POWER MINIMIZATION IN THE MULTIUSER MIMO-OFDM BROADCAST CHANNEL WITH IMPERFECT CSI

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ABSTRACT

This work addresses the design of linear precoders and receivers in multiuser *Multiple-Input Multiple-Output* (MIMO) downlink channels using *Orthogonal Frequency Division Multiplexing* (OFDM) modulation when only partial *Channel State Information* (CSI) is available at the transmitter. Our aim is to minimize the total transmit power subject to per-user *Quality-of-Service* (QoS) constraints expressed as per-user rates. We propose a gradient-projection algorithm to optimally distribute the per-user rates among the OFDM subcarriers. Then, another algorithm is used to obtain the per-subcarrier precoders and receivers that minimize the overall transmit power. Based on the *Minimum Mean Square Error* (MMSE) duality between the MIMO *Broadcast Channel* (BC) and the MIMO *Multiple Access Channel* (MAC), both algorithms perform an *Alternating Optimization* (AO).

1. INTRODUCTION

The radio interface of the current broadband wireless communication standards, such as *Long-Term Evolution* (LTE) or LTE-Advanced [1, 2], combine the MIMO technique with OFDM to achieve the demanded high data rates.

This paper focuses on the MIMO-OFDM BC, a suitable model for the downlink of such broadband wireless communication systems. Linear precoding is used at the centralized transmitter in the MIMO-OFDM BC and linear receivers at the user terminals. We also assume perfect CSI at the receivers but the transmitter has partial knowledge of the CSI.

The optimization of MIMO-OFDM BCs has been previously considered, e.g., weighted sum rate maximization algorithms have been proposed for MIMO-OFDM systems in [3,4]. The power minimization with QoS constraints has been already studied in [3,5,6]. However, the case of imperfect CSI at the transmitter was not considered in these papers.

In this work, we will follow an MMSE approach to design the linear precoders and receivers in a MIMO-OFDM BC. Optimal precoders and receivers that minimize the sum MSE in multiuser MIMO-OFDM systems for the perfect CSI have been studied in [7,8]. Moreover, the design of linear precoders for single-user MIMO-OFDM systems with imperfect CSI assuming a certain channel estimation error model is considered in [9]. Another error model is employed in [10] where data is transmitted only to the best user for each subcarrier.

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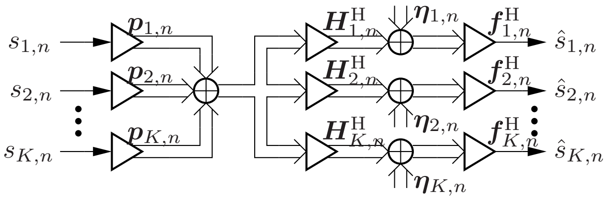

Fig. 1. Per-subcarrier discrete-time equivalent model

We formulate the problem of optimizing the linear precoders and receivers of a MIMO-OFDM BC such that the total transmit power is minimized under per-user rate constraints. We develop a gradient-projection algorithm to optimally transform the per-user rate constraints into per-subcarrier rate constraints following an approach similar to that presented in [5]. We then develop another algorithm to determine the optimal power allocation among users and subcarriers that minimizes the overall transmit power while satisfying the *Quality-of-Service* (QoS) constraints. This second algorithm follows an approach similar to that in [11].

2. SYSTEM MODEL

Figure 1 plots the discrete-time equivalent model corresponding to the n-th subcarrier of a MIMO-OFDM BC. We assume a centralized transmitter sends the data signal $s_{k,n}$ to user $k \in \{1,\ldots,K\}$ over subcarrier $n \in \{1,\ldots,N\}$. Data signals corresponding to different users and/or subcarriers are mutually independent and normalized to unit power (i.e. $\mathrm{E}[|s_{k,n}|^2] = 1$). We also assume that the transmitter is equipped with T antennas while all receivers are equipped with the same number R of receiving antennas. Hence, each data signal $s_{k,n}$ is precoded by $\boldsymbol{p}_{k,n} \in \mathbb{C}^T$ and propagates over the MIMO channel $\boldsymbol{H}_{k,n} \in \mathbb{C}^{T\times R}$. The received signal is perturbed by the additive Gaussian noise $\boldsymbol{\eta}_{k,n} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \boldsymbol{C}_{\boldsymbol{\eta}_{k,n}})$ and filtered with the linear receiver $\boldsymbol{f}_{k,n} \in \mathbb{C}^R$. Along this work we will assume that perfect CSI is available at the receivers whereas the transmitter knows partial CSI modelled by v, and the conditional PDFs $f_{\boldsymbol{H}_k|v}(\boldsymbol{H}_k|v)$ associated to each v. Moreover we assume the cyclic prefix of the OFDM modulation is large enough to avoid intercarrier interference.

