Numerical Characterization for Optimal Designed Waveform to Multicarrier Systems in 5G

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***Abstract*—High mobility of terminals constitutes a hot topic that is commonly envisaged for the next Fifth Generation (5G) of mobile communication systems. The wireless propagation channel is a time-frequency variant. This aspect can dramatically damage the waveforms orthogonality that is induced in the Orthogonal frequency division multiplexing (OFDM) signal. Consequently, this results in oppressive Inter-Carrier Interference (ICI) and Inter-Symbol Interference (ISI), which leads to performance degradation in OFDM systems. To efficiently overcome these drawbacks, we developed in [1] an adequate algorithm that maximizes the received Signal to Interference plus Noise Ratio (SINR) by optimizing systematically the OFDM waveforms at the Transmitter (TX) and Receiver (RX) sides.**

In this paper, we go further by investigating the performance evaluation of this algorithm. We start by testing its robustness against time and frequency synchronization errors. Then, as this algorithm banks on an iterative approach to find the optimal waveforms, we study the impact of the waveform initialization on its convergence. The obtained simulation results confirm the efficiency of this algorithm and its robustness compared to the conventional OFDM schemes, which makes it an appropriate good candidate for 5G systems.

***Index Terms*—OFDM, Optimazed Waveforms, Inter-Carrier Interference, Inter-Symbol Interference, SINR**

I. Introduction

The maximization of the overall data rate transmission in communication systems represents actually the main challenge and objective of many research works. This objective generally faces a high interference level that exists in the received signal and that dramatically disturbs and degrades its quality. Interesting solutions were proposed in order to overcome this problem, including the Orthogonal Frequency Division Multiplexing (OFDM) on which latest cellular technologies rely on [2]. This modulation offers several advantages over conventional single carrier approaches such as an enhanced capacity of the OFDM based system, a high spectral efficiency for it and immunity to Inter-Symbol Interference (ISI). In addition to these benefits, equalization on a narrow band subcarrier is less complex in terms of processing than other broadband schemes that don't use the OFDM transmission [3], [4].

Although the advantages of the OFDM transmission technique, several research studies shed light on various drawbacks of this technique, including mainly the spectral leakage of Digital Fourier Transform (DFT)-based OFDM systems that can result in an important interference level through the OFDM subcarriers, awareness to carrier frequency offsets and also constrained bandwidth efficiency due to the junction of Cyclic Prefix (CP) used for the channel equalization in the frequency domain. However, the OFDM approach efficiently solves the problem of the frequency selective fading channel by the mean of low-complexity equalizers and this does represent a crucial aspect in the case of high frequency-selective channels. Contrary to high mobility situations that are commonly envisaged for the next Fifth Generation (5G) of mobile communication systems, the wireless propagation channel is a time-frequency variant where the time dispersion emerges from the multipath characteristic and the time-selectivity arises from the Doppler spread that damages the orthogonality induced in the OFDM signal and consequently results in oppressive Inter-Carrier Interference (ICI) [5], [6], [7]. Thus a non-orthogonal future wireless multi-carrier scheme with malleable waveforms would represent an interesting solution that potentially reduces the ISI and ICI and also minimizes the energy spreading. This could also be a powerful candidate to be used in the 5G systems.

Several solutions were proposed in the literature to mitigate the ICI and ISI when the propagation channel is doubly dispersive. One of the envisaged solution is the one proposed in [5], that banks on waveform based on Hermite-Gaussian function whose time-frequency density is equal to 2 and that leads to a better time-frequency localization compared to the OFDM basic rectangular pulse. This solution reduces the ICI/ISI levels but at the same time results in a spectral efficiency reduction and remains more complex from a processing point of view than the CP-OFDM scheme. Another attractive alternative technique is the Lattice-OFDM [9] that consists in adapting the signal waveforms and the lattice feature to the propagation channel conditions. Other approaches to reduce ICI/ISI are based on further equalization treatment on the receiver side such as the Least-Squares QR (LSQR) algorithm [10].

