Iterative Grid Search for RSS-Based Emitter Localization

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***Abstract*—In this paper, we present a reduced complexity iterative grid-search technique for locating non-cooperating primary emitters in cognitive radio networks using received signal strength (RSS) measurements. The technique is based on dividing the search space into a smaller number of candidate subregions, selecting the best candidate that minimizes a cost function and repeating the process iteratively over the selections. We evaluate the performance of the proposed algorithm in independent shadowing scenarios and show that the performance closely approaches to that of the full search, particularly at small shadowing spread values with significantly reduced computational complexity. We also look at the performance of our algorithm when the initial search space is specified based on two different data-aided approaches using sensor measurements. Our simulation results show that the data-aided initialization schemes do not provide performance improvement over blind initialization.**

I. Introduction

Flexible radio technologies that enable opportunistic access to unused spectrum have been a focal point of recent research in addressing spectrum shortage problem. Opportunistic radio systems are expected to adhere to spectrum regulations in the region they are deployed and they should avoid harmful interference to primary spectrum holders in their exclusive region. Primary exclusive region is the area within which opportunistic users are not allowed to transmit [1]. Predicting primary exclusive region of a primary emitter requires the knowledge of primary emitter's location and its transmit power, which in most cases are not readily available due to non-cooperative nature of primary networks.

Emitter localization problem in general has been considered extensively in the literature; see for example [2] for an overview of localization techniques. Localization may be accomplished via many techniques, such as received signal strength (RSS), time of arrival (TOA), time difference of arrival (TDOA) and angle of arrival (AOA). Although TOA and TDOA are generally more accurate, RSS based techniques are often of interest as they require simpler hardware. The challenge of RSS based localization is due to numerous factors affecting the energy decay between the transmitter and emitter such as shadowing, multipath, path loss exponent estimation errors, geometric configuration of the nodes and antenna orientation. Despite having several sources of error, RSS based techniques are expected to perform satisfactorily when a large number of spatially separated sensors are employed.

The literature on RSS based emitter localization has been on developing efficient algorithms for accurate location estimation. Among the others, maximum-likelihood estimation (MLE) method offers an attractive approach for the localization problems since it is asymptotically efficient, unbiased and it does not require prior information [3],[4]. In fact, it has been shown that MLE method achieves the Cramer-Rao lower bound (CRLB) at small shadowing variances [3]. The maximum likelihood estimator of the emitter location requires the minimization of a non-convex cost function. Since this cost function exhibits numerous local minima, its global minimization is usually realized by means of numerical approaches. One approach is to employ grid search where the algorithm scans all possible grid points in the localization space. The grid point that maximizes the likelihood is selected as the location of the emitter. In a grid-search algorithm, the size of the grid elements must be chosen small to obtain more accurate location estimates. However, smaller grid size increases the computational complexity.

The need for reduced complexity grid search techniques arises in several fields. For example, a variable-mesh, derivative-free optimization algorithm, namely contracting-grid search method, is used to derive interaction locations in compact gamma cameras in [5]. In [6], the location of a sound source in a distributed sensor network is estimated using a grid-based multi-resolution search to reduce the complexity of an exhaustive maximum likelihood estimator and a smarter multi-resolution search is proposed based on searching around the highest energy reading sensor. [7] proposes a low-complexity positioning procedure that simply searches for the global minimum around the sensor exhibiting the smallest local maximum of the cost function and it is shown that it outperforms the naive approach that searches for the global minimum around the sensor reporting the largest signal strength. In [8], a tree search algorithm (TSA) is used to reduce the computational complexity of grid search algorithm in sensor networks assuming that the power of the transmitter to be located is known and it is shown that the performance of the TSA algorithm closely achieves the performance of least squares estimator with significantly reduced computational complexity.

In this paper, we propose a reduced complexity iterative grid-search algorithm for locating primary emitters with unknown power in cognitive radio networks. The proposed technique is based on refining search space based on the minimization of a cost function. We show that a significant complexity reduction is achieved by the proposed method at the cost of a small performance loss. The performance of this iterative grid-search method is also studied under different grid-spacing values and/or with different number of iterations and the limits of the proposed solution is also determined.