Let us now collect the N symbols transmitted during one OFDM symbol towards user k into $\boldsymbol{s}_k = [s_{k,1}, \ldots, s_{k,N}]^{\mathrm{T}}$. Next we define the following matrices to represent the MIMO-OFDM precoder, $\boldsymbol{P}_k = \mathrm{blockdiag}(\boldsymbol{p}_{k,1}, \ldots, \boldsymbol{p}_{k,N})$, channel, $\boldsymbol{H}_k = \mathrm{blockdiag}(\boldsymbol{H}_{k,1}, \ldots, \boldsymbol{H}_{k,N})$, receiver filter, $\boldsymbol{F}_k = \mathrm{blockdiag}(\boldsymbol{f}_{k,1}, \ldots, \boldsymbol{f}_{k,N})$, and channel noise $\boldsymbol{\eta}_k = \mathrm{blockdiag}(\boldsymbol{\eta}_{k,1}, \ldots, \boldsymbol{\eta}_{k,N})$, corresponding to user k. Accordingly, the output of the k-th user receive filter (i.e. the k-th user estimated symbols) are as follows

$$\hat{\boldsymbol{s}}_k = \boldsymbol{F}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \sum_{i=1}^{K} \boldsymbol{P}_i \boldsymbol{s}_i + \boldsymbol{F}_k^{\mathrm{H}} \boldsymbol{\eta}_k. \tag{1}$$

The k-th user average Mean Square Error (MSE) is given by

$$\begin{aligned} \overline{\mathrm{MSE}}_k^{\mathrm{BC}} &= \mathrm{E}[\|\boldsymbol{s}_k - \hat{\boldsymbol{s}}_k\|_2^2 \,|\, v] \\ &= \mathrm{E}\left[\left\|\mathbf{I}_N - \boldsymbol{F}_k^{\mathrm{H}} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{P}_k\right\|_{\mathrm{F}}^2 + \mathrm{tr}\left(\boldsymbol{F}_k^{\mathrm{H}} \boldsymbol{X}_k \boldsymbol{F}_k\right) | \, v\right] \end{aligned} \tag{2}$$

where $\boldsymbol{X}_k = \boldsymbol{H}_k^{\mathrm{H}} \sum_{i \neq k} \boldsymbol{P}_i \boldsymbol{P}_i^{\mathrm{H}} \boldsymbol{H}_k + \boldsymbol{C}_{\boldsymbol{\eta}_k}$. Note that $\boldsymbol{P}_k$ depends on v but $\boldsymbol{F}_k$ depends on the instantaneous CSI $\boldsymbol{H}_k$. For given precoder $\boldsymbol{P}_k$ and channel $\boldsymbol{H}_k$, the receiver filter that minimizes (2) is $\boldsymbol{F}_{k,\mathrm{MMSE}} = (\boldsymbol{H}_k^{\mathrm{H}} \sum_{i=1}^{K} \boldsymbol{P}_i \boldsymbol{P}_i^{\mathrm{H}} \boldsymbol{H}_k + \boldsymbol{C}_{\boldsymbol{\eta}_k})^{-1} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{P}_k$. Substituting $\boldsymbol{F}_{k,\mathrm{MMSE}}$ into (2) and applying the matrix inversion lemma we get the following expression

$$\overline{\mathrm{MMSE}}_k^{\mathrm{BC}} = \mathrm{E}\left[\mathrm{tr}\left(\mathbf{I}_N + \boldsymbol{P}_k^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{X}_k^{-1} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{P}_k\right)^{-1} | \, v\right]. \tag{3}$$

We next address the joint optimization of the linear precoders and equalizers $\{\boldsymbol{P}_k, \boldsymbol{F}_k\}_{k=1}^{K}$ to minimize the sum transmit power $\sum_{i=1}^{K} \|\boldsymbol{P}_i\|_{\mathrm{F}}^2$ while ensuring that the k-th user's average rate

$$\mathrm{E}[R_k | \, v] = \mathrm{E}\left[\log_2 |\mathbf{I}_N + \boldsymbol{P}_k^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{X}_k^{-1} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{P}_k| \, | \, v\right] \tag{4}$$

satisfies the constraint $\mathrm{E}[R_k | \, v] \geq \rho_k$ where ρ_k is the minimum rate assigned to user k.

