\sigma_X^2(t) = \sigma_c^2 \sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} |g_m(t-nT)|^2$. Hence:

$$|X(t)|^2 \sim \chi^2 \text{ with two degrees of freedom.} \quad (10)$$

$$\text{Denoting} \quad x(t_i) = \frac{P_{c,mean}}{\sigma_X^2(t_i)},$$

we get from Eq.(6) and Eq.(10), the following result:

Approximate PAPR distribution of continuous GWMC signal for a finite observation duration of NT

For large M, with the considered simplifying assumptions, we have:

$$\Pr(\text{PAPR}_c^N \leq \gamma) \approx \prod_{i\in[\![0,Q_N-1]\!]} [1 - e^{-x(t_i)\gamma}], \quad (11)$$

$$\text{with} \quad x(t_i) = \frac{\sum_{m=0}^{M-1} \|g_m\|^2}{T \sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} |g_m(t_i - nT)|^2}.$$

Note that the approximate distribution of the PAPR depends on the family of modulation functions $(g_m)_{m\in[\![0,M-1]\!]}$, therefore the PAPR performance can be changed by choosing the appropriate modulation system. In addition, it depends also on the parameter Q_N which is proportional to N the number of GWMC symbols considered in the observation. The approximate expression of the CDF of the PAPR will be compared to the empirical CDF in Section 4.

3. A DISCRETE-TIME ANALYSIS OF THE PAPR

In [10], A. Skrzypczak studies the discrete-time PAPR for any waveform used with Fourier exponential basis. We generalize this discrete-time study by considering any family of modulation functions. Our functions must only satisfy Lyapunov's condition, the choice is then wider. Thus, for an infinite time of transmission, we can express the GWMC discrete signal at the output of the modulator as:

Definition 3. (GWMC discrete signal)

$$X(k) = \sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} C_{m,n} \underbrace{g_m(k-nP)}_{g_{m,n}(k)}, \quad (12)$$

- *P: number of samples in period T.*

3.1. General definition of the discrete-time PAPR

For a finite observation period of NP, the PAPR of the discrete GWMC signal can be defined as follows:

Definition 4. (Discrete-time PAPR of GWMC signal for a finite observation duration)

$$\begin{aligned}\mathit{PAPR}_d^N &= \frac{\max_{k\in[\![0,NP-1]\!]} |X(k)|^2}{P_{d,mean}} \quad (13) \\ P_{d,mean} &= \frac{\sigma_c^2}{P} \sum_{m=0}^{M-1} \|g_m\|^2, \quad (14) \\ \textit{where} \quad \|g_m\|^2 &= \sum_{k=-\infty}^{+\infty} |g_m(k)|^2.\end{aligned}$$

The subscript d corresponds to the discrete-time context. In fact, the discrete mean power is defined as:

$$P_{d,mean} = \lim_{K\to+\infty} \frac{1}{2K+1} \sum_{k=-K}^{K} E(|X(k)|^2). \quad (15)$$

C. Siclet has derived in his thesis [11], the mean power of a discrete BFDM/QAM (Biorthogonal Frequency Division Multiplexing) signal that is expressed as: $X[k] = \sum_{m=0}^{M-1} \sum_{n=-\infty}^{+\infty} f_m[k-nP]$, such that $f_m[k]$ is an analysis filter. His derivation does not use the exponential property of $f_m[k]$. Then, in our case, we can follow the same method to get Eq.(14).

3.2. General approximation of the CDF of the discrete-time PAPR

By considering an observation duration limited to N GWMC symbols of P samples each, and by approximating the samples $X(0)$, $X(1)$, $X(2)$,..., $X(NP-1)$ as being independent, the CDF of the PAPR for the GWMC discrete signal defined in Eq.(12), can be expressed as follows:

$$\begin{aligned}\Pr(\mathrm{PAPR}_d^N \leq \gamma) &= \Pr[\max_{k\in[\![0,NP-1]\!]} |X(k)|^2 \leq \gamma P_{d,mean}] \\ &= \Pr(\forall k \in [\![0, NP-1]\!]\ |X(k)|^2 \leq \gamma P_{d,mean}) \\ &\approx \prod_{k\in[\![0,NP-1]\!]} \Pr(|X(k)|^2 \leq \gamma P_{d,mean}). \quad (16)\end{aligned}$$

In order to look for the distribution of $|X(k)|^2$, we can proceed in the same way as the continuous time context, then we get:

$$\begin{aligned}\Pr(|X(k)|^2 \leq z) &\approx 1 - \mathrm{e}^{-\frac{z}{\sigma_X^2(k)}}, \quad (17)\\ \text{with} \quad \sigma_X^2(k) &= \sigma_c^2 \sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} |g_{m,n}(k)|^2.\end{aligned}$$

$$\text{Denoting} \quad x(k) = \frac{P_{d,mean}}{\sigma_X^2(k)},$$

and from Eq.(16) and Eq.(17), we obtain:

PAPR distribution of discrete-time GWMC signal for a finite observation duration

For large M, with the considered simplifying assumptions, we have:

$$\begin{aligned}\Pr(\mathrm{PAPR}_d^N \leq \gamma) &\approx \prod_{k\in[\![0,NP-1]\!]} [1 - \mathrm{e}^{-x(k)\gamma}], \quad (18)\\ \text{with} \quad x(k) &= \frac{\sum_{m=0}^{M-1} \|g_m\|^2}{P\sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} |g_m(k-nP)|^2}.\end{aligned}$$

Note that the expression of the approximate PAPR distribution in the discrete case is similar to its expression in the continuous case, with all parameters explicitly defined. Thus, taking into account the simplifying assumption of our derivations, we can easily study the PAPR performance of any Multi-Carrier Modulation (MCM) system.

4. APPLICATIONS

The closed-form approximations derived in this paper give us the possibility to find the CDF of the PAPR of any modulation waveform that satisfies our assumptions. We illustrate this fact by the following examples:

4.1. Conventional OFDM

We want to check the expression derived by Van Nee in [1] for the conventional OFDM for the discrete case. Then let us consider:

- Fourier basis is used for the modulation, with a rectangular waveform: $g_m(k) = \mathrm{e}^{\frac{j2\pi mk}{P}} \Pi_{[0,P]}(k)$, with $\Pi_{[0,P]}(k) = \begin{cases} 1 & \text{if } 0 \leq k \leq P \\ 0 & \text{else} \end{cases}$,
- The observation is limited to one OFDM symbol and the number of samples considered for this symbol is M.

The expression of the GWMC signal in Eq.(12) becomes $X(k) = \sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} C_m \mathrm{e}^{\frac{j2\pi m(k-nP)}{P}} \Pi_{[0,P]}(k-nP)$. We apply Eq.(18) of the PAPR distribution with $N=1$. We get:

$$x(k) = \frac{MP}{P\sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} |\mathrm{e}^{\frac{j2\pi m(k-nP)}{P}} \Pi_{[0,P]}(k-nP)|^2} = 1,$$

$$\text{and then,} \quad \Pr(\mathrm{PAPR}_d^1 \leq \gamma) = [1 - \mathrm{e}^{-\gamma}]^M. \quad (19)$$

It is similar to the expression derived by Van Nee for the discrete case.

4.2. FBMC systems

We consider the OFDM/OQAM as being an example of the FBMC system, and we check the expression derived by A.Skrzypczak [12]. Then let us consider:

- The modulation functions are:
$$g_m(k-nP) = h_{OQAM}(k-nP)\mathrm{e}^{j2\pi\frac{m}{M}(k-\frac{D}{2})}\mathrm{e}^{j\theta_{m,n}},$$
$$\text{then} \quad |g_m(k-nP)|^2 = h_{OQAM}^2(k-nP),$$
h_{OQAM} is the prototype filter (IOTA, SRRC, PHYDYAS..),
- The observation is limited to one block of symbols and the number of samples considered in this block is M, then $P = M$,
- $\|g_m\|^2 = 1$.

The expression of the GWMC signal in Eq.(12) becomes $X(k) = \sum_{n\in\mathbb{Z}} \sum_{m=0}^{M-1} C_{m,n} h_{\mathrm{OQAM}}(k-nP)\mathrm{e}^{j2\pi\frac{m}{M}(k-\frac{D}{2})}\mathrm{e}^{j\theta_{m,n}}$. We apply Eq.(18) of the discrete-time PAPR for a finite observation duration with $N=1$. Then we get:

$$x(k) = \frac{1}{M\sum_{n\in\mathbb{Z}} h_{OQAM}^2(k-nP)},$$

and then:

$$\Pr(\mathrm{PAPR}_d^1 \leq \gamma) = \prod_{k=0}^{M-1} [1 - \mathrm{e}^{\overline{M\sum_{n\in\mathbb{Z}} h_{OQAM}^2(k-nP)}^{-\gamma}}]. \quad (20)$$

This result is similar to the one obtained by A. Skrzypczak.

