ANALYSIS OF COLOURED NOISE IN RECEIVED SIGNAL STRENGTH USING THE ALLAN VARIANCE

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ABSTRACT

The received signal strength (RSS) of wireless signals has been widely used in communications, localization and tracking. Theoretical modelling and practical applications often make a white noise assumption when dealing with RSS measurements. However, as we will show in this paper, the noise present in RSS measurements has time-dependency properties. In order to study these properties and provide noise characterisation, we propose to use the Allan Variance (AVAR) and show its better performance in comparison with direct analysis in the frequency domain using a periodogram. We further study the contribution of each component by testing real RSS data. Our results confirm that the noise associated with RSS signals is actually coloured and demonstrate the appropriateness of AVAR for the identification and characterisation of these components.

Index Terms— RSS, coloured noise, Allan variance, noise characterisation, 802.11

1. INTRODUCTION

The application of wireless signals for the purpose of localization, communication and tracking has recently shown increased popularity [1, 2]. Among the various properties of wireless signals, the received signal strength (RSS) constitutes a key performance metric and is often used to implement main functionalities of the design. For example, a localization system can measure the RSS of the signals received at a destination and use these values to estimate the distance between source and destination [3]. In this context, inaccurate RSS modelling and measurement can lead to a number of problems such as communication outage, poor localization, loss of tracking, vehicle collision [4, 5] etc.

The work in [6] set the basis of the study of RSS and its properties by proposing the free space propagation model. More recent work, e.g. [7, 8], focused on the radio propagation channels and characterised the reflecting surface, terrain type and fading as the three major factors affecting RSS values. The random variation of RSS was further modelled as white noise processes. For simplicity reasons, an assumption of the RSS noise as a white process following a lognormal distribution in the logarithmic domain, is widely adopted in the literature and practised in the real world [1, 9].

The white noise assumption is based on the hypothesis that the noise power spectral density (PSD) approximately equally spreads across the whole bandwidth. Such assumption provides a manageable model to describe RSS and makes its mathematical expression tractable. However, if meaningful variations in the PSD do exist, the noise can no longer be considered as white and this issue should be further investigated. In this paper, we test the aforementioned hypothesis using real data and discover that besides white noise components, there exist several types of noise that show correlation time-dependencies, which are commonly referred to as coloured noise. These types of noise are also present in a number of physical processes. For example, the measurements from some instruments include such noise components with both short and long term correlations due to circuit drift leading to nonuniform spectral distributions [10].

A commonly employed characterisation procedure for coloured noise relies on the definition of parametric models for the PSD which consider specific contribution from each noise component. By adjusting the parametric equation in the frequency domain (e.g. using the periodogram), the different noise levels can be estimated. However, as we will show with our experiments, the use of the periodogram (specially when estimated through the Discrete Fourier Transform) is prone to errors when the PSD shows a significant frequency-dependency behaviour, especially for power-law densities [11]. Exploring alternative approaches is thus required.

In order to overcome the barrier in identifying different noise in RSS measurements, this paper introduces the Allan variance (AVAR) [12]. AVAR is a standard method to characterise noise processes [13] in precise clock systems. Our experiments will demonstrate that the use of AVAR outperforms direct characterisation in the frequency domain and is a promising tool to reveal the contribution from noise compo-

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nents which follow a power-law property [13].

The contribution of this paper is three-fold: 1) We first show that the noise present in RSS measurements is actually coloured by testing the null hypothesis of white noise on real RSS data. 2) We then introduce the AVAR as an appropriate tool to characterise the main noise components by applying Least Squares fitting over a parametrized model directly mapped to the PSD. 3) We provide noise characterisation of RSS data to conclude that the three major types of existing noise components correspond to Brownian motion, Flicker and white noises.

The paper continues as follows: Section 2 presents the RSS model and introduces the AVAR for noise characterisation. Experimental evidence of the appropriateness of using AVAR is also provided. Section 3 uses real data to show the time-dependency of the noise and evaluates the analysis tool by providing noise characterisation and comparing the obtained performance to direct estimation in the frequency domain. Section 4 discusses these results together with the relevant issues of RSS measurement and noise modelling. Section 5 concludes this paper.