Introducing the matrix $\boldsymbol{C}_k = \mathbf{I}_N + \boldsymbol{P}_k^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{X}_k^{-1} \boldsymbol{H}_k^{\mathrm{H}} \boldsymbol{P}_k$, Eq. (3) and (4) can be rewritten as $\overline{\mathrm{MMSE}}_k^{\mathrm{BC}} = \mathrm{E}[\mathrm{tr}(\boldsymbol{C}_k) | \, v]$ and $\mathrm{E}[R_k | \, v] = \mathrm{E}[\log_2(|\boldsymbol{C}_k^{-1}|) | \, v]$, respectively. Note that, due to the block diagonal structure of $\boldsymbol{H}_k$, $\boldsymbol{C}_k$ is diagonal and the k-th user rate can be expressed as the sum of the per-subcarrier rates,

$$\mathrm{E}[R_k | \, v] = \sum_{j=1}^{N} \mathrm{E}[R_{k,j} | \, v] = \sum_{j=1}^{N} \mathrm{E}[\log_2([\boldsymbol{C}_k]_{j,j}^{-1}) | \, v]. \tag{5}$$

Hence, per-user rate constraints can be satisfied by imposing a set of per-subcarrier rate constraints $\boldsymbol{\varrho}_k = [\varrho_{k,1}, \ldots, \varrho_{k,N}]^{\mathrm{T}}$ such that $\sum_{j=1}^{N} \varrho_{k,j} = \rho_k$. Due to Jensen's inequality, we can introduce new lower bounds $-\mathrm{E}[\log_2([\boldsymbol{C}_k]_{n,n}) | \, v] \geq -\log_2(\mathrm{E}[[\boldsymbol{C}_k]_{n,n} | \, v])$. Moreover, the average MSE per subcarrier is $\overline{\mathrm{MMSE}}_{k,n}^{\mathrm{BC}} = \mathrm{E}[[\boldsymbol{C}_k]_{n,n} | \, v]$. Therefore, we can equivalently express the rate requirements as MMSE upper bounds as follows

$$\overline{\mathrm{MMSE}}_{k,n}^{\mathrm{BC}} \leq 2^{-\varrho_{k,n}} \tag{6}$$

where $\varrho_{k,n}$ holds for the rate constraint for user k and subcarrier n. Notice that the MMSE constraints are more restrictive that the original ones. For further details see [12].

Using the MMSE based constraints, the power minimization problem for given per-carrier rates reads as

$$\min_{\{\boldsymbol{P}_k, \boldsymbol{F}_k\}_{k=1}^{K}} \sum_{i=1}^{K} \|\boldsymbol{P}_i\|_{\mathrm{F}}^2 \;\; \text{s.t.} \;\; \overline{\mathrm{MMSE}}_{k,n}^{\mathrm{BC}} \leq 2^{-\varrho_{k,n}} \, \forall k, n \tag{7}$$

3. PROBLEM FORMULATION

As explained in the previous section, before solving the optimization problem (7) it is necessary to find the optimal per-subcarrier targets $\{\varrho_{k,n}\}_{k=1,n=1}^{K,N}$.

