INSTANTANEOUS PARAMETERS ESTIMATION ALGORITHM FOR NOISY AM-FM OSCILLATORY SIGNALS

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ABSTRACT

The paper addresses the problem of estimation of amplitude envelope and instantaneous frequency of an amplitude and frequency modulated (AM-FM) signal in noisy conditions. The algorithm proposed in the paper utilizes derivatives of the signal and is analogous to well-known Energy Separation Algorithms (ESA) based on Teager-Kaiser energy operator (TEO). The formulation of the algorithm is based on Prony's method that provides estimates of phase and damping factor as well. Compared to ESA the proposed algorithm has a very close performance for pure oscillatory signals and a better performance for signals with additive white noise.

Index Terms— Time-frequency analysis, estimation of instantaneous frequency, Teager-Kaiser energy operator, Prony's method

1. INTRODUCTION

One of the main approaches to estimation of time-varying amplitude and frequency parameters of real-valued signals is Energy Separation Algorithm [1]. The method is based on the nonlinear differential Teager-Kaiser Energy Operator [2]

$$\Psi[x(t)] \triangleq \dot{x}^2(t) - x(t)\ddot{x}(t) \qquad (1)$$

where $\dot{x}(t) = dx(t)/dt$. As has been shown in [3] the operator can be used for short-term energy estimation in additive noise with a better performance than the squared operator

$$\mathrm{S}[x(t)] \triangleq x^2(t).$$

According to ESA two TEO's outputs are separated into amplitude modulation and frequency modulation components. As shown in [4] the third-order energy operator

$$\Upsilon_3[x(t)] \triangleq x(t)x^{(3)}(t) - \dot{x}(t)\ddot{x}(t) \qquad (2)$$

(where $x^{(3)}(t) = d^3x(t)/dt^3$) can be used for estimating damping factor.

The Discrete Energy Separation Algorithms (DESAs) derived in [1] utilize discrete TEO's approximation

$$\Psi[x(n)] \triangleq x^2(n) - x(n-1)x(n+1). \qquad (3)$$

DESAs have been used in many signal processing applications such as demodulation [5], speech/music source separation [6], event detection and other. The algorithms are notable for low computational complexity and represent a real-valued alternative to analysis methods based on Hilbert transform.

The aim of the work presented here is to find a noise robust counterpart of DESAs which has the same (or very close) estimation accuracy and capability of estimating instantaneous phase and damping factor. Our study focuses on the formulation of the algorithm which is derived using Prony's method [7]. Original Prony's method fits complex exponents into sampled data assuming invariability of their parameters during observation interval. An application of Prony's method for instantaneous frequency estimation was given in [8] where a 5-point algorithm is presented. The idea is to combine time-shifted versions of the original Prony's approach to achieve increased accuracy at high signal-to-noise ratios (SNR). While the present study applies Prony's method for direct estimation of the required parameters from instantaneous derivatives of the signal [9]. At first the proposed algorithm is derived for continuous-time signals and then its discretization is performed. Noise robustness of the algorithm is achieved by approximation of differential operator with 3-point symmetric differences.

2. ESTIMATION ALGORITHM FOR CONTINUOUS-TIME SIGNALS

2.1. General case

Let us consider a continuous signal $x(t)$ which can be represented as a sum of damped complex exponents:

$$x(t) = \sum_{k=1}^{p} h_k \, z_k^t$$

where p is the number of exponents, $h_k = A_k e^{j\theta_k}$ is an initial complex amplitude and $z_k = e^{\alpha_k + jf_k}$ is a time-dependent damped complex exponent with damping factor α_k and normalized angular frequency f_k. Then let us introduce a time shift t_0 and obtain *n*-th order derivatives of $x(t)$:

$$x^{(n)}(t) = \left(\sum_{k=1}^{p} h_k\, z_k^{t-t_0}\right)^{(n)} = \sum_{k=1}^{p} h_k\,(\alpha_k + jf_k)^n z_k^{t-t_0} = \sum_{k=1}^{p} l_k(t)\, y_k^n \tag{4}$$

where (n) denotes order of derivative,
$l_k(t) = h_k z_k^{t-t_0} = A_k e^{\alpha_k(t-t_0)+j(\theta_k+f_k(t-t_0))}$,
$y_k = (\alpha_k + jf_k) = e^{(\ln|y_k|+j\arg(y_k))}$.