2. ANALYSIS OF RSS NOISE

2.1. RSS Signal Model

Our study is based on the standard wireless RSS signal model given by [8]

$$P_y(d)[dB] = P_x - \bar{PL}(d_0) - 10n\log(\frac{d}{d_0}) + X_\sigma, \quad (1)$$

where n is the path loss factor whose value is associated with the propagation environment [1], P_x is the transmitting power, d and d_0 are the transmitter-receiver distance and reference distance respectively. X_σ is the random noise term, and it is usually modelled in the literature (e.g. [14, 15]), as a zero-mean Gaussian distributed random variable (in dB) with standard deviation σ and (band-limited) power spectral density (PSD) given by

$$S_w(f) = \frac{N_0}{2} \quad (2)$$

where $N_0 = 2\sigma^2$.

Equation (1) can be rewritten for a fixed position of transmitter and receiver as

$$P_y(t)[dB] = P_x - \bar{PL}(d_0) - 10n\log(\frac{d}{d_0}) + X_\sigma(t), \quad (3)$$

leading to a signal model where only the noise term is time varying.

The white noise assumption given by (2) significantly simplifies the mathematical treatment and analysis of the present noise. However, it can be an over-simplified makeshift in describing the underlying noise components when significant correlations do exist. In this case, the PSD is no longer constant, and this type of noise is often referred to as coloured.

2.2. Allan Variance

The AVAR was proposed in 1966 by David W. Allan as an effective tool for the characterisation of the underlying noise processes in precise clock systems [12]. Since then, it has been widely used for this purpose and, together with some modifications, has been recommended as a standard [13]. Let $X(t)$ be the noise signal to be analyzed. The AVAR at different time instants t_m is:

$$\sigma_\tau^2 = \frac{1}{2}\text{Var}\{\overline{X}(t_m+\tau) - \overline{X}(t_m)\} = \frac{1}{2}\text{Var}\{\overline{X}_{m+1} - \overline{X}_m\} \quad (4)$$

where

$$\overline{X}_m = \frac{1}{\tau}\int_{t_m}^{t_m+\tau} X(t)dt \quad (5)$$

and, for stationary processes, is only a function of τ - the correlation time. In order to estimate AVAR, (5) is numericaly computed as a sample average for different time instants $t_m,\ m = 1, ..., M-1$ and the sample variance is used for final estimation (note that $E\{\overline{X}_{m+1} - \overline{X}_m\} = 0$ for stationary processes):

$$\hat{\sigma}_\tau^2 = \frac{1}{2(M-1)}\sum_{m=1}^{M-1}(\mu_{m+1} - \mu_m)^2 \quad (6)$$

where

$$\mu_m = \frac{1}{K}\sum_{k=1}^{K} X(t_m + kTs)$$

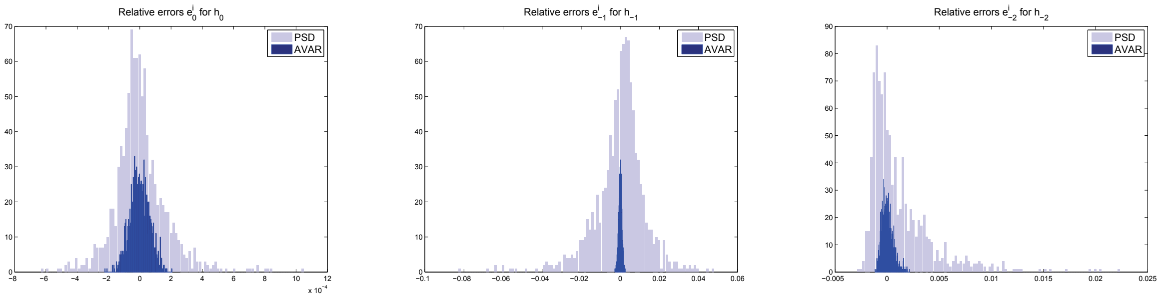
with T_s the sampling time and $K = \lfloor \tau/T_s \rfloor$. The number of available points M to calculate the sample variance is a function of the total length of the signal $N \cdot T_s$ and the integration interval τ