In [1], we proposed a novel algorithm to minimize the ICI and ISI through an adequate design of the OFDM waveforms. This algorithm has the advantage to remain powerful even the waveform orthogonality is destroyed.

In this paper, we evaluate numerically the robustness of our algorithm [1]. First, we study its sensibility to waves initializations. Second, we characterize its efficiency against synchronization errors.

This paper is organized as follows: in Section III, we will present the multicarrier system model where we specify its transmitter and receiver blocks. We also detail the propagation channel models that will be used in this paper. In Section IV, we will describe the optimization technique for waveform design. In Section V, we will study the algorithm sensibility to waves initializations. Section VI will evaluate the robustness of our algorithm against to time and frequency synchronisation errors. Finally, Section VII will draw conclusions and perspectives of our work .

II. Notations

The boldface lower case letters denote vectors and boldface upper case letters refer to matrices. The superscripts $.^T$ and $.^*$, denote the transpose and the element-wise conjugation, respectively. In addition, $\mathbb{E}$ and $\odot$ refer to the expectation operator and the component-wise product of two vectors or matrices, respectively. $\mathbf{I}$ denotes the identity matrix with ones in the main diagonal and zeros elsewhere.

III. System Model

This section provides preliminary concepts and notations related to the considered multicarrier scheme and the channel assumptions and model. Starting from the symbols, lattices, and waveform inspired from the framework of Weyl-Heisenberg, frames are discussed through this section. Furthermore, we consider their discrete time version in order to simplify the theoretical derivations that will be investigated.

A. OFDM Transmitter and receiver blocks

We denote by T the OFDM symbol duration and by F the frequency separation between two adjacent subcarriers. Let $a_{m,n}$, $m,n \in \mathbb{Z}$, be the transmitted symbol at time nT using subcarrier mF and assumed to be independent identically distributed (i.i.d.) with zero mean and energy equal to E. The baseband transmitted signal is given by the following expression:

$$e(t) = \sum_{m,n} a_{mn}\varphi_{mn}(t),$$

where $\varphi_{mn}(t) = \varphi(t-nT)e^{j2\pi mFt}$ denotes the time and frequency shifted version of the OFDM transmitter prototype waveform $\varphi(t)$ used to transmit the symbol a_{mn} and that is assumed to have a unitary energy, means $\|\varphi(t)\| = 1$.

The transmitted signal is sampled at a sampling rate $R = \frac{1}{T_s}$, where $T_s = \frac{T}{N}$ is the sampling period such that $N \in \mathbb{N}$, $\frac{1}{T_sF} = Q \in \mathbb{N}$ and $N > Q$. Let Q denotes the number of the OFDM signal sub-carriers. The sub-carrier frequencies correspond to $mF = \frac{m}{QT_s}$, where $m = 0, 1, \ldots, Q-1$. Note that the parameter $(N-Q)T_s$ can be confused with the notion of Cyclic Prefix (CP) in a conventional OFDM system and the time-frequency plane density refers to $\delta = \frac{1}{FT} = \frac{N}{Q}$.

Let the infinite vector $\mathbf{e} = [\ldots, e_{-2}, e_{-1}, e_0, e_1, e_2, \ldots]^T = (e_q)_q$ where e_q, also denoted $[\mathbf{e}]_{\mathbf{q}}$, be the sampled version of the transmitted signal at time qT_s with $q \in \mathbb{Z}$ such that $\mathbf{e} = \sum_{m,n} a_{mn}\varphi_{mn}$. We denote by $\varphi_{mn} = (\varphi_{q-nN})_q \odot (e^{j2\pi\frac{mq}{Q}})_q$ the vector that results from a time shift of $nNT_s = nT$ and a frequency shift of $mF = \frac{m}{QT_s}$ of the transmission prototype vector $\varphi = (\varphi_q)_q$. The waveform unitary energy assumption is also maintained in the discrete version, i.e., $\|\varphi\|^2 = \sum_{q\in\mathbb{Z}} |\varphi_q|^2 = 1$.