The organization of the paper is as follows: In Section II, we introduce the network and signal model and assumptions used in this work. In section III, we provide the theoretical background of ML approach for emitter localization and introduce full and reduced complexity grid search techniques. We present simulation results in Section IV and conclude the paper in Section V.

II. Assumptions

A. Network model

We consider a cognitive radio network model consisting of a number of secondary users (nodes) deployed at known but arbitrary locations in a given geographical area. These radio nodes monitor the received power level due to a primary transmitter whose location and power is not known to the secondary nodes. We assume that only a single primary emitter is active at any given time. Each node is equipped with an omni-directional antenna and the nodes report their location information and the received signal strength (RSS) measurements due to the primary transmission to a fusion center through a common channel. We assume that there is no information loss in the delivery of sensor measurements to the fusion centre. The fusion center processes RSS measurements to estimate the location of the primary emitter.

B. Signal model

We assume that one primary user is active at any given time and the objective is to estimate its location. Transmission from the primary emitter to the sensors is assumed to be omni-directional and the signal propagation is governed by a log-distance path loss model such that the noise-free mean received power (in dBm) at the ith sensor is given by

$$m_i = 10\log_{10} P_T - 10\rho\log_{10} d_i \tag{1}$$

where $i = 1, 2, ..., N_s$ is the sensor number, N_s is the number of sensors, P_T is the transmit power of the primary emitter, ρ is the path-loss exponent, and d_i is the distance between the transmitter and the ith sensor, $d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2}$, and (x_i, y_i) and (x_0, y_0) are ith sensor's and the emitter's locations, respectively.

We assume that each sensor experiences log-normal shadowing. If the fast fading effects are sufficiently averaged over time then the resulting unknown measured power from the emitter to the ith sensor is given by

$$r_i = m_i + w_i \tag{2}$$

where $w \sim N(0, \sigma^2)$ is the gain/loss in dB due to shadowing and σ is called the shadowing spread. The received power r_i at a distance d_i from the emitter is then a normal random variable with mean m_i and variance σ^2 and its pdf is given by:

$$p(r_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(r_i - m_i)^2}{2\sigma^2}} \tag{3}$$

III. Emitter location estimation

Assuming that the received signal strength values are independently distributed and each having a log-normal shadowing, the conditional probability of observing all N_s sensor outputs given θ is written as

$$p(r|\theta) = \prod_{i=1}^{N_s} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(r_i - m_i)^2}{2\sigma^2}} \tag{4}$$

where θ=(x_0, y_0) is the emitter location parameter to be estimated. An estimate of the emitter location can be obtained by minimizing a cost function which requires calculating the sum of squared differences between each sensor's emitter power estimate and average of all sensor estimates [9]:

$$\hat{\theta} = \min_{x,y} \sum_{i=1}^{N_s} \Big(\log\left(p_i d_{T,i}^{\rho}\right) - \frac{1}{N_s} \sum_{j=1}^{N_s} \log\left(p_j d_{T,j}^{\rho}\right) \Big)^2 \tag{5}$$

where $p_i = 10^{\frac{r_i}{10}}$ and $d_{T,i}$ is the distance between ith sensor and the test location T. A closed-form solution of this minimization problem is intractable. A numerical solution of $\hat{\theta}$ can be obtained by grid search algorithms. In a grid search algorithm, the solution space is divided into a number of grids and a cost function is calculated for each grid center. The center of the grid that minimizes the cost function is selected as the solution.

A. Full grid search

Assuming that the region of interest (ROI) within which the primary emitter and the sensors lie in a square of length L, and the ROI is divided into smaller squares of search elements (grids) of length K, the total number of elements needed for an exhaustive search is $(L/K)^2$. A sample configuration is illustrated in Fig. 1 to show all 100 possible grid elements in a scenario for which L/K ratio is 10. In this figure, 5 sensor locations are shown with circles whereas the actual emitter location is marked with a diamond shape. The value of the cost function calculated at each grid location are also shown in the figure. As seen from the figure, the minimum value of the cost function which is 3 for this example, is obtained around the actual emitter location. The estimated emitter location is shown with a square at the center of the grid for which the cost is minimum.