We start considering the MSE duality between the BC and the MAC filters as described in [13]. Such duality provides conversion formulas to switch from one domain to another since we can always find MAC receivers $\boldsymbol{g}_{k,n}$ (BC precoders) and precoders $\boldsymbol{t}_{k,n}$ (BC receivers) such that $\overline{\mathrm{MSE}}_{k,n}^{\mathrm{BC}} = \overline{\mathrm{MSE}}_{k,n}^{\mathrm{MAC}}$, $\forall k, n$. Moreover, the same transmit power is used in both the BC and the MAC. The dual MAC channel is $\boldsymbol{H}_{k,n} \boldsymbol{C}_{\eta_{k,n}}^{-\mathrm{H}/2}$, and $\boldsymbol{\eta}_n$ is the AWGN. To compute the expectations introduced by the imperfect CSI we perform the Montecarlo Method, i.e. the M channel realizations $\hat{\boldsymbol{H}}_{k,n}^{(1)}, \ldots, \hat{\boldsymbol{H}}_{k,n}^{(M)}$ resulting from the PDF $f_{\boldsymbol{H}_k|v}(\boldsymbol{H}_k|v)$, together with their precoders $\boldsymbol{t}_{k,n}^{(1)}, \ldots, \boldsymbol{t}_{k,n}^{(M)}$ are collected into $\boldsymbol{\tau}_{k,n} = \frac{1}{\sqrt{\xi_{k,n}}}[\hat{\boldsymbol{H}}_{k,n}^{(1)} \boldsymbol{C}_{\eta_{k,n}}^{-\mathrm{H}/2} \boldsymbol{t}_{k,n}^{(1)}, \ldots, \hat{\boldsymbol{H}}_{k,n}^{(M)} \boldsymbol{C}_{\eta_{k,n}}^{-\mathrm{H}/2} \boldsymbol{t}_{k,n}^{(M)}]$. Now, let us define the average transmit power $\xi_{k,n} = \frac{1}{M} \sum_{m=1}^{M} ||\boldsymbol{t}_{k,n}^{(m)}||_2^2$. Therefore, the average MSE in the dual MAC can be written as

$$\begin{aligned} \overline{\mathrm{MSE}}_{k,n}^{\mathrm{MAC}} = 1 &- \frac{2}{M} \mathrm{Re}\left\{\boldsymbol{g}_{k,n}^{\mathrm{H}} \boldsymbol{\tau}_{k,n} \mathbf{1} \sqrt{\xi_{k,n}}\right\} \\ &+ \boldsymbol{g}_{k,n}^{\mathrm{H}} \sum_{i=1}^{K} \frac{\xi_{i,n}}{M} \boldsymbol{\tau}_{i,n} \boldsymbol{\tau}_{i,n}^{\mathrm{H}} \boldsymbol{g}_{k,n} + \|\boldsymbol{g}_{k,n}\|_2^2 \end{aligned} \tag{8}$$

where $\mathbf{1}$ is the all ones vector.

The optimal MAC receivers can be computed as the MMSE filters $\boldsymbol{g}_{k,n}^{\mathrm{MMSE}}$ (see [12]). However, to allow for the power adaptation we introduce the scalar receiver $r_{k,n}$ so that

$\boldsymbol{g}_{k,n}^{\mathrm{MMSE}} = r_{k,n}\tilde{\boldsymbol{g}}_{k,n}$ and rewrite (8) as

$$\overline{\mathrm{MSE}}_{k,n}^{\mathrm{MAC}} = 1 - \frac{2}{M}\,\mathrm{Re}\left\{r_{k,n}^{*}\tilde{\boldsymbol{g}}_{k,n}^{\mathrm{H}}\boldsymbol{\tau}_{k,n}\mathbf{1}\sqrt{\xi_{k,n}}\right\} + |r_{k,n}|^{2}\,x_{k,n}^{-1} \tag{9}$$

where $x_{k,n}^{-1} = \tilde{\boldsymbol{g}}_{k,n}^{\mathrm{H}}\sum_{i=1}^{K}\frac{\xi_{i,n}}{M}\boldsymbol{\tau}_{i,n}\boldsymbol{\tau}_{i,n}^{\mathrm{H}}\tilde{\boldsymbol{g}}_{k,n} + \|\tilde{\boldsymbol{g}}_{k,n}\|_2^2$ and $\mathbf{1}$ is the all-ones vector. It is straightforward to obtain the optimal scalar receivers by $r_{k,n}^{\mathrm{MMSE}} = \frac{1}{M}\tilde{\boldsymbol{g}}_{k,n}^{\mathrm{H}}\boldsymbol{\tau}_{k,n}\mathbf{1}\sqrt{\xi_{k,n}}x_{k,n}$. Substituting $r_{k,n}^{\mathrm{MMSE}}$ into (9) gives

$$\overline{\mathrm{MMSE}}_{k,n}^{\mathrm{MAC}} = 1 - \frac{\xi_{k,n}}{M^2}|\tilde{\boldsymbol{g}}_{k,n}^{\mathrm{H}}\boldsymbol{\tau}_{k,n}\mathbf{1}|^2 x_{k,n}. \tag{10}$$