Assuming a linear time-varying multipath channel $h(p,q)$ with q and p standing respectively for the normalized observation time and the time delay, the received signal has the following expression:

$$\begin{aligned} r_q &= \sum_{m,n} a_{mn} \sum_p h(p,q)[\varphi_{mn}]_{q-p} + n_q \\ &= \sum_{m,n} a_{mn}[\tilde{\varphi}_{mn}]_{q-p} + n_q, \end{aligned}$$

where p denotes the channel taps, $[\tilde{\varphi}_{mn}]_q = \sum_p h(p,q)[\varphi_{mn}]_q$ is the channel distorted version of φ_{mn} and $\mathbf{n} = (n_q)_q$ denotes the discrete complex Additive White Gaussian Noise (AWGN), the samples of which are centered, uncorrelated with common variance N_0. The decision variable

$$\Lambda_{kl} = \langle \Psi_{kl}, \mathbf{r} \rangle = \Psi_{kl}^H \mathbf{r} \tag{1}$$

on the transmitted symbol a_{mn} is obtained by projecting the received signal $\mathbf{r}$ on the receiver pulse Ψ_{kl}, where $\Psi_{kl} = (\Psi_{q-lN})_q \odot (e^{j2\pi\frac{kq}{Q}})_q$ is the time and frequency shift version of the received vector dedicated to demodulate a_{mn}. Perfect demodulation is achieved since the transmitter waveform and the receiver one are bi-orthogonal. Note that this condition could not be perfectly satisfied and we will consider a general framework for it in this paper.

B. Channel model

For simplification sake, assume that the Linear Time Variant (LTV) channel $h(p,q)$ is Wide-Sense Stationary with Uncorrelated Scattering (WSSUS). Then, the channel discrete scattering function has the following expression:

$$S(p,\nu) = \sum_{\Delta q} \phi_h(p,\Delta q) e^{-j2\pi\nu T_s \Delta q},$$

where $\phi_h(p_1,p_2;\Delta q) = \mathbb{E}[h^*(p_1,q)\,h(p_2,q+\Delta q)] = \phi_h(p_1,\Delta q)\delta_K(p_2-p_1)$ defines the corresponding autocorrelation function with δ_K being the Kronecker symbol.

To simplify the derivations, we presume a channel with a finite path number, K, with the following channel impulse response:

$$h(p,q) = \sum_{k=0}^{K-1} h_k e^{j2\pi\nu_k T_s q}\delta_K(p-p_k),$$

where h_k, ν_k and p_k are respectively the amplitude, Doppler frequency and the time delay of the k^{th} path. The paths amplitudes h_k are assumed to be i.i.d. complex Gaussian variables with zero mean and average powers equal to $\pi_k = \mathbb{E}[|h_k|^2]$ such that $\sum_{k=0}^{K-1} \pi_k = 1$. We choose an exponential truncated decaying model. Let $0 < b < 1$ be the decaying factor, such that the paths powers can be expressed as $\pi_k = \frac{1-b}{1-b^K} b^k$.

In this paper, we consider a radio mobile channel where the scattering function $S(p,\nu)$ has periodical classical Doppler spectral density $\alpha(\nu)$, with period $\frac{1}{T_s}$. This scattering function obeys to the Jakes model that is decoupled from the dispersion in the time domain denoted $\beta(p)$. This means that $S(p,\nu) = \beta(p)\alpha(\nu)$, such that $\beta(p) = \sum_{k=0}^{K-1} \pi_k \delta_K(p - p_l)$ and

$$\alpha(\nu) = \begin{cases} \frac{1}{\pi B_d} \frac{1}{\sqrt{1-(\frac{2\nu}{B_d})^2}} & \text{if } |\nu| < \frac{B_d}{2} \\ 0 & \text{if } \frac{B_d}{2} \leq |\nu| \leq \frac{1}{2T_s} \end{cases} \quad (2)$$

where B_d is the Doppler speard and $\int \alpha(\nu) e^{j2\pi \frac{\nu}{T_s} k} \, d\nu = J_0(\pi B_d T_s k)$.

