GREEDY ORTHOGONAL MATCHING PURSUIT FOR SPARSE TARGET DETECTION AND COUNTING IN WSN

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ABSTRACT

The recently emerged Compressed Sensing (CS) theory has widely addressed the problem of sparse targets detection in Wireless Sensor Networks (WSN) in the aim of reducing the deployment cost and energy consumption. In this paper, we apply CS approach for both sparse events recovery and counting. We first propose a novel Greedy version of the Orthogonal Matching Pursuit (GOMP) algorithm allowing to account for the decomposition matrix non orthogonality. Then, in order to reduce the GOMP computational load, we propose a two-stages version of GOMP, the 2S-GOMP, which separates the events detection and counting steps. Simulation results show that the proposed algorithms achieve a better tradeoff between performance and computational load when compared to the recently proposed GMP algorithm and its two stages version denoted 2S-GMP.

***Index Terms*—** Wireless sensor network, rare events detection, Compressed Sensing.

1. INTRODUCTION

Compressed Sensing (CS) theory has recently become widely popular to improve system efficiency in several fields. It has interesting applications in image processing [1], signal detection [2], channel estimation [3], etc. The CS indeed provides a theoretical framework allowing, under some conditions, to recover signals which have sparse representations in some basis from a reduced number of measurements [4, 5]. With the emergence of CS theory, we have seen a new avenue of research in the field of Wireless Sensor Networks (WSN) for events detection [6,7]. CS application in WSN is based on the sparse nature of events occurrence within the monitored area. In this context, CS allows not only to enhance the events detection and counting performance in WSN but also to reduce the deployment cost and energy consumption. Several works considered the problem of events detection in WSN under binary detection model [8] in which a sensor reports value '1' if one or more targets are detected in its sensed area and '0' otherwise. To detect the sparse events locations, a complex bayesian approach is proposed in [8]. However, counting the number of point events or targets is very interesting to many sensors network applications like for target tracking [9, 10] as animals in forests and robots in industry.

In this paper, we consider the application of the CS theory for sparse targets detection and counting in WSN. In this framework, the monitored area is partitioned into cells, each equipped with one sensor. The goal is to position the cells where events occur and to count the number of events per cell. CS theory allows, under the assumption of rare events, to detect and count them from just a few number of sensors (measurements). Then, in order to reduce the energy consumption, only a subset of sensors are activated, each of which delivers one measurement. In [11], counting and positioning events have been investigated where a novel Greedy Matching Pursuit (GMP) algorithm is proposed which has the advantage not to require any knowledge about the signal sparsity level. In this contribution, we propose a novel Greedy Orthogonal Matching Pursuit (GOMP) algorithm that outperforms the GMP algorithm in both detection and counting performance. Like GMP, the GOMP detects in each iteration, the cell which contributes the most to the residual observation. The proposed enhancement is basically based on accounting for the non orthogonality of the decomposition basis. Further, a two-stages reduced complexity version 2S-GOMP of GOMP is proposed which separates the detection and counting stages to reduce complexity.

The remainder of this paper is organized as follows. After formulating the problem of events detection and counting in WSN in section 2, the proposed GOMP algorithm and its modified version 2S-GOMP are developed in section 3. Then, the performance evaluation through numerical results is given in section 4 before the conclusion.

2. PROBLEM FORMULATION

We consider a wireless sensor network deployed in the monitored area for counting and localizing potential events. Some sources may be present and may generate point events. The aim is here to detect (localize) the events and to count their number per cell. For this purpose, a grid partitions the monitored area into N regular cells. Let S_i, denote the number of events in the cell i which is an integer. Denoting by m an upper bound on the possible number of events a cell can hold, $S_i \in \{0, 1, \ldots, m\}$ for $i = 1, \ldots, N$. We consider the case of rare events, that is events occurring in only a small portion K of the N cells. Let $\mathbf{s} = [S_1, S_2, \ldots, S_N]^T$ denote the N elements vector containing the events number at each cell. Then, events rareness assumption implies that $\mathbf{s}$ is K-sparse with $K \ll N$ not zero valued elements corresponding to cells holding events. Each sensor j receives the signals generated by targets, affected by both the path loss and the Rayleigh fading. The sensed data energy measured by sensor j, supposing that signals from different targets are uncorrelated, is expressed as [11]