$$M = \lfloor \frac{NT_s}{\tau} \rfloor.$$

Let us consider a (one-sided) power-law spectral density which can be reasonably used to model the random fluctuations in RSS signals [12]:

$$S_X(f) = \sum_\alpha h_\alpha f^\alpha. \quad (7)$$

In practice, these random fluctuations can often be represented by the sum of five noise processes $-2 \leq \alpha \leq 2$ assuming to be independent [12, 13]. In this paper, we follow the approach in [12], and only consider the major components which contribute the most part of the noise and essentially shape the AVAR curve. Specifically, the following 5 components are analyzed [16]: Brownian (random walk) noise ($\alpha = -2$), Pink (flicker) noise ($\alpha = -1$), White noise



(a) (b) (c)

Fig. 1. Distribution of the relative estimation errors of h_0 (a), h_{-1} (b), and h_{-2} (c) using AVAR. Results obtained from direct estimation using the periodogram are also presented for comparison.

($\alpha = 0$), Blue noise ($\alpha = 1$) and Violet noise ($\alpha = 2$). The reference to actual colour names is related to the higher or lower frequency content in comparison to the visible light spectra (except for the Brown noise, which is named after Robert Brown, the discoverer of Brownian motion).

The power-law identities present in RSS noise components can lead to simpler analyses if they are handled correctly. This is achieved by taking advantage of the one-to-one relationship between the parametrization of AVAR and the noise PSD.

Specifically, for the model in (7), and considering the bounds $-2 \leq \alpha \leq 2$, AVAR can be expressed in the time domain as:

$$\sigma_\tau^2 \approx Ah_{-2}\tau + Bh_{-1} + \frac{Ch_0}{\tau} + \frac{Dh_1 + Eh_2}{\tau^2} \qquad (8)$$

with the mapping coefficients given by

$$A = \frac{2\pi^2}{3}, \; B = 2\ln(2), \, C = \frac{1}{2},$$
$$D = \frac{1.038 + 3\ln(w_h\tau)}{4\pi^2}, E = \frac{3f_h}{4\pi^2}$$

where $f_h = w_h/(2\pi)$ is the bandwidth of the measurement system.

The $h_{-\alpha}$ coefficients can be estimated using Least Squares (LS) algorithms as in [17]. The use of AVAR to characterise the noise PSD may be preferable to (for instance) directly fitting (7) using the periodogram. To illustrate this, we conducted a simple experiment: We generated 1000 realizations (each with $N = 2^{14} = 16384$ samples) of a noise process whose power spectral density is given by (7) with $h_{-2} = 0.01, h_{-1} = 1, h_0 = 100, h_1 = 0, h_2 = 0$ as in [17]. For each of them, we estimated the h_α using LS to fit both the AVAR curve ($\hat{h}_\alpha^\sigma$) and the PSD directly estimated with the periodogram ($\hat{h}_\alpha^{PSD}$) for the sake of comparison. The histograms for the relative errors $e_\alpha^i = (\hat{h}_\alpha^i - h_\alpha)/h_\alpha$, $i = \{\sigma, PSD\}$, are presented in Fig. 1. The results demonstrate that the AVAR method has lower estimation errors and dispersions than the direct estimation of PSD for all the three coefficients.

3. EXPERIMENTS ON REAL DATA

We have collected a dataset of RSS measurements from an 802.11 system using omnidirectional antennas on both the transmitter and receiver. The positions of the transmitter and receiver were fixed with the distance of 1.016m (40 inch) and both of them were equipped with a WiFi interface (Gigabyte GN-WI01GT). The carrier frequency (central frequency) was set to 5.22GHz (WLAN Channel 44) which was known and tested to be free from any adjacent frequency interferences. The transmitter constantly broadcasts BEACON signals every 100ms, which were received and demodulated by the receiver. We used an off-line application to extract the RSS measurements with time stamps from the original received packets.