We now transform the optimization problem (7) into the following N decoupled optimization problems

$$\min_{\{\boldsymbol{g}_{k,n},\boldsymbol{\tau}_{k,n},\xi_{k,n}\}_{k=1}^{K}} \sum_{i=1}^{K}\xi_{i,n} \text{ s.t. } \overline{\mathrm{MMSE}}_{k,n}^{\mathrm{MAC}} \le 2^{-\varrho_{k,n}}\forall k. \tag{11}$$

Note that the total transmit power $\sum_{i=1}^{K}\sum_{j=1}^{N}\xi_{i,j} = P_{tx}$, is a function of the targets for each user and subcarrier, $\{\varrho_{k,n}\}_{k=1,n=1}^{K,N}$. Therefore, prior to solving the optimization problems (11), it is necessary to find the optimal per-subcarrier constraints $\{\varrho_{k,n}\}_{k=1,n=1}^{K,N}$,

$$\min_{\{\varrho_{k,n}\}_{k=1,n=1}^{K,N}} P_{tx} \quad \text{s.t.} \quad \sum_{j=1}^{N}\varrho_{k,j} = \rho_k\,\forall k. \tag{12}$$

We propose that this minimization problem be solved by means of the following gradient algorithm

$$\varrho'_{k,n} = \varrho_{k,n} - \mu\frac{\partial P_{\mathrm{tx}}}{\partial\varrho_{k,n}}. \tag{13}$$

where μ is the algorithm step-size. To calculate the gradient in (13), we first compute the partial derivatives of the $\overline{\mathrm{MMSE}}^{\mathrm{MAC}}$ in (10) with respect to the rate targets as

$$\frac{\partial\overline{\mathrm{MMSE}}_{k,n}^{\mathrm{MAC}}}{\partial\varrho_{q,c}} = \sum_{i=1}^{K}\sum_{j=1}^{N}\frac{\partial\overline{\mathrm{MMSE}}_{k,n}^{\mathrm{MAC}}}{\partial\xi_{i,j}}\frac{\partial\xi_{i,j}}{\partial\varrho_{q,c}}. \tag{14}$$

The left side of (14) can be straightforwardly obtained from

$$\frac{\partial\overline{\mathrm{MMSE}}_{k,n}^{\mathrm{MAC}}}{\partial\xi_{q,c}} = \begin{cases} y_{k,n}^2(z_{k,n}^{k} - x_{k,n}^{-1})x_{k,n}^2, & q = k, c = n \\ y_{k,n}^2 z_{k,n}^{q} x_{k,n}^2, & q \ne k, c = n \end{cases}$$

where we introduced $y_{k,n} = \frac{1}{M}|\tilde{\boldsymbol{g}}_{k,n}^{\mathrm{H}}\boldsymbol{\tau}_{k,n}\mathbf{1}|$ and $z_{k,n}^{l} = \frac{\xi_{k,n}}{M}||\tilde{\boldsymbol{g}}_{k,n}^{\mathrm{H}}\boldsymbol{\tau}_{l,n}||_2^2$. Notice that this derivative is 0 when $c \ne n$, i.e. the derivatives for different subcarriers are independent.

At the optimum of (11), $\overline{\mathrm{MMSE}}_{k,n}^{\mathrm{MAC}} = 2^{-\varrho_{k,n}}$ and the derivative (14) is equal to $-\ln(2)2^{-\varrho_{k,n}}$ when $q = k$ and $c = n$, and 0 otherwise. Using matrix notation, $\boldsymbol{\Upsilon}_n = -\ln(2)\,\mathrm{diag}(2^{\varrho_{1,n}},\ldots,2^{\varrho_{K,n}})$ for the nth subcarrier.