According to (4) for any fixed moment of time $t = t_0$ series of derivatives $x, \dot{x}, \ddot{x}, \dots, x^{(n)}$ can be represented as a sum of damped complex exponents with initial complex amplitudes $l_k(t_0) = h_k$, damping factors $\ln|y_k|$ and normalized angular frequencies $\arg(y_k)$. The required parameters of the model h_k and y_k can be found using original Prony's method applied to series of derivatives as briefly summarized below.

In order to estimate exact model parameters $2p$ complex samples of the sequence are required. The solution is obtained using the following system of equations:

$$\begin{pmatrix} y_1^0 & y_2^0 & \cdots & y_p^0 \\ y_1^1 & y_2^1 & \cdots & y_p^1 \\ \vdots & \vdots & & \vdots \\ y_1^{p-1} & y_2^{p-1} & \cdots & y_p^{p-1} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{pmatrix} = \begin{pmatrix} x \\ \dot{x} \\ \vdots \\ x^{(p-1)} \end{pmatrix} \tag{5}$$

The required exponents $y_1, y_2, \dots, y_p$ are estimated as roots of the polynomial

$$\psi(z) = \sum_{m=0}^{p} a_m z^{p-m} \tag{6}$$

with complex coefficients a_m which are the solution of the system

$$\begin{pmatrix} x^{(p-1)} & x^{(p-2)} & \cdots & x \\ x^{(p)} & x^{(p-1)} & \cdots & \dot{x} \\ \vdots & \vdots & & \vdots \\ x^{(2p-2)} & x^{(2p-3)} & \cdots & x^{(p-1)} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix} = -\begin{pmatrix} x^{(p)} \\ x^{(p+1)} \\ \vdots \\ x^{(2p-1)} \end{pmatrix}$$

and $a_0 = 1$. Each damping factor α_k and frequency f_k are calculated using the following equations:

$$\alpha_k = \mathrm{Re}(y_k), f_k = \mathrm{Im}(y_k).$$

Using the extracted values of $y_1, y_2, \dots, y_p$ system (4) is solved with respect to $h_1, h_2, \dots, h_p$. From each of these parameters initial amplitude A_k and phase θ_k are calculated as:

$$A_k = |h_k|, \theta_k = \arctan\left[\frac{\mathrm{Im}(h_k)}{\mathrm{Re}(h_k)}\right].$$

For real-valued signals the solution gives pairs of complex conjugate exponents. In order to identify parameters of b real-valued sinusoids we should calculate $4b - 1$ derivatives.

2.2. Real-valued one component case

Considering $x(t)$ as a single real-valued damped sinusoid it is possible to identify its parameters using its instantaneous value and three derivatives. Using the equations that have been given above we can formulate the following estimation algorithm.

1) Calculate three derivatives of the signal: $\dot{x}, \ddot{x}, x^{(3)}$;
2) Calculate coefficients of polynomial (6)

$$a_1 = \frac{x x^{(3)} - \dot{x}\ddot{x}}{\dot{x}^2 - x\ddot{x}} = \frac{\Upsilon_3[x]}{\Psi[x]},$$

$$a_2 = \frac{\ddot{x}^2 - \dot{x}x^{(3)}}{\dot{x}^2 - x\ddot{x}} = \frac{\Psi[\dot{x}]}{\Psi[x]};$$

3) Calculate roots of polynomial (6)

$$y_{1,2} = \frac{1}{2}\left(-a_1 \pm \sqrt{a_1^2 - 4a_2}\right) = -\frac{\Upsilon_3[x]}{2\Psi[x]} \pm \sqrt{\frac{\Upsilon_3^2[x]}{4\Psi^2[x]} - \frac{\Psi[\dot{x}]}{\Psi[x]}};$$

4) Calculate initial complex amplitude

$$h = \frac{x y_2 - \dot{x}}{y_2 - y_1} = \frac{1}{2}\left(x + \frac{\frac{\Upsilon_3^2[x]}{2\Psi[x]} x + \dot{x}}{\sqrt{\frac{\Upsilon_3^2[x]}{4\Psi^2[x]} - \frac{\Psi[\dot{x}]}{\Psi[x]}}}\right);$$

5) Calculate required parameters of the sinusoid

$$\alpha = \mathrm{Re}(y_1) = -\frac{\Upsilon_3[x]}{2\Psi[x]},$$

$$f = \mathrm{Im}(y_1) = \sqrt{\frac{\Psi[\dot{x}]}{\Psi[x]} - \frac{\Upsilon_3^2[x]}{4\Psi^2[x]}}, \tag{7}$$

$$A = 2|h|, \qquad \theta = \arctan\left[\frac{\mathrm{Im}(h)}{\mathrm{Re}(h)}\right].$$

Note that the resulting equation for damping factor is exactly the same as given in [4] and the equation for frequency can be derived from the case of cosine with exponential amplitude discussed in [1].