4.3. NOFDM

For NOFDM system, considering Hamming window, the waveform is expressed as:

$$g_m(t) = e^{\frac{j2\pi mt}{T}} w(t),$$
$$w(t) = \begin{cases} 0.54 - 0.46\cos(2\pi \frac{t}{T}) & \text{if } 0 \leq t \leq T \\ 0 & \text{else.} \end{cases}$$

Fig.1 shows the theoretical CCDF of the PAPR based on Eq.(18) with $P = M$ and $N = 1$, and the experimental CCDF simulated by generating 10000 realizations of NOFDM symbols, for different number of carriers. We observe that the

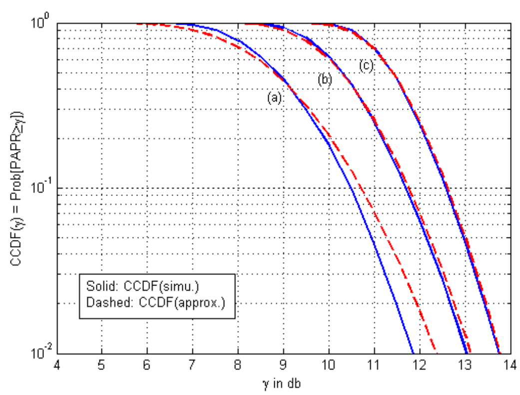

Fig. 1. Experimental and Theoretical CCDF of the PAPR for NOFDM system using Hamming window for different number of carriers: (a) $M = 64$, (b) $M = 256$, (c) $M = 1024$.

larger the number of carriers gets, the more accurate the theoretical curve gets. And this is due to the fact that our derivation are based on the G-CLT which assumes a large number of carriers.

5. CONCLUSION

In this paper, we make several derivations to achieve the more general CDF approximation of the PAPR in the sense that transmission symbols are carried by any functions. The analysis is performed in both continuous and discrete time. To illustrate the theoretical results, we express the PAPR distribution of different multi-carrier systems using the general equation.
The future study is to establish an optimization problem to find the optimal family of modulation functions that maximize the CDF of the PAPR. The constraints of this problem vary each time the application requirements vary, giving each time a new optimization problem and therefore a new family of modulation functions as an optimal solution, which can lead to a new system of Multi-Carrier Modulation.

6. ACKNOWLEDGEMENTS

This work has received a French state support granted to the Comin-Labs excellence laboratory and managed by the National Research Agency in the "Investing for the Future" program under reference Nb. ANR-10-LABX-07-01. The authors would also like to thank the Region Bretagne, France, for its support of this work.

7. REFERENCES

[1] R. V. Nee and R. Prasad, *OFDM for wireless multimedia communications*, Artech House, Inc., 2000.

[2] Yves Louët and Sajjad Hussain, "Peak-to-mean envelope power ratio statistical analysis of continuous ofdm signal," in *Vehicular Technology Conference, 2008. VTC Spring 2008. IEEE*. IEEE, 2008, pp. 1681–1685.

[3] Lalchandra Patidar and Ashish Parikh, "BER Comparison of DCT-based OFDM and FFT-based OFDM using BPSK Modulation over AWGN and Multipath Rayleigh Fading Channel," *International Journal of Computer Applications*, vol. 31, no. 10, 2011.

[4] M. Hoch, S. Heinrichs, and J. B Huber, "Peak-to-average power ratio and its reduction in wavelet-OFDM," in *International OFDM Workshop*, 2011, pp. 56–60.

[5] P. Chevillat and G. UngerBoeck, "Optimum FIR transmitter and receiver filters for data transmission over band-limited channels," *Communications, IEEE Transactions on*, vol. 30, no. 8, pp. 1909–1915, 1982.

[6] B. Le Floch, M. Alard, and C. Berrou, "Coded orthogonal frequency division multiplex," *Proceedings of the IEEE*, vol. 83, no. 6, pp. 24–182, 1995.

[7] Ari Viholainen, Maurice Bellanger, and Mathieu Huchard, "Prototype filter and structure optimization," *website: www. ict-phydyas. org: Document D5. 1 deliverable*, 2009.

[8] Sourabh Mahajan, Parveen Kumar, and Anita Suman, "Analysis of ofdm and owdm system with various wavelets families," *IJERGS*, vol. 2, April-May 2014.

[9] Hanna Bogucka, Alexander M Wyglinski, Srikanth Pagadarai, and Adrian Kliks, "Spectrally agile multicarrier waveforms for opportunistic wireless access," *Communications Magazine, IEEE*, vol. 49, no. 6, pp. 108–115, 2011.

[10] A. Skrzypczak, P. Siohan, and J.P. Javaudin, "Peak-to-Average Power Ratio Issues for Pulse-Shaped Multicarrier Modulations," in *Advances on Processing for Multiple Carrier Schemes: OFDM and OFDMA*, F. Bader and N. Zorba, Eds., chapter 2, pp. 43–90. Nova Science Publishers, ECAM Rennes - Louis de Broglie, France and others, 2012.

[11] C. Siclet, *Application de la théorie des bancs de filtres à l'analyse et à la conception de modulations multiporteuses orthogonales et biorthogonales*, Ph.D. thesis, Rennes I University, France, 2002.

[12] A. Skrzypczak, *Contribution à l'étude des modulations multiporteuses OFDM/OQAM et OFDM suréchantillonnées*, Ph.D. thesis, Rennes I University, France, 2007.