Let us next define the Jacobian matrix, $\boldsymbol{J}_n$, of $\overline{\mathrm{MMSE}}^{\mathrm{MAC}}$ with respect to the powers at subcarrier n as $[\boldsymbol{J}_n]_{a,b} = \frac{\partial\overline{\mathrm{MMSE}}_{a,n}^{\mathrm{MAC}}}{\partial\xi_{b,n}}$, and the diagonal matrix $\boldsymbol{D}_n = \mathrm{diag}(\xi_{1,n},\ldots,\xi_{k,n})$. Note that $-\boldsymbol{J}_n$ is a Z-matrix and $\boldsymbol{J}_n\boldsymbol{D}_n$ is strictly diagonal dominant. Due to that, $-\boldsymbol{J}_n$ is non-singular and inverse positive, i.e. increasing the MMSE targets reduces the total power. We introduce the matrix $[\boldsymbol{J}_{\boldsymbol{\xi}_n}]_{a,b} = \frac{\partial\xi_{a,n}}{\partial\varrho_{b,n}}$, containing the necessary derivatives to compute (13). Multiplying $\boldsymbol{J}_n$ times $\boldsymbol{J}_{\boldsymbol{\xi}_n}$ we get $\frac{\partial\overline{\mathrm{MMSE}}_{k,n}^{\mathrm{MAC}}}{\partial\varrho_{q,n}} = [\boldsymbol{J}_n\boldsymbol{J}_{\boldsymbol{\xi}_n}]_{k,q} = [\boldsymbol{\Upsilon}_n]_{k,q}$, and that leads, eventually, to $\boldsymbol{J}_{\boldsymbol{\xi}_n} = \boldsymbol{J}_n^{-1}\boldsymbol{\Upsilon}_n$. Therefore, the gradient in (13) is given by

$$\frac{\partial P_{\mathrm{tx}}}{\partial\varrho_{k,n}} = \mathbf{1}^{\mathrm{T}}\boldsymbol{J}_{\boldsymbol{\xi}_n}\boldsymbol{e}_k \tag{15}$$

with $\mathbf{1}$ the all ones vector and $\boldsymbol{e}_k$ the canonical vector.

We mentioned previously that the rate requirements distribution is solved like N decoupled problems because the structure of the MIMO-OFDM channel allows us to split the overall optimization into N parallel subproblems. However, it should be noticed that after every step in the gradient algorithm, the per-subcarrier rate targets do not fulfill the per-user constraints. Therefore, we need to project the updated targets $\varrho'_{k,n}$ onto a set of values where $\sum_{j=1}^{N}\varrho_{k,j} = \rho_k$ holds. To do so, we propose to minimize the Euclidean distance over all the subcarriers subject to the per-user rate restrictions, i.e.

$$\min_{\{\varrho_{k,i}\}_{i=1}^{N}} \sum_{j=1}^{N}\left(\varrho_{k,j} - \varrho'_{k,j}\right)^2 \text{ s.t. } \varrho_{k,n} \ge 0 \text{ and } \sum_{j=1}^{N}\varrho_{k,j} = \rho_k.$$

Finally, we get the updated per-subcarrier rate targets as $\varrho_{k,n} = \max\{0, \varrho'_{k,j} - \beta_k\}$, where $\beta_k = \frac{1}{N}\left(\sum_{j=1}^{N}\varrho'_{k,j} - \rho_k\right)$.

4. PROPOSED ALGORITHM

Alg.1 presents the algorithm that solves the problem of minimizing the transmit power in the MIMO-OFDM BC subject to QoS constraints. Lines 3 and 16 solve the power minimization problem (7) for given target rates. To do so we employ the algorithm presented in [11] to solve the N parallel subproblems. Notice that between lines 5 and 24 we have two nested loops. The outer one computes the gradient for each subcarrier in line 7 while the inner updates the target for every subcarrier (lines 9 to 11) and they are projected to get the new targets in lines 12 to 14. Afterwards, if the total transmit power is lower than the one in the previous iteration, the per-stream targets, the filters, and the power allocation are updated. If not, the step size μ is reduced in line 21.

It is important to note that problem feasibility depends on the channel and the number of antennas, as shown in [11]. If the rate constraints are feasible for each subcarrier, convergence to a local minimum is guaranteed since in every iteration the power decreases or remains unchanged.