$$\mathbf{x}_j = \sum_{i=1}^{N} P_0 \frac{|h_{ij}|^2 S_i}{d_{ij}^{\alpha}} + \mathbf{n}_j, \tag{1}$$

where P_0 is the transmitted signal energy, supposed the same for all targets. $|h_{ij}|^2$ traduces the Rayleigh fading effect where h_{ij} is modeled as complex Gaussian noise with zero mean and unit variance. d_{ij} is the distance between the target at location i and the sensing device at cell j and α is the path loss exponent, $\mathbf{n}_j$ refers to the noise component energy where the noise follows a gaussian and complex distribution with variance σ^2.

Concatenating $\mathbf{x}_j$ for $j = 1, \ldots, N$ leads to the N elements vector

$$\mathbf{x} = \Phi \mathbf{s} + \mathbf{n}, \tag{2}$$

where Φ is a $N \times N$ target energy decay matrix given by

$$\Phi = P_0 \begin{pmatrix} \frac{|h_{11}|^2}{d_{11}^{\alpha}} & \frac{|h_{21}|^2}{d_{21}^{\alpha}} & \cdots & \frac{|h_{N1}|^2}{d_{N1}^{\alpha}} \\ \frac{|h_{12}|^2}{d_{12}^{\alpha}} & \frac{|h_{22}|^2}{d_{22}^{\alpha}} & \cdots & \frac{|h_{N2}|^2}{d_{N2}^{\alpha}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{|h_{1N}|^2}{d_{1N}^{\alpha}} & \frac{|h_{2N}|^2}{d_{2N}^{\alpha}} & \cdots & \frac{|h_{NN}|^2}{d_{NN}^{\alpha}} \end{pmatrix}.$$

Due to energy and deployment cost limitations, the number of activated sensors should be very limited.

Our goal is to recover the sparse signal $\mathbf{s}$ from a small subset $M \ll N$ of the measurements contained in $\mathbf{x}$. In our framework, only M among the N sensors are randomly activated (or deployed). Let $\mathbf{y}$ denote the $M \times 1$ vector recording the M measurements of the selected sensors. Then, we have

$$\begin{aligned} \mathbf{y} &= \Psi \Phi \mathbf{s} + \check{\mathbf{n}}, & (3) \\ &= \mathbf{A}\mathbf{s} + \check{\mathbf{n}}, & (4) \end{aligned}$$

where Ψ is an $M \times N$ selection submatrix, $\check{\mathbf{n}} = \Psi \mathbf{n}$ is a subvector of $\mathbf{n}$ and $\mathbf{A} = \Psi \Phi$.

3. PROPOSED GREEDY ORTHOGONAL MATCHING PURSUIT (GOMP) ALGORITHM

We here propose a novel Greedy Orthogonal Matching Pursuit (GOMP) algorithm for sparse targets counting and localization in WSN. In [11], the proposed iterative scheme GMP jointly detects a new active (with events) cell and counts the number of events in the detected cell in each iteration. Then, the residual observation is updated accordingly. Respecting this algorithm and accounting for the sparse vector to be with integer entries, the herein proposed algorithm is also greedy. However, observing that the decomposition basis $\mathbf{A}$ is not orthogonal, we here propose to update the iterative projection procedure according to the Orthogonal Matching Pursuit (OMP) algorithm [12].

In this way, the number of events values in the cells already detected as active (entries of $\mathbf{s}$) are updated after each new active cell detection, thus following the OMP algorithm principle [12]. We hereafter provide the details of GOMP algorithm application for target counting and detection.

3.1. GOMP Principle

- **Input:**
  An $M \times N$ measurement matrix $\mathbf{A}$.
  An $M-$ dimensional signal measurement vector $\mathbf{y}$.