3. ESTIMATION ALGORITHMS FOR DISCRETE-TIME SIGNALS

In order to process a discrete-time signal we use a discrete differentiator at the first step of the algorithm. Similarly to DESAs there are two main alternatives that produce counterparts of DESA-1 and DESA-2 respectively: we can choose either the 2-point or 3-point differences.

3.1. Using the 2-point differentiator

If $x(n)$ is a discrete-time signal we can replace first derivative with 2-sample difference $d_1(n) = [x(n) - x(n-1)]$. For each derivative order we get the correspondent differentiators with the following impulse responses h_{1-3} and frequency responses $H_{1-3}(\omega)$ as follows:

$$h_1 = [1;\ -1];$$
$$H_1(e^{j\omega}) = (1 - e^{-j\omega})e^{j\frac{\omega}{2}} = 2\,j\sin\left(\frac{\omega}{2}\right);$$
$$h_2 = [1;\ -2;\ 1];\ H_2(e^{j\omega}) = \left[2\,j\sin\left(\frac{\omega}{2}\right)\right]^2; \qquad (8)$$
$$h_3 = [1;\ -3;\ 3;\ -1];\ H_3(e^{j\omega}) = \left[2\,j\sin\left(\frac{\omega}{2}\right)\right]^3.$$

The frequency responses of the differentiators are not ideal and introduce frequency distortion. From (7) and (8) we get corrected frequency value:

$$f = 2\arcsin\left({}^{\mathrm{Im}(y_1)}/_2\right).$$

We call this algorithm DIPA-1 (Discrete Instantaneous Prony's Algorithm). The algorithm requires four consecutive samples of the signal.

3.2. Using the 3-point differentiator

Using the 3-sample symmetric difference $d_1(n) = [x(n+1)/2 - x(n-1)/2]$ we obtain the following impulse and frequency responses of the differentiators:

$$h_1 = \left[\frac{1}{2};\ 0;\ \frac{-1}{2}\right];$$
$$H_1(e^{j\omega}) = \left(\frac{1}{2} - \frac{e^{-j2\omega}}{2}\right)e^{j\omega} = j\sin(\omega);$$
$$h_2 = [\tfrac{1}{4};\ 0;\ \tfrac{-1}{2};\ 0;\ \tfrac{1}{4}];\ H_2(e^{j\omega}) = [j\sin(\omega)]^2; \qquad (9)$$
$$h_3 = [\tfrac{1}{8};\ 0;\ \tfrac{-3}{8};\ 0;\ \tfrac{3}{8};0;\tfrac{-1}{8}];\ H_3(e^{j\omega}) = [j\sin(\omega)]^3.$$

Note that according to (9) calculations of x and $\dot{x}$ as well as of $\ddot{x}$ and $x^{(3)}$ are done independently using nonintersecting sets of samples that is beneficial for robustness of the algorithm. Using (7) and (9) we can calculate correction for estimated frequency as follows:

$$f = \arcsin(\mathrm{Im}(y_1)).$$

We call this DIPA-2. The algorithm requires seven consecutive samples of the signal. Note that just like DESA-2 it can be used to estimate instantaneous frequencies $\leq 1/4$ of the sampling frequency because of the chosen differentiator [1].

3.3. Increased robustness to additive noise of DIPA-2

Here we give some considerations why the proposed DIPA-2 algorithm can be more robust compared to DESAs in case of additive white noise.

It is known that discrete TEO is sensitive to wideband noise [1], however the effects of noise can be greatly reduced using lowpass filtering of TEO's output [2]. The idea originates from the fact that TEO's output normally has a lower bandwidth comparing to the components of the signal itself.