Algorithm 1 Gradient Algorithm

$l \leftarrow 0$, random init.: $\boldsymbol{P}_k, \forall k$ and $\varrho_{k,n}^{(0)}, \forall n$
for $n = 1$ to N **do**
    find the optimum $\boldsymbol{\tau}_{k,n}^{(0)}$, $\boldsymbol{g}_{k,n}^{(0)}$ and $\xi_{k,n}^{(0)}$ $\forall k$
end for
repeat
    $l \leftarrow l + 1$
    $\boldsymbol{J}_{\boldsymbol{\xi}_n}^{(l)} \leftarrow \boldsymbol{J}_n^{(l),-1} \boldsymbol{\Upsilon}_n^{(l)}$, $\forall n$, $exit \leftarrow 0$, $\mu \leftarrow \mu_0$
    repeat
        for $n = 1$ to N **do**
            $\varrho_{k,n}^{'} \leftarrow \varrho_{k,n}^{(l-1)} - \mu \mathbf{1}^{\mathrm{T}} \boldsymbol{J}_{\boldsymbol{\xi}_n}^{(l)} \boldsymbol{e}_k$, $\forall k$
        end for
        for $k = 1$ to K **do**
            $\varrho_{k,n}^{(l)} \leftarrow \max\{0, \varrho_{k,n}^{'} - \beta_k^{(l)}\}$, $\forall n$
        end for
        for $n = 1$ to N **do**
            find the optimum $\boldsymbol{\tau}_{k,n}^{(l)}$, $\boldsymbol{g}_{k,n}^{(l)}$ and $\xi_{k,n}^{(l)}$ $\forall k, \forall n$
        end for
        if $P_{\text{tx}}^{(l-1)} - P_{\text{tx}}^{(l)} > 0$ **then**
            $exit \leftarrow 1$
        else
            $\mu \leftarrow \frac{\mu}{2}$
        end if
    until $exit$
until $\sum_{k=1}^{K} \sum_{i=1}^{d_k} \xi_{k,i}^{(l-1)} - \sum_{k=1}^{K} \sum_{i=1}^{d_k} \xi_{k,i}^{(l)} \leq \delta$

5. SIMULATION RESULTS

In this section we present the results of simulation experiments carried out to evaluate the performance of the proposed power minimization algorithm. Simulations considered the I-METRA case D channel model for the point-to-point links in the BC [14]. Assuming proper cyclic insertion [15], the I-METRA MIMO-OFDM channel model is given by

$$\boldsymbol{H}_n = \sum_{l=1}^{L} \boldsymbol{R}^{1/2} \boldsymbol{H}(l) \boldsymbol{T}^{1/2} \exp\left(\frac{-j2\pi l n}{N}\right) \tag{16}$$

where $\boldsymbol{H}(l)$, $l \in \{1, \ldots, L\}$ is a sequence of spatially uncorrelated time-domain $T \times R$ MIMO channel matrices, and $\boldsymbol{T}$ and $\boldsymbol{R}$ represent the transmit and receive spatial-correlation matrices, respectively. The I-METRA model assumes Rayleigh fading, i.e. the entries to $\boldsymbol{H}(l)$ are complex valued zero-mean circularly-symmetric Gaussian random variables. In I-METRA case D, the power delay profile is that of the ITU Pedestrian B channel model whereas a specification is given for matrices $\boldsymbol{T}$ and $\boldsymbol{R}$ when $T = R = 4$ (see [14] for further details). Finally, the noise covariance matrix is $\boldsymbol{C}_{\boldsymbol{\eta}_{k,n}} = \mathbf{I}\, \forall k, n$. We considered $K = 2$ independent MIMO-OFDM channels were generated using the above described model $\boldsymbol{H}_{k,n} \forall k, n$.

The CSI available at transmission are estimates of the MIMO-OFDM channels, $\boldsymbol{H}_{k,n}$, i.e.

$$\hat{\boldsymbol{H}}_{k,n}^{(m)} = \boldsymbol{H}_{k,n} + \boldsymbol{E}_{k,n}^{(m)} \tag{17}$$

where $\boldsymbol{E}_{k,n}^{(m)}$ is the channel estimation error. We assume that $\boldsymbol{E}_{k,n}^{(m)} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, 0.1\mathbf{I})$ and $m = 1, \ldots, M$. The gradient initial step is $u_0 = 1$ and the threshold value δ is set to 10^{-4}.