A full grid search algorithm may result in an irreducible error floor for given non-zero grid sizes due to quantization even when there is no noise in the sensor measurements. The amount of the quantization noise is limited by the grid size.

B. Iterative grid search

A full search algorithm is brute-force therefore it does not use any scheme to reduce the number of computations.

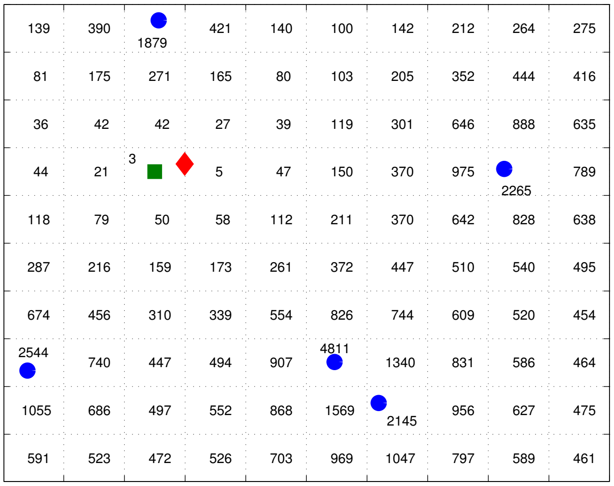

Fig. 1. An example of full grid search containing 100 grid elements.

However, in an emitter localization problem many cost function computations may not actually be required. Iterative grid search algorithm can be initialized in different configurations based on the initial selection of the search partitions. For example, for a 2x2 partitioning, ROI is divided into four quadrants and initial candidate locations are set at the centres of each quadrant. After calculating the cost function at each candidate location, search space is reduced to one fourth of the previous search area and it is re-centred at the location that provides the minimum of the calculated cost values. The iterations are run in this way for a given number of levels to achieve desired grid resolution. Final grid size at the end of the last level is given by $\frac{L}{Q^{M/2}}$ where Q is the number of partitions and M is the number of levels.

An example of 2x2 partitioning ($Q = 4$) is illustrated in Fig. 2 for a 3 level iteration. In this example, the same sensor and emitter placement of Fig. 1 is used. As seen from the figure, the minimum cost values of 42, 19 and 1 are achieved at successive iterations. At the end of the third iteration, the emitter location is estimated to be the centre of the grid that results in minimum cost value.

Similarly higher number of partitions can be obtained by dividing the ROI into smaller areas. Increased partitioning reduces the grid resolution, however it results in increased number of computations as well. The total number of required cost function calculations for the proposed iterative search algorithm is QxM.

The proposed algorithm is summarized below:

Algorithm 1 Iterative grid search

`Initialize the center of the search space`
$x_c = 0$, $y_c = 0$
repeat
`Place candidate locations`
`Evaluate the cost function`
`Select min of cost function`
`Set next candidates`
$m \leftarrow m + 1$
until $m = M$

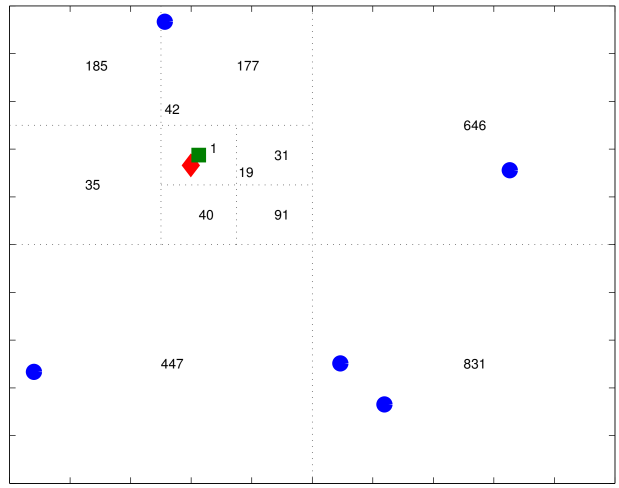

Fig. 2. An example of 3 level 2x2 partitioning for iterative grid search.