3.1. Testing the Hypothesis of White Noise

To verify that the noise present in RSS measurements is actually coloured, we tested $N = 15000$ RSS measurements against the null hyphotesis of whiteness of the noise. We employed the Ljung-Box Q-test [18] for the sample autocorrelation coefficient of the noise process $X_\sigma(j), j = 1, \ldots, N$:

$$\hat{\rho}(k) = \frac{\sum_{j=k+1}^{N} X_\sigma(j)X_\sigma(j-k)}{\sum_{j=1}^{N} X_\sigma^2(j)} \qquad k \in \mathbb{Z}$$

with null and alternative hypothesis:

$$\mathcal{H}_0(\text{white noise}): \hat{\rho}(k) = 0 \quad \forall k \neq 0$$
$$\mathcal{H}_1(\text{coloured noise}): \hat{\rho}(k) \neq 0 \quad \text{for some } k \neq 0.$$

Under $\mathcal{H}_0$, the statistic

$$Q_m = N(N+2)\sum_{k=1}^{m} \frac{\hat{\rho}^2(k)}{N-k}$$

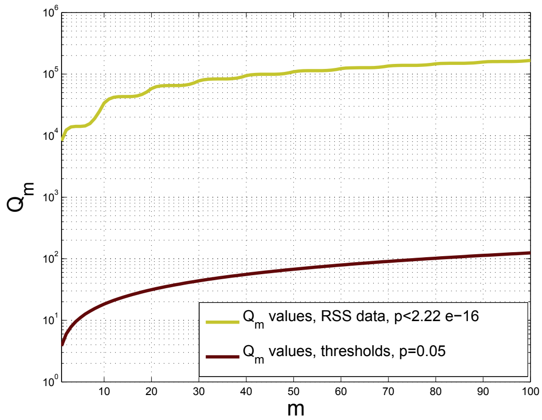


Fig. 2. Ljung-Box Q-test statistics obtained for $m = 1, 2, \ldots, 100$ time lags from $N = 15000$ RSS samples. Threshold values for $\alpha = 0.05$ are also presented

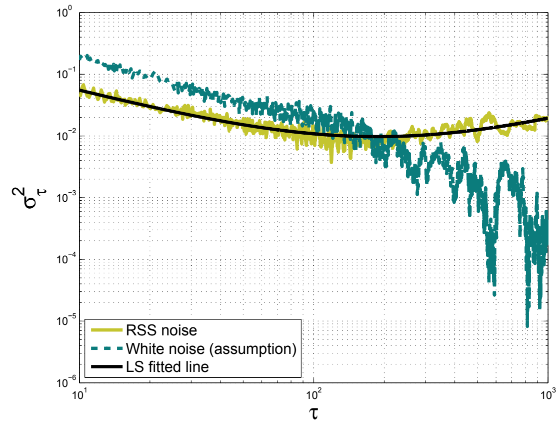


Fig. 3. AVAR and LS fitted line for the real RSS dataset.

asymptotically follows a Chi-squared distribution with m degrees of freedom (χ^2_m). The results of the test are shown in Fig. 2, where the obtained Q_m values for $m = 1, 2, \ldots, 100$ together with the threshold values at which, we would reject the null hypothesis at $\alpha = 0.05$ significance level are presented. The obtained values are far above the thresholds which means that the null hypothesis can be rejected with a very low error probability. Actually, the p values obtained from our data are all below $2.22 \cdot 10^{-16}$.

The results of this test show that the noise present in the RSS measurements has a time-dependant statistical behaviour.

3.2. Noise Analysis using AVAR

Fig. 3 shows the AVAR estimation obtained from the acquired dataset. The coloured noise coefficients from LS estimation are $h_{-2} = 2.217 \cdot 10^{-5}, h_{-1} = 3.105 \cdot 10^{-3}, h_0 = 1.017, h_1 < 1e-7, h_2 < 1e-7$. The LS fitted curve is also presented as well as a realization of a white noise Gaussian process of the same power for the sake of comparison. Results show that, because of the existence of coloured noise components, the AVAR curves of the RSS measurements do not decrease linearly with the increase of correlation time τ on the log-log diagram, whereas those with artificial white noise do follow this linear law. As we have expected from the results in the previous section, we can conclude that the traditional white noise assumption is inaccurate.