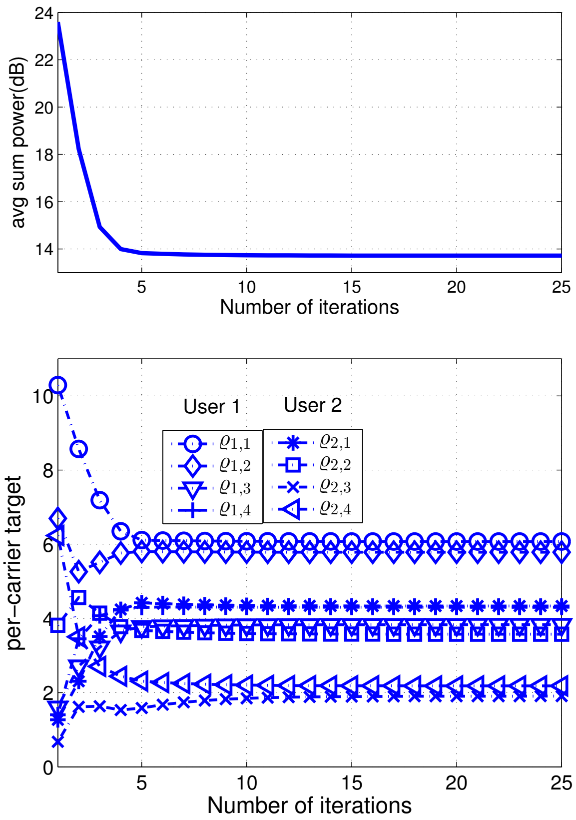

Fig. 2. Example of algorithm execution for $K = 2$, $N = 4$.

Fig. 2 depicts the evolution of the algorithm for a given BC channel realization $\boldsymbol{H}_{k,n}$, with $M = 500$, $K = 2$ users and $N = 4$ subcarriers. The per-user target rates where set to $\rho_1 = 5 \times N$ and $\rho_2 = 3 \times N$. The top figure shows how the total sum power diminishes with the number of iterations. The bottom figure displays the evolution of the per-subcarrier target rates with the number of iterations and how they converge to an optimal value. Note that at all iterations the sum of all per-sucarrier rates gives the per-user target rates.

Fig. 3 represents the converged transmit power per subcarrier after averaging over 100 channel realizations, $\boldsymbol{H}_{k,n}$ and $M = 100$. Average transmit power per subcarrier (P_{tx}/N) is expressed logarithmically (dB) for $N = 8, 16, 32$ and 64 subcarriers. The figure Fig. 3 considers the per-user target rates $\rho_1 = 5 \times N$ and $\rho_2 = 3 \times N$ bits. Note that the per-user target rates increase proportionally with the number of subcarriers in order to keep the system spectral efficiency

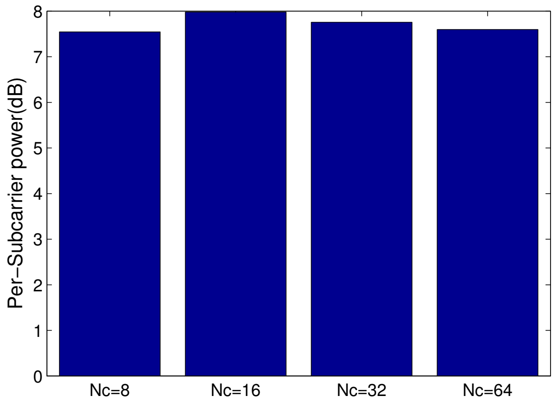

Fig. 3. Resulting Average Per-Subcarrier Power.

constant. It is apparent from Fig. 3 that the proposed power minimization algorithm produces similar results irrespective of the number of subcarriers, hence indicating that the algorithm works correctly.

6. CONCLUSION

We have developed a gradient-projection iterative algorithm to minimize the transmit power in a MIMO-OFDM BC while fulfilling a given set of target user rates. Such an algorithm first distributes each user target rate among the different subcarriers, and then determines the optimum filters and the power allocated at each of the user subcarriers.

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