IV. Simulations

We ran Monte Carlo trials to simulate sensor measurements based on the described scenarios. In all simulations, one primary emitter and a number of sensors were placed randomly in a 1km by 1km square area. We assumed independent log-normal shadowing of given dB spread and log-distance path loss model with a fixed path-loss exponent of 3.5. The performance results are presented in terms of mean of the location estimation error, μ, which is defined by

$$\mu = \frac{1}{S}\sum_{i=1}^{S}\sqrt{(x_0 - \hat{x}_{i0})^2 + (y_0 - \hat{y}_{i0})^2} \tag{6}$$

where (x_0, y_0) is the actual emitter location, $(\hat{x}_{i0}, \hat{y}_{i0})$ is the estimated emitter location in ith simulation and S is the number of Monte Carlo simulations.

A. Full search

In order to evaluate the performance of the full search method with different grid sizes, we simulated random networks with 50 nodes (sensors) and ran 1000 simulations at each dB spread values ranging from 0 to 12dB for the grid-size values of 10, 20, 25, 40 and 50m. The mean localization error is shown in Fig. 3. As seen from this figure, the grid search method performs better with decreased grid size. However, the number of cost function calculations increases significantly with reduced grid size. For grid sizes of 10, 20, 25, 40 and 50m, the full search requires 10000, 2500, 1600, 625 and 400 cost functions calculations, respectively.

B. Iterative search

We ran 1000 Monte Carlo simulations for 4 and 9 levels of 2x2 and 4x4 partitions. We evaluated the performance of the algorithm at different shadow spread values ranging from 0 to 12dB. The results are shown in Fig. 4. Also shown in the figure

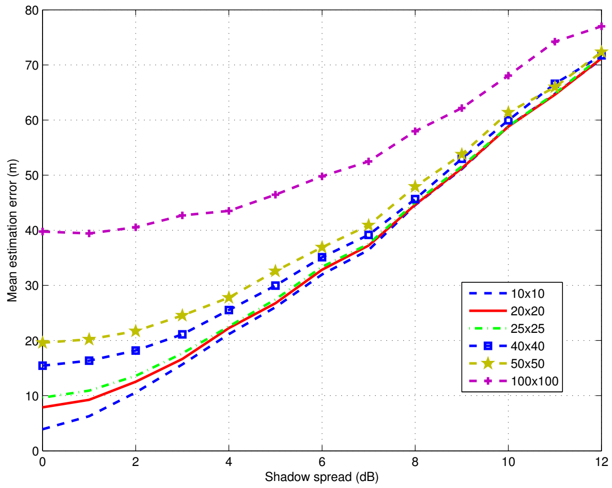

Fig. 3. Performance of full grid search using different grid sizes

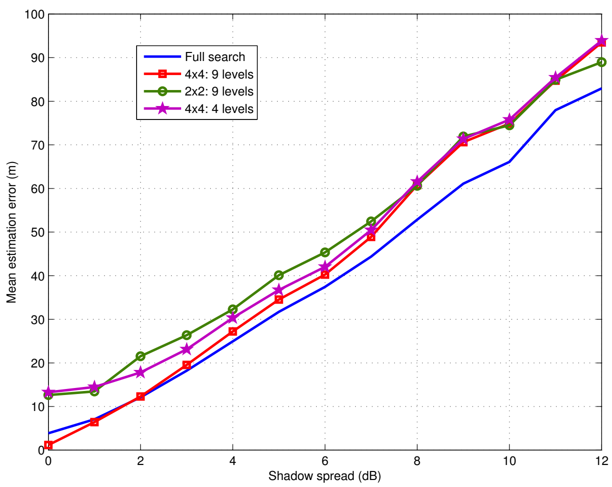

Fig. 4. Performance comparison of full and iterative grid search at various partitioning and iteration levels versus shadowing spread

is the performance of full grid search with a grid size of 10m. As seen from this figure, the performance of the iterative grid search improves with an increase in the number of partitions and levels when the dB spread is less than 6dB. However, when the dB spread is higher than about 6dB, increasing the number of levels and/or the number of partitions does not improve the performance. For lower dB spread values the performance of 9 level 4x4 iterative grid search is very close to that of full search. The full search requires 10000 calculations whereas 9 level 4x4 iterative search requires only 144 calculations.