IV. Waveforms Design

Without loss of generality, we will focus on the evaluation of the SINR for the symbol a_{00}. Referring to (1), the decision variable on a_{00} has the following expression:

$$\Lambda_{00} = a_{00} \langle \Psi_{00}, \tilde{\varphi} \rangle + \sum_{(m,n)\neq(0,0)} a_{mn} \langle \Psi_{00}, \tilde{\varphi}_{mn} \rangle + \langle \Psi_{00}, \mathbf{n} \rangle .$$

This decision variable is the sum of three elements: a useful element, an interference one and a noise one. The purpose of our optimization approach is to minimize the mean power of the interference element, resulting from symbols a_{mn} such that $(m,n) \neq (0,0)$, for a fixed value of the useful element power.

A. Average Useful Power

The useful term corresponding to a_{00} is given by $U_{00} = a_{00} \langle \Psi_{00}, \tilde{\varphi}_{00} \rangle$. For a given realization of the channel, the average power of the useful terms is given by $P_S^h = E | \langle \Psi_{00}, \tilde{\varphi}_{00} \rangle |^2$. Therefore, the average of the conditional useful power over channel realizations is $P_S = \mathbb{E}[P_S^h]_h$, where $[\tilde{\varphi}_{00}]_q = \sum_{k=0}^{K-1} h_k [\tilde{\varphi}_{00}]_{q-p_k} e^{j2\pi\nu_k T_s (q-p_k)}$. Let $\sigma_p(\mathbf{v})$ denote the time shift operator by p sample durations of the vector $\mathbf{v} = (\nu_q)_q$, i.e. $\sigma_p(\mathbf{v}) = (\nu_{q-p_k})_q$ and Φ_ν denote the Hermitian matrix with $(p,q)^{th}$ entry $e^{j2\pi\nu T_s (p-q)}$. Under these notions, we deduce that:

$$P_S = E \, \Psi^H \mathbf{KS}^{\varphi}_{S(p,\nu)} \Psi,$$

where we define the useful signal Kernel matrix as $\mathbf{KS}^{\varphi}_{S(p,\nu)} = \sum_{k=0}^{K-1} \pi_k \Phi_{\nu_k} \odot (\sigma_{p_k}(\varphi_{00}) \sigma_{p_k}(\varphi_{00})^H)$. Since P_S is a positive entity, then the Kernel matrix is a positive Hermitian matrix. Hence, given any choice of the transmitter prototype φ, one can maximize the useful signal power by choosing the receiver prototype vector Ψ as the eigenvector of the $\mathbf{KS}^{\varphi}_{S(p,\nu)}$ that corresponds to its maximum eigenvalue.

B. Average Interference Power

The interference term within the decision variable Λ_{00} is given by $I_{00} = \sum_{(m,n)\neq(0,0)} a_{mn} \langle \Psi_{00}, \tilde{\varphi}_{mn} \rangle$ that results from the contribution of a_{mn} such that $(m,n) \neq (0,0)$. The mean power of P_I^h over channel realizations is given by

$$P_I = \mathbb{E}[P_I^h]_h = E \sum_{(m,n)\neq(0,0)} \mathbb{E}[| \langle \Psi_{00}, \tilde{\varphi}_{mn} \rangle |^2]_h.$$

By re-iterating the same derivation as the one in Section IV-A, we find that:

$$P_I = E \, \Psi^H \mathbf{KI}^{\varphi}_{S(p,\nu)} \Psi.$$

where the interference Kernel matrix is expressed as $\mathbf{KI}^{\varphi}_{S(p,\nu)} = \sum_{k=0}^{K-1} \pi_k \Phi_{\nu_k} \odot (\sum_{(m,n)\neq(0,0)} \sigma_{p_k}(\varphi_{mn}) \sigma_{p_k}(\varphi_{mn})^H)$. Since P_I is always positive, then $\mathbf{KI}^{\varphi}_{S(p,\nu)}$ is also a positive semidefinite matrix. We deduce that for any choice of the transmitter prototype vector φ, one can minimize the interference power by choosing the receiver prototype vector Ψ as the eigenvector of $\mathbf{KI}^{\varphi}_{S(p,\nu)}$ that corresponds to its smallest eigenvalue.