- **Output:**
  An $N-$ dimensional reconstructed signal $\hat{\mathbf{s}}$ with integer entries.
- **Procedure:**
  1) Initialization: $\Omega = \{1, 2, \ldots, N\}$, $D^{(0)} = \emptyset$.
  At the i^{th} iteration
  2) $D^{(i-1)}$ presents the detected positions at the $i - 1^{th}$ iteration.
  Search all combinations (possibilities) $C^{(i)}$ of i values (numbers of events) taken from the set $\{1, 2, \ldots, m\}$. Note that $i - 1$ positions are known ($D^{(i-1)}$) and the i^{th} position is to be determined within Ω. We also incorporate the case where the new detected position is zero valued (no more events to detect). The number of tested combinations is then $Card(C^{(i)}) = (N - i + 1)(m^{(i-1)}(m + 1))$.
  3) New active cell detection step

$$g_i, c_{opt} = arg \min_{j \in \Omega, c \in C^{(i)}} \|\mathbf{y} - \mathbf{A}\mathbf{z}_i^c\|_2^2, \tag{5}$$

  where $\mathbf{z}_i^c$ is a vector with non zero elements at positions in $D^{(i-1)}$ and one more position $j \in \Omega$ and corresponding values in $c \in C^{(i)}$. g_i then gives the position of the new active cell and c_{opt} the updated number of events values in the i detected cells. If the new cell has zero events, then break. Else, we get $D^{(i)} = D^{(i-1)} \cup \{g_i\}$ and we remake steps 2 and 3.

3.2. Two Stages GOMP (2S-GOMP)

As presented in subsection 3.1., the GOMP algorithm is based on joint detection of new active cell position and counting the targets in the set of already detected cells. Despite its potentials for improved performance compared to the recently proposed GMP algorithm, the GOMP incurs a high computational load due to the implied multidimensional optimization. Then, to reduce this computational load, we here propose a

two-stages modified version of GOMP (2S-GOMP) which aims to separate the detection and counting steps.
2S-GOMP Principle

- Same input and output than GOMP.
- **Procedure:**
  1) Initialization: the residual $\mathbf{y}^{'} = \mathbf{y}$, $\Omega = \{1, 2, \ldots, N\}$, $D^{(0)} = \emptyset$.
  At the i^{th} iteration
  2) Detection step: find the position g_i that solves the following optimization
  $$g_i = arg\max_{j\in\Omega} \frac{| < \mathbf{y}^{'}, \mathbf{a}_j > |}{\|\mathbf{a}_j\|_2.\|\mathbf{y}^{'}\|_2},$$
  where $\mathbf{a}_j$ is the j^{th} column vector of matrix $\mathbf{A}$.
  3) $D^{(i)} = D^{(i-1)} \cup \{g_i\}$ denotes the active detected cells positions.
  Like for GOMP the same set $C^{(i)}$ is to be determined and tested.
  4) Counting step: find the number of targets in the detected active cells
  $$c_{opt} = arg\min_{c\in C^{(i)}} \|\mathbf{y} - \mathbf{A}\mathbf{z}^{c}_{j_{opt}}\|_2^2. \quad (6)$$
  If the optimal set $c_{opt} \in C^{(i)}$ in (6) corresponds to a zero value in the i^{th} last detected cell then break. Else, update the residual as $\mathbf{y}^{'} = \mathbf{y}^{'} - \mathbf{A}\mathbf{z}^{c_{opt}}_{g_i}$ and iterate steps 2 to 4.

The two above proposed approaches are iterative and aim to count and localize targets from a small number of measurements. In order to accelerate their convergence, we can distinguish two cases. The first assumes the Network Level of Sparsity (NLS) K knowledge, which is not true in practice. Rather, the Restricted Isometry Property (RIP) is considered which, if verified by the random matrix $\mathbf{A}$, it guarantees the decomposition uniqueness if the number of measurements M fulfills the following $M = O(Klog(\frac{N}{K}))$. Since M should verify $M \geq 2K$, this leads to $K \leq Ne^{\frac{-2}{\beta}}$, which can be considered as the maximal allowed number of iterations in GOMP and 2S-GOMP.

3.3. Complexity Evaluation

Joint detection and counting by GOMP involves a search over a set of Card $(\Omega) \times Card(C^{(i)})$. As 2S-GOMP operates detection and counting separately, it involves a search over a reduced set of Card $(\Omega) + Card(C^(i))$.