We also evaluated the performance of the algorithm with different number of sensors ranging from 20 to 80 at a shadow spread of 6 dB and the results are shown in Fig.5. As seen from the figure, the gap between full and iterative grid searches decreases as the number of sensors increases.

C. Search space initialization

Our regular iterative algorithm sets a search space centred at the origin of the region of interest and it does not rely

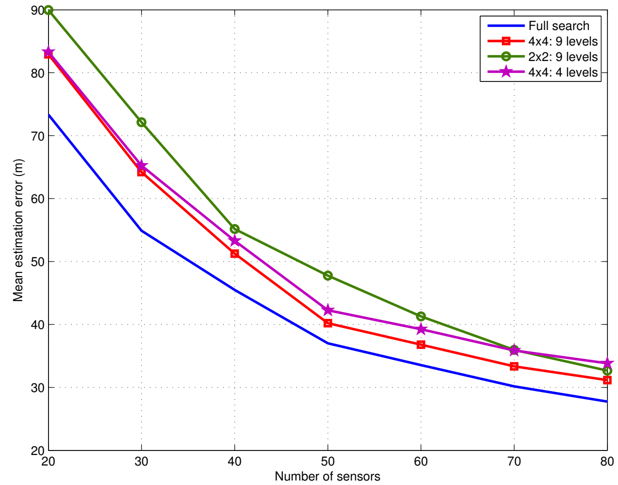

Fig. 5. Performance comparison of full and iterative grid search at various partitioning and iteration levels versus number of sensors

on any sensor measurements or sensor measurement related metric in setting the initial search space. In this section, we look at the performance of the iterative algorithm when the initial search space is set based on two different approaches. The first approach sets the search space around the sensor that reports the highest RSS (the maximum approach). The rational is that the sensor reporting the maximum RSS is more likely to be closer to the emitter than the rest of sensors. Even though this is true when the shadow spread is low, the condition may not be satisfied when the closest sensor is blocked by obstacles (hidden node problem). This approach is used in [6] to locate sound sources. In the second approach, the cost metric in (5) is calculated at a very close neighbourhood of each sensor location. This metric generates local maxima at the sensor neighbourhood and the sensor location that results in the minimum of these maxima is set as the centre of the search space (the minimax approach). It is reported in [7] that this approach outperforms the naive approach that searches around the receiver reporting the largest RSS in emitter localization in shadow fading.

We compared the performances of search space initialization schemes at different shadow spread values ranging from 0 to 12dB for 9 level 2x2 partitions. At each dB spread value, we have performed 1000 Monte Carlo simulations and mean estimation errors are plotted in Fig. 6. In this figure, the maximum approach sets the search space around the sensor that reports the maximum RSS whereas the minimax approach sets the search space around the sensor that results in the minimum of maximum cost values. Both techniques use a search space of $L/2$ by $L/2$ whereas our proposed regular iterative approach performs the search inside L by L square centred at the origin. As seen from this figure, the minimax approach performs better than the maximum approach; however the regular approach outperforms both the minimax and the maximum approaches at higher dB spread values. Please note that the minimax approach requires N_s additional cost function calculations compared to the maximum and the regular approaches.