C. Average noise power

The noise average power is given by:

$$P_N = \mathbb{E}[| \langle \Psi_{00}, n \rangle |^2] \quad (3)$$
$$= \Psi_{00}^H \mathbb{E}[\mathbf{n}\mathbf{n}^H] \Psi_{00}^H. \quad (4)$$

Since the noise is assumed to be white, therefore its covariance matrix is equal to $\mathbf{R}_{nn} = \mathbb{E}[\mathbf{n}\mathbf{n}^H] = N_0 \mathbf{I}$. Consequently, as the prototypes are of unitary energy, then $P_N = N_0$.

D. Optimization step

Using the obtained expressions of the useful power P_S, the interference power P_I and the noise power N_0, the SINR has the following expression:

$$SINR = \frac{P_S}{P_I + P_N} = \frac{\Psi^H \mathbf{KS}^{\varphi}_{S(p,\nu)} \Psi}{\Psi^H \mathbf{KI}^{\varphi}_{S(p,\nu)} \Psi + \frac{N_0}{E}}. \quad (5)$$

This expression is valid for normalized transmitted energy and Tx/Rx prototype functions φ and Ψ.

Our main objective consists in determining the couple (φ, Ψ) that maximizes the SINR for a given SNR value. A direct optimization method consists in diagonlizing the SINR denominator of expression (5) and then perform a basis change that will simplify the expression of this denominator, so that our optimization problem becomes a maximization one that implies to find the eigenvector of the SINR numerator that corrresponds to its maximum eigenvalue. More precisely, we first introduce the Kernel function $\mathbf{KIN}^{\varphi}_S(p,\nu) = \mathbf{KI}^{\varphi}_S(p,\nu) + (\frac{N_0}{E})\mathbf{I}$. The eigendecomposition of $\mathbf{KIN}_S^{\varphi}(p,\nu)$ is $\mathbf{KIN}_S^{\varphi}(p,\nu) = \mathbf{U}\Lambda\mathbf{U}^H$, where $\mathbf{U}$ is a unitary matrix, Λ is a diagonal one with nonnegative real numbers on the diagonal. Then, the SINR denominator can be written as $\Psi^H\mathbf{KIN}_S^{\varphi}(p,\nu)\Psi = \Psi^H\mathbf{U}\Lambda\mathbf{U}^H\Psi = \mathbf{u}^H\mathbf{u}$ where $\mathbf{u} = \Lambda^{\frac{1}{2}}\mathbf{U}^H\Psi$. Since $\mathbf{KI}_S^{\varphi}(p,\nu)$ is a positive semidefinite matrix, then all the entries of Λ are positive and greater than $\frac{N_0}{E} > 0$. Therefore, $\Psi = \mathbf{U}\Lambda^{-\frac{1}{2}}\mathbf{u}$ and the SINR expression becomes the following:

$$SINR = \frac{\mathbf{u}^H\Phi\mathbf{u}}{\mathbf{u}^H\mathbf{u}},$$

where $\Phi = \Lambda^{-\frac{1}{2}}\mathbf{U}^H\mathbf{KS}_S^{\varphi}(p,\nu)\mathbf{U}\Lambda^{-\frac{1}{2}}$ is a positive matrix. Hence, maximizing the SINR is equivalent to determine the maximum eigenvalue of Φ and its associated eigenvector $\mathbf{u}_0$. Therefore, $\Psi^{opt} = \mathbf{U}\Lambda^{-\frac{1}{2}}\mathbf{u}_0$.