Let us denote outputs of differentiators derived from 3-sample symmetric difference as

$$d_1[n] = (x[n+1] - x[n-1])/2,$$
$$d_2[n] = \frac{1}{4}x[n+2] - \frac{1}{2}x[n] + \frac{1}{4}x[n-2], \qquad (10)$$
$$d_3[n] = \frac{1}{8}x[n+3] - \frac{3}{8}x[n+1] + \frac{3}{8}x[n-1] - \frac{1}{8}x[n-3].$$

Using (1) and (9) we get an alternative discrete approximation of TEO:

$$\widetilde{\Psi}(x[n]) = (d_1[n])^2 - x[n]d_2[n] = \frac{1}{4}\Psi(x[n-1]) + \frac{1}{2}\Psi(x[n]) + \frac{1}{4}\Psi(x[n+1])$$
$$\widetilde{\Psi}(d_1[n]) = (d_2[n])^2 - d_1[n]d_3[n] = \frac{1}{4}\Psi(d_1[n-1]) + \frac{1}{2}\Psi(d_1[n]) + \frac{1}{4}\Psi(d_1[n+1]) \qquad (11)$$

From (11) it is evident that TEO approximation $\widetilde{\Psi}$ based on 3-sample differentiator is actually a lowpass filtered output of original TEO (3).

4. EXPERIMENTAL EVALUATION

We have made some experimental comparisons with known energy separation algorithms (DESA-1 and DESA-2 [1]) and known Prony's-based estimation algorithms (original 4-point Prony's method [7] denoted as 'Prony' and modified 5-point Prony's method [8] denoted as 'Prony.m'). These known estimation techniques are compared to 4-point DIPA-1 and 7-point DIPA-2 presented in the paper. The performance comparisons are made using artificial oscillatory signals which are given by the following equation [8]:

$$\left[1 + k\cos\left(\frac{\pi}{100}n\right)\right]\cos\left[\frac{\pi}{5}n + 20\lambda\sin\left(\frac{\pi}{100}n\right)\right]$$

where $(k,\lambda)\epsilon\{(0.05i, 0.05j) :\ i = 1,\ldots,10; j = 1,\ldots,5\}$ and $n = 1,\ldots,400$. The initial test set consists of 50 sample sequences with different AM and FM ratios in the range from 5 to 50%. Then Gaussian white noise was added forming 5 test sets with different SNRs. For each sequence mean absolute errors of estimated frequency and amplitude values were calculated. The errors were classified as gross errors (GE) and fine errors (FE). Then total percentage of gross errors was calculated as

$$GE(\%) = \frac{N_{GE}}{N_s} \times 100,$$

where N_{GE} - the number of frames with errors higher than $\pm 20\%$ of the true values and N_s - total number of frames. Fine errors were normalized by the true values and averaged over their total quantity:

$$FE_P = \frac{1}{N_{FE}} \sum_{n=1}^{N_{FE}} \frac{|P^{true}(n) - P^{est}(n)|}{P^{true}(n)} \times 100,$$

where N_{FE} - number of fine errors, P^{true} - true value of the parameter, P^{est} - estimated value of the parameter, P stands for corresponding parameter (instantaneous frequency or amplitude).

	GE frequency %		FE frequency Mean Abs(%)		GE amplitude %		FE amplitude Mean Abs(%)	
Median filter	no	yes	no	yes	no	yes	no	yes
clear cosine signals								
DESA1 (5pt)	**0**	**0**	0.19	0.18	**0**	**0**	0.25	0.21
DESA2 (5pt)	**0**	**0**	0.22	0.20	**0**	**0**	0.28	0.23
Prony (4pt)	**0**	**0**	0.22	0.22	**0**	**0**	0.99	0.99
Prony.m (5pt)	**0**	**0**	**0.08**	**0.08**	-	-	-	-
DIPA1 (4pt)	**0**	**0**	0.38	0.21	0.36	**0**	9.86	3.21
DIPA2 (7pt)	**0**	**0**	0.15	0.15	**0**	**0**	**0.19**	**0.17**
cosine signals with noise, SNR 40dB								
DESA1 (5pt)	0.01	**0**	1.5	0.68	**0**	**0**	1.71	0.90
DESA2 (5pt)	**0**	**0**	2.04	1.09	0.02	**0**	2.70	1.26
Prony (4pt)	2.49	0.02	3.48	1.56	3.12	0.20	4.12	1.97
Prony.m (5pt)	8.40	0.89	4.85	2.72	-	-	-	-
DIPA1 (4pt)	2.76	0.20	3.79	1.84	6.49	0.12	9.68	3.72
DIPA2 (7pt)	**0**	**0**	**0.58**	**0.36**	**0**	**0**	**0.59**	**0.46**
cosine signals with noise, SNR 30dB								
DESA1 (5pt)	2.70	0.08	4.13	2.02	2.64	0.54	4.90	2.69
DESA2 (5pt)	4.95	0.99	5.50	3.14	10.2	1.95	6.28	3.44
Prony (4pt)	19.1	5.98	6.63	4.18	24.3	7.74	7.24	4.52
Prony.m (5pt)	33.3	16.6	8.08	5.54	-	-	-	-
DIPA1 (4pt)	21.0	6.41	7.19	4.64	23.7	4.48	8.90	4.59
DIPA2 (7pt)	**0.05**	**0**	**1.80**	**0.96**	**0.01**	**0**	**1.77**	**1.22**
cosine signals with noise, SNR 20dB								
DESA1 (5pt)	25.8	7.81	7.44	5.30	31.6	13.4	8.38	5.86
DESA2 (5pt)	37.3	14.5	8.93	6.48	48.6	20.2	8.92	6.84
Prony (4pt)	57.0	37.9	8.86	6.82	60.3	33.4	9.17	7.81
Prony.m (5pt)	69.8	57.2	10.0	8.01	-	-	-	-
DIPA1 (4pt)	59.8	31.8	9.33	7.62	46.5	14.2	8.81	7.46
DIPA2 (7pt)	**3.60**	**0.30**	**4.90**	**2.94**	**1.42**	**0.37**	**5.50**	**3.80**
cosine signals with noise, SNR 15dB								
DESA1 (5pt)	45.8	23.1	8.68	7.30	53.7	30.3	9.08	7.69
DESA2 (5pt)	58.3	34.7	9.50	7.97	66.2	40.9	9.92	8.43
Prony (4pt)	74.3	56.3	9.24	7.85	73.9	47.1	9.70	9.28
Prony.m (5pt)	82.5	76.5	10.23	9.93	-	-	-	-
DIPA1 (4pt)	76.3	51.2	9.52	8.56	57.6	27.7	9.46	8.88
DIPA2 (7pt)	**12.6**	**2.70**	**6.99**	**4.88**	**11.6**	**4.25**	**8.35**	**6.27**