In order to evaluate the effect of initial search space size

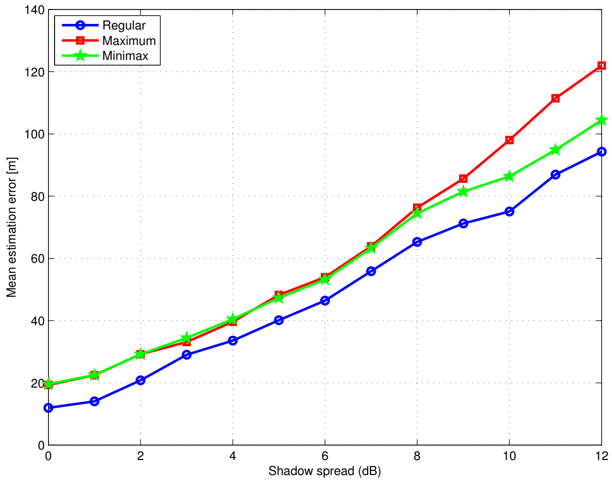

Fig. 6. Performance comparison of regular, maximum and minimax approaches versus shadow spread

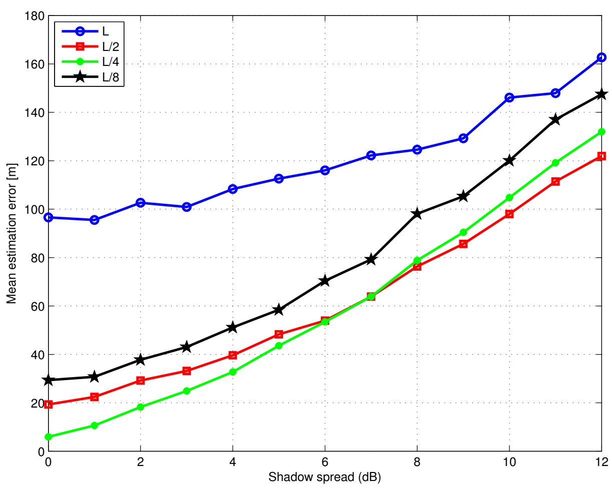

Fig. 7. Performance of maximum approach for different search space sizes

on the performance of maximum and minimax approaches, we have performed simulations using different search space sizes of L, $L/4$, and $L/8$ in addition to $L/2$. The performances of the maximum and the minimax techniques with different search space sizes are shown in Figs. 7 and 8, respectively. As seen from these figures, setting the initial search space too large or too small deteriorates the performances significantly. Comparing these figures to Fig. 6 reveals that the regular approach outperforms the two techniques irrespective of the initial search space size.

V. Conclusions

In this paper, we presented a reduced complexity iterative grid-search algorithm for RSS-based localization of primary emitter in cognitive radio networks. The performance of the proposed method closely approached to the performance of full grid search, particularly at small shadowing spread values with significantly reduced computational complexity. Because the grid search has coarser resolution at earlier stages, it is possible that the global minimum of the cost function is

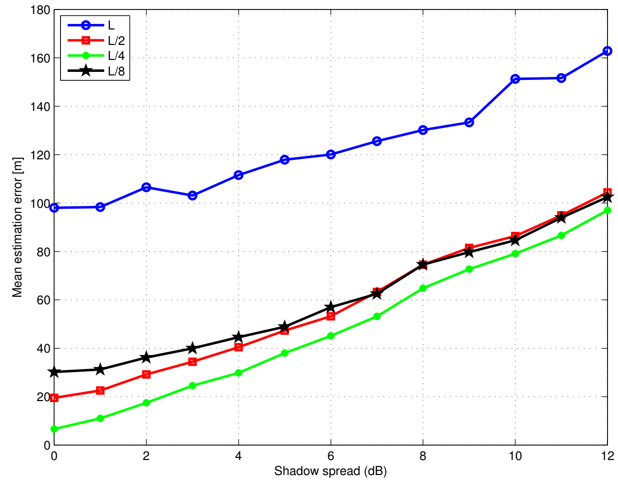

Fig. 8. Performance of minimax approach for different search space sizes

missed, especially when the uncertainty is high due to larger shadowing spreads. We also show that restricting the search space based on the maximum and minimax criteria do not improve the performance of the iterative search algorithm. Future work will include devising smarter algorithms to close the gap between the full search and the lower complexity grid search at higher dB spread values. Future work will also cover extending the reduced complexity grid search technique to multiple emitter localization problem where the size of the search space grows exponentially.

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