V. Sensitivity to Waveforms initializations

In this section, we study the performance of our algorithm in terms of SINR for different waveforms initializations. We consider Gaussian waveforms where we vary the mean and the standard variation as illustrated in Fig.1. Then, for each Gaussian waveform, we apply our optimization algorithm (See Section IV-D) and we represent the SINR evolution that is depicted in Fig.2. From this figure, we can conclude that whatever the waveform initialization that we consider, our algorithm converges to the same SINR. This highlights that our algorithm isn't sensitive to the used initialization.

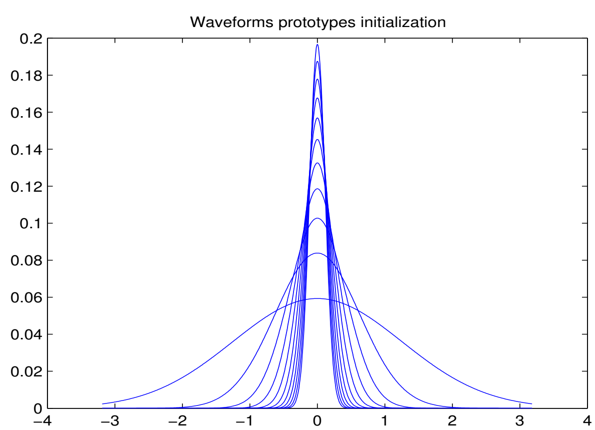

Fig. 1: Gaussian Waveforms initializations

VI. Robustness

As it is known, the synchronization is a crucial indicator for efficiency of wireless communication systems and eventually for 5G [6], [7]. Usually, such systems are too sensitive to any synchronization error. As our algorithm was essentially conceived for non-orthogonal future wireless multi-carrier, it is worth meaning to evaluate its vulnerability against time and frequency synchronization errors.

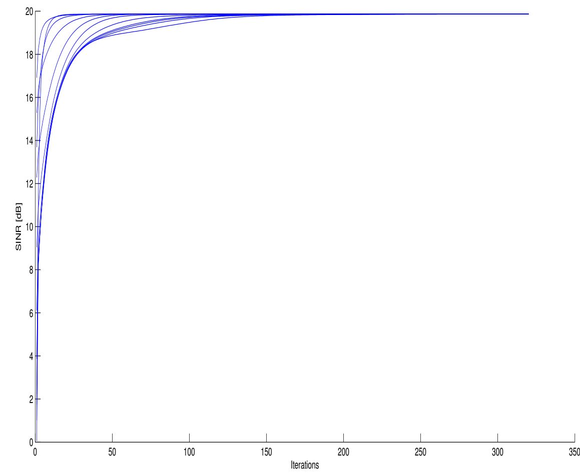

Fig. 2: Optimized SINR as a function of waves initializations ($CP = 32$, $SNR = 30$dB, $B_dT_m = 10^{-2}$ and waveform support duration equal to $3T$).

In this section, we investigate this aspect and then we focus on the sensitivity of the optimized waveforms for any variation around the optimal B_dT_m where T_m and B_d are respectively the delay and Doppler spreads.
In Fig.3, we can conclude that our proposed algorithm significantly outperforms the conventional OFDM one in terms of robustness against the time synchronization errors when $CP = 32$ and $CP = 16$. For the frequency synchronization errors, the obtained results show that our algorithm doesn't degrade the SINR performance compared to the conventional OFDM one (See Fig.4).