From the obtained values, we observe that the three contributions to the RSS measurement noise are from random walk (h_{-2}), flicker (h_{-1}) and white noise (h_0). Theoretically the other two components – h_1 and h_2 – cannot be excluded. However, the overall AVAR (and thus, the noise) is dominated by these three types. We can also see from the figure that the fluctuations in the curve become more intense for higher τ values. The explanation for this comes from the intrinsic nature of AVAR calculation: the larger the correlation lag, the lower the number of samples available to estimate the variance [see (6)], and thus the curve becomes less smooth.

Fig. 4 shows an estimation of the PSD of the noise obtained using a raw FFT-based periodogram. The flicker ($\alpha = -1$) and white noise ($\alpha = 0$) contributions can be easily appreciated in the figure while the random walk contribution ($\alpha = -2$) is not so obvious due to its low value. The significant frequency peak at $f = 1$ Hz prevents proper identification of the white noise contribution, which leads to a poor fitting of the PSD model if the periodogram estimation is directly employed. The theoretical fittings obtained from both the AVAR and PSD estimations are also shown in the figure. Actually, the values obtained from the latter ($h_{-2} = 6.527 \cdot 10^{-7}, h_{-1} = 1.793 \cdot 10^{-3}, h_0 = 3.809, h_1 < 1e-7, h_2 < 1e-7$) tend to overestimate the white noise component and underestimate the other $1/f^\alpha$ coefficients. This can be observed in the figure where the negative slope of the PSD fitting is lower in absolute value, and the constant noise floor is reached at lower frequencies. On the other hand, the fitting obtained using AVAR better follows the PSD curve, following the same slope and reaching the noise floor level at the expected frequencies.

4. DISCUSSION AND FUTURE WORK

Even though this paper used 802.11 signals to test the hypothesis and the analysis tools, a similar methodology can be easily applied to analyze other types of wireless signals following the propagation model (3). Our future work aims at further investigating the contribution from each noise component and research on the corresponding source. To this end, we will acquire and analyze more datasets from different settings and environments and employ AVAR for their character-

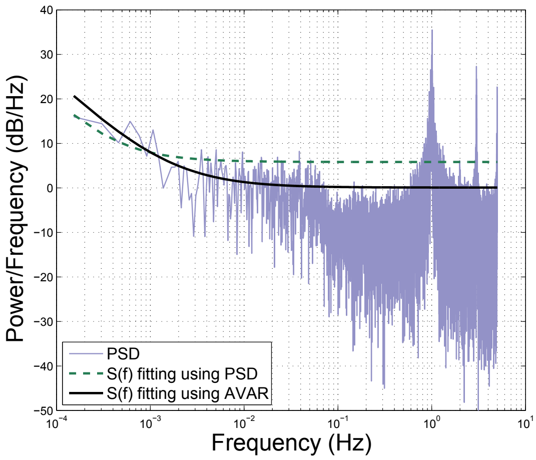


Fig. 4. PSD estimation of the RSS signal with theoretical fittings using both Allan and straightforward estimation.

isation.

From the paper, we observe that RSS measurements demonstrate clear existence of coloured noise components. However, it might not be always the case in other measurements. This raises an interesting topic which is worth further investigation: what is the proper way to generate the noise components and reconstruct the coloured noise, given the discovery of this paper? The successful solution to this problem can be significantly beneficial for testing and evaluation purposes. Another interesting topic is to propose improved practical applications, e.g. localization, based on the methods in this paper.

5. CONCLUSION

This paper studies the noise of RSS measurements which commonly exists in current wireless systems. We find that there certainly exist coloured noise components that contribute to the RSS measurement. A comparison of AVAR and direct estimation in the frequency domain for the analysis of noise components is presented with the conclusion that AVAR is more suitable due to its lower estimation error and higher accuracy. The analysis method and results of this paper can be used in scenarios requiring accurate modelling and reconstruction of RSS data.

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