In this part, table 1, where $\hat{K}_1$ and $\hat{K}_2$ denote respectively the iterations number of GOMP and 2S-GOMP algorithms, presents a comparison of their computational complexities, evaluated in terms of multiplications number.

Considering that $N \gg K$, $N \gg \hat{K}_1$ and $N \gg \hat{K}_2$, and the complexities of GOMP and 2S-GOMP respectively given by C_1 and C_2 in table 1, we notice that C_1 proportionality coefficient with N is $M\sum_{i=1}^{\hat{K}_1} i(m^i + m^{(i-1)})$ whereas it is of $M\sum_{i=1}^{\hat{K}_2} 1 = M\hat{K}_2$ for C_2 which is much lower, thus showing that even for equivalent numbers of iterations $\hat{K}_1$ and $\hat{K}_2$, 2S-GOMP provides an important complexity reduction. This gain still increases if 2S-GOMP attains the stop condition more rapidly than GOMP, i.e.$\hat{K}_1 > \hat{K}_2$.

Algorithm	Complexity
GOMP	$C_1 = M\sum_{i=1}^{\hat{K}_1}(N - i + 1)m^{(i-1)}(m + 1)i.$
2S-GOMP	$C_2 = M\sum_{i=1}^{\hat{K}_2}\Big(N - i + 1 + m^{(i-1)}(m + 1)i\Big).$

Table 1. Computational complexity comparison of GOMP and 2S-GOMP in terms of multiplications number.

For the values $N = 64$, $M = 20$, $m = 3$ and $K = 2$, which will be further used in simulations and supposing the number of iterations $\hat{K}_1 = \hat{K}_2 = K$, the computational loads of GOMP and 2S-GOMP are respectively $C1 = 35360$ and $C_2 = 3100$ which is lower than $\frac{C_1}{11}$ showing the important computational burden reduction allowed by 2S-GOMP.

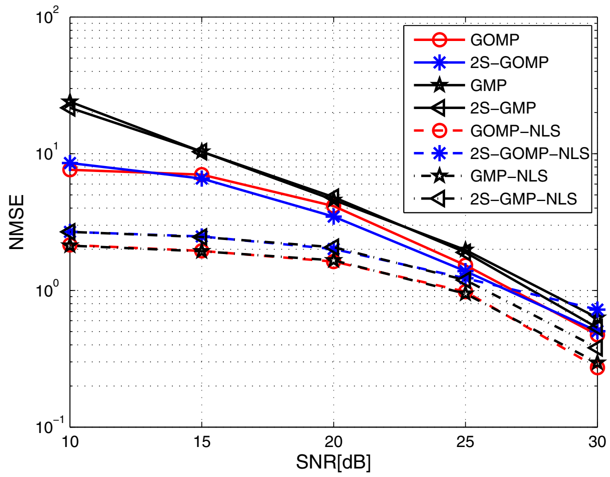
4. SIMULATION AND EVALUATION

4.1. Simulation parameters

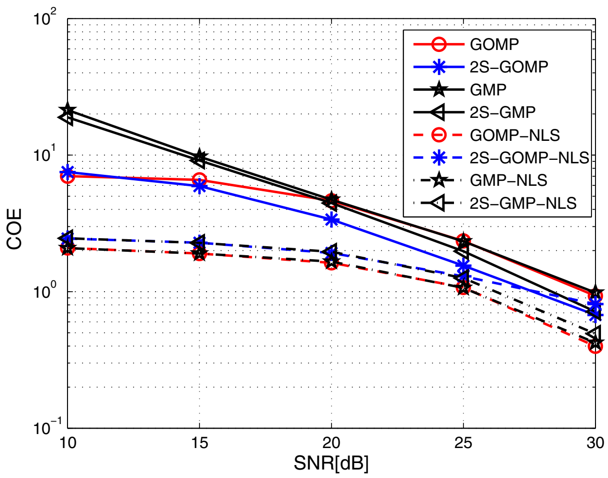
For all presented results, we randomly deploy $M = 20$ sensors over a monitored area regularly divided into $N = 64$ cells. We randomly consider only $K = 2$ ($K < M$) cells as active, i.e. where some events occur. The number of events in these active cells is chosen uniformly at random from $\{1, 2, \ldots, m\}$ ($m = 3$). In the same cell, the distance between a target and the sensor follows a uniform distribution on the interval $[0.1, 1]\frac{d}{2}$ where d is the regular spacing between two adjacent sensors. The path loss exponent is fixed to $\alpha = 2$ and the transmitted power P_0 is normalized to 1.