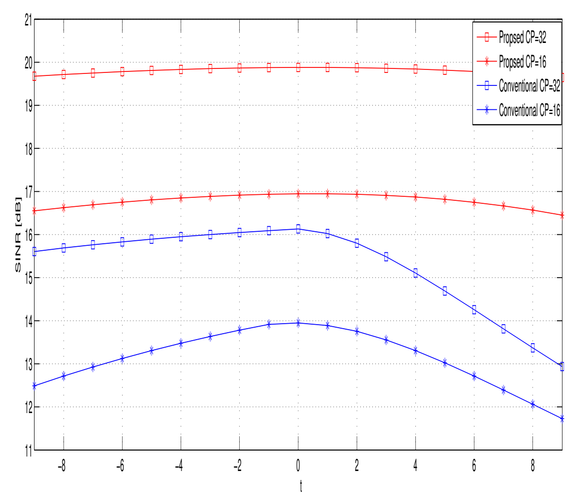

Fig. 3: Optimized SINR as a function of time Synchronization errors (Δt)(SNR=30dB, B$_dT_m = 10^{-2}$ and waveform support duration equal to $3T$).

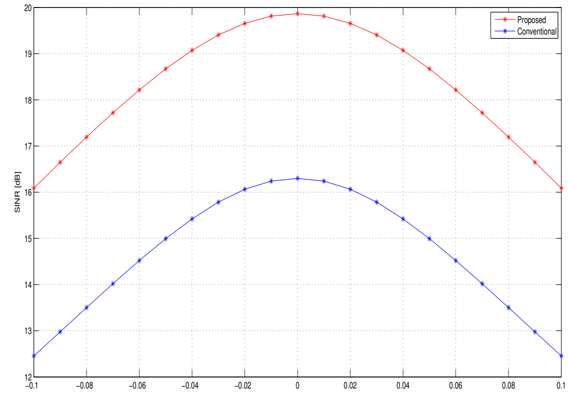

Fig. 4: Optimized SINR as a function of frequency synchronization errors $(\Delta\nu)$(CP=32, SNR=30dB,$B_dT_m = 10^{-2}$ and waveform support duration equal to $3T$).

Fig.5 illustrates the sensitivity of our algorithm when we assume a synchronization error on B_dT_m ranging between 0.001 and 0.01. In this figure, we represent the SINR obtained after optimizing the waveforms when $B_dT_{m1} = 0.001$ and $B_dT_{m2} = 0.01$, respectively. We remark that the obtained SINR performance for B_dT_{m2} is slightly degraded compared to the situation where the optimization is performed for B_dT_{m1}. Therefore, it is better to consider our waveforms optimization for large B_dT_m values when we do not know in advance its optimal value.

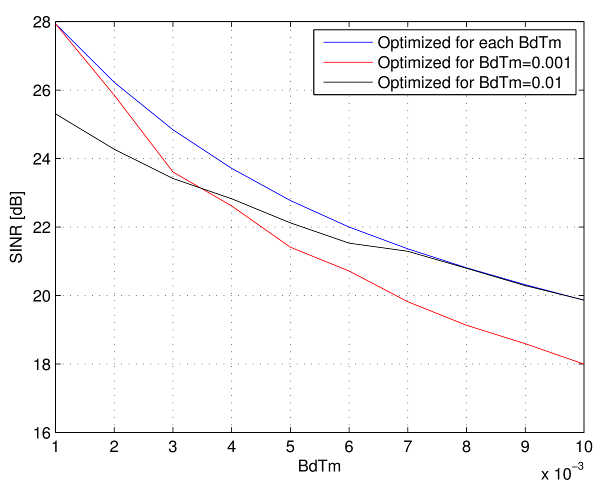

Fig. 5: Optimized SINR as a function of BdTm ($CP = 32$, $SNR = 30$dB, and waveform support duration equal to $3T$).

VII. Conclusion

In this paper, we evaluate numerically the performance of our proposed waveforms optimization algorithm in terms of robustness to time and frequency synchronization errors. We also studied its sensitivity to waveforms initializations. The obtained results showed the good performance of our waveforms optimization algorithm even in cases where the OFDM waveforms orthogonality was not respected (this situation is often in high mobility propagation channels). This includes good robustness against time and frequency synchronization errors and insensitivity to waveforms initializations.
This confirms well that our proposed solutions can be considered an attractive candidate to 5G systems. A possible challenging research axis that could be investigated consists in extending our optimization algorithm to the case of OQAM/OFDM systems.

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