Table 1 - Performance evaluation

In order to make our evaluations comparable to the results presented in [1] we optionally apply 5-point median post smoothing. The evaluation results are summarized in Table 1. For clear cosine signals performance of all algorithms is very close, however, 5-point modified Prony's algorithm gives the most accurate frequency estimates and DIPA-2 gives the most accurate amplitude estimates. When noise is added conventional estimation techniques based on Prony's method ('Prony' and ' Prony.m') are much worse compared to DESAs and 4-point DIPA-1 does not show any improvement as well. While 7-point DIPA-2 performs very well – its gross error rate and values of mean fine errors are much smaller for SNR≤ 30 dB. Median filtering extends analysis window for additional 4-points, however despite that overall accuracy of all methods with median filtering is still worse than that of 7-point DIPA-2 without median smoothing. A short example of parameters estimation at SNR 20dB is given in figure 1.

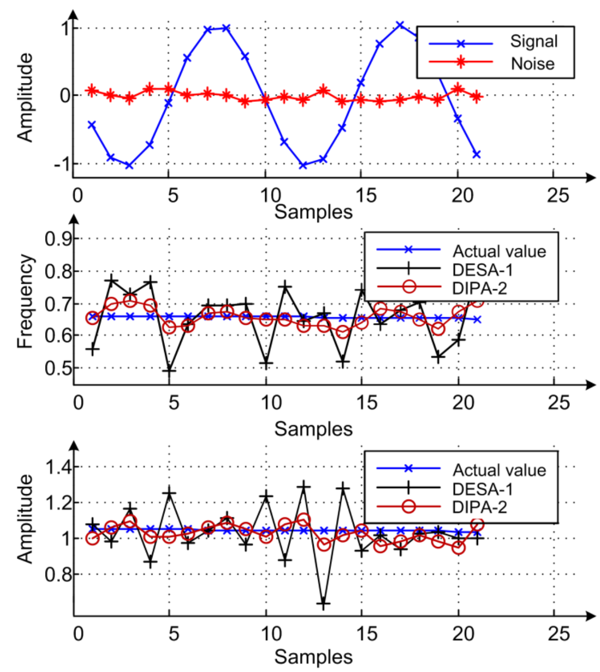

Figure 1 – Instantaneous amplitude and frequency estimation, SNR 20dB

5. CONCLUSIONS

An algorithm for instantaneous parameters (frequency, amplitude, phase and damping factor) estimation which can be applied to AM-FM oscillatory signals has been presented. The algorithm is based on Prony's method and interprets the signal in terms of instantaneous parameters using its instantaneous derivatives as inputs. According to experimental evaluations the algorithm is significantly more robust to Gaussian white noise compared to DESAs and original Prony's method.

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