The performance is evaluated in terms of Normalized Mean Squares Error (NMSE) on s and of correct detection rate of targets positions for varying signal to noise ratio defined as $SNR = \frac{P_0}{\sigma^2}$. Also, the Counting Error (COE) given by $COE = \frac{\sum_{i=1}^{N} |n_i - n_i^{'}|}{\sum_{i=1}^{N} n_i}$, where n_i and $n_i^{'}$ denote respectively the actual and the estimated numbers of targets at cell i, is studied. Likewise, the rates of active cell missing and of false alarm are presented. In order to test the ability of the proposed algorithms in sparse targets recovery, a comparative study with the recent GMP algorithm and its two stages version (2S-GMP) where targets detection and counting are separated as in 2S-GOMP, is established. The NLS version (Network Level of Sparsity) of the considered algorithms assumes that K is known. For unknown K, the stop condition is here taken as $K_{max} = Ne^{-2} \simeq 8$ (case $\beta = 1$).

4.2. Numerical results



(a) Normalized Mean Squares Error versus SNR.

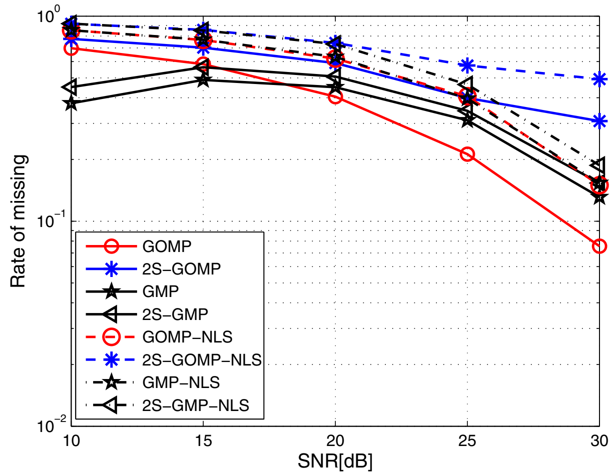


(b) Counting Error rate versus SNR.

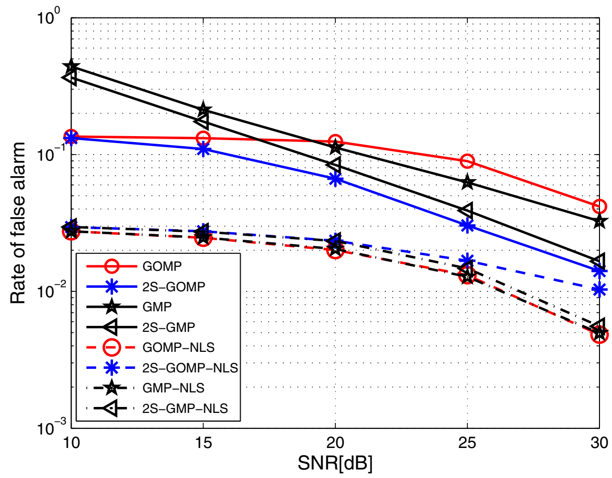
The Normalized MSE (NMSE) performance on s obtained by the compared techniques are depicted in figure (a). It is clear that an enhancement of recovery accuracy, by exploiting the information about the Network Level of Sparsity (NLS) is achieved over the whole SNR range. However, in this case, the 2S versions of GMP and GOMP lead to higher NMSE compared to the joint one stage processing where detection and counting are simultaneously operated. It is worth to note that GMP and GOMP achieve the same NMSE for known NLS, whereas 2S-GOMP shows a degradation w.r.t. 2S-GMP at high SNR. Then, for unknown NLS, the two proposed algorithms GOMP and 2S-GOMP achieve an enhanced performance compared to the recently proposed GMP and 2S-GMP version especially at low SNR. Still for unknown K, the 2 stages versions lead to sensibly the same NMSE performance than their one stage counterparts.

Counting Error rates are displayed in figure (b). Still, the known NLS scheme leads to the lowest COE. Then, for unknown NLS, it appears that the 2S-GOMP is the most relevant and efficient for targets counting. Indeed, separating the detection and counting steps prevents the occurrence of simultaneous errors in detection and counting.

Examining the active cells missing and false alarm rates, dis-



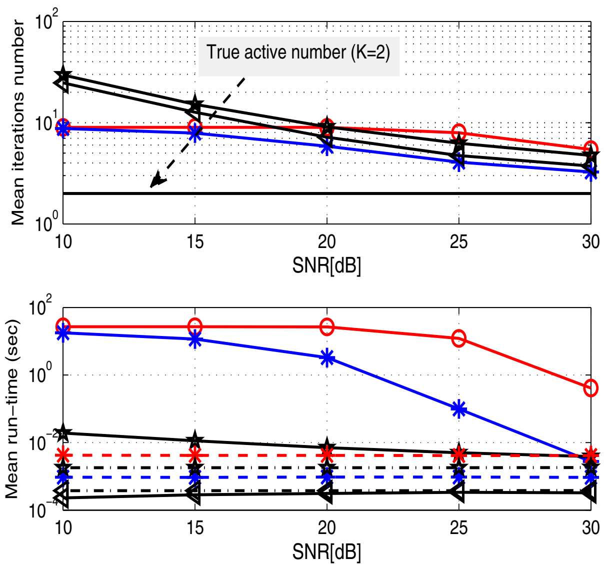
(c) Rate of active cell missing versus SNR.



(d) Rate of false cell detection versus SNR.

played respectively in figures (c) and (d), shows the obvious compromise between these two detection performance measures. Indeed, for the active cell missing rate, the techniques supposing known NLS lead to the poorest results, followed by 2S-GOMP, then 2S-GMP and GMP. Then, the GOMP algorithm leads to the lowest missing rates. This order is exactly reversed in the false alarm cell detection rates. For known NLS, GOMP and GMP lead to the same rates, the 2S-GOMP-NLS has however higher missing and false alarm rates than the 2S-GMP which explains the observed NMSE behavior.

Figure (e) illustrates the mean-run time and iterations number of all compared algorithms. The legends are those of previous figures, for readability they are omitted. Figure (e), subfig.1 shows that the number of iterations before reaching stop condition is lower for 2S-GOMP compared to GOMP, i.e. : $K = 2 < \hat{K}_1 < \hat{K}_2$ (which results from its lower false alarm rates). The lowest mean iterations number of 2S-GOMP is obtained thanks to its lowest false alarm rates (refer to figure (d)). We can see, based on subfig.2 of figure (e), that GOMP has the highest mean-run time which corroborates with the computational complexity analysis of section 3.3. The 2S-GOMP achieves a noticeable reduction in computational load w.r.t. GOMP. Also, we note that the 2S-GOMP mean-run time decreases with SNR, following the decrease of the false alarm rate. At high SNR, it achieves the same order than that of GMP which is less efficient, in terms of NMSE and COE (refer to figures (a) and (b)).



(e) Mean iterations number and run-time versus SNR. Refer to legends of figures (a)-(d).

5. CONCLUSION

In this paper, we consider a distributed Wireless Sensor Network where compressible sensors measurements are supposed available at a fusion center whose aim is to detect events and to count them per monitored area unit. The adopted theoretical framework is the Compressed Sensing theory which is justified by the occurrence rareness of the events to be detected. In this context, two versions of a novel Greedy Orthogonal Matching Pursuit algorithm, which do not require prior knowledge of signal sparsity level, are proposed for targets detection and counting. The enhancement w.r.t. the GMP scheme is indeed obtained by accounting for the CS decomposition basis non orthogonality. The two proposed versions GOMP and 2S-GOMP respectively correspond to joint and separate events detection and counting steps. Simulation results validate their superiority over the recently proposed GMP algorithm in terms of mean squares error, counting error and correct events positions detection. The two-stages GOMP is also shown to provide a better performance at lower complexity load when compared to the algorithm GOMP which jointly processes events detection and counting.

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