RETINAL BLOOD VESSEL EXTRACTION USING CURVELET TRANSFORM AND CONDITIONAL FUZZY ENTROPY

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ABSTRACT

This work employs multiple thresholds on matched filter response for automatic extraction of blood vessels, specially from a low contrast and non-uniformly illuminated background of retina. Curvelet transform is used first to enhance the finest details along the vessels followed by matched filtering to intensify the blood vessels' response. The conditional fuzzy entropy is then maximized to find the set of optimal thresholds to extract different types of vessel silhouettes from the background. Differential Evolution algorithm is used to specify the optimal combination of the fuzzy parameters. Thresholds thus obtained extract the thin, the medium and the thick vessels from the enhanced image which are then logically OR-ed to obtain the entire vascular tree. Performance evaluated on publicly available DRIVE database is compared with the existing blood vessel extraction methods. Experimental runs demonstrate that the proposed method outperforms the existing methods in detecting various types of vessels.

Index Terms— Retinal vessel segmentation, Curvelet, Matched filter, Conditional Fuzzy Entropy, Differential Evolution

1. INTRODUCTION

Diabetic retinopathy (DR) is a common complication of diabetes and a leading cause of blindness. It occurs when the blood vessels that nourish the retina are blocked and hence deprive of supplying several areas of retina with blood and oxygen. Normal retinal images are rich with the large number of thick and thin vessels. The blood vessels become thinner as they travel radially outward from the optic disc at different orientations. Moreover, the contrast between the background and the vessels along their directions gradually become poorer. The images are also degraded during acquisition due to noise, blurring etc. and sometimes they are non-uniformly illuminated. Furthermore, in proliferative diabetic retinopathy (PDR), new blood vessels start growing along the retina. These fragile and abnormal blood vessels appear as very thin vascular nets in retinal images. Existing retinal screening programs mostly employ manual extraction of blood vessels which is time consuming, prone to errors and highly depend on personal skills of the opthalmologists. In order to provide proper treatment to the PDR patients it is very much necessary to determine the grade of PDR i.e. whether PDR is in its initial stage or in the advanced stage. This puts a pressing demand to design an automated system capable of accurate extraction and classification of blood vessels, specially the abnormal and very thin vascular nets for the diagnosis and treatment of PDR.

Several methods for retinal blood vessel extraction are proposed in the literature that include pixel based classification techniques using artificial neural network [1], mathematical morphology [2], vessel tracking/ tracing [3], matched filtering [4] etc. Some of these techniques view retinal blood vessel extraction as an analogous problem to image segmentation. On image segmentation, entropy based method and its extension based on fuzzy set theory are applied extensively using single or multiple thresholds. To the best of our knowledge, the use of multiple thresholds in retinal blood vessel extraction is not reported. Multiple thresholding is relevant to determine the different types of blood vessels' extraction as these extracted blood vessel information to be used later for identifying the stages of PDR. Since there lies always some uncertainty among different classes, like the non-vessel and the thin vessel, the thin and the medium vessel and the medium and the thick vessels, conditional entropy based thresholding seems to be more appropriate than absolute entropy to improve the overall vessel extraction accuracy as well as identifying the individual ones in reduced overlapping computation space.

To this goal, this paper uses curvelet transform for enhancement of retinal images since it is very much efficient in identifying curves, contours, missing and imprecise edges (thin vessels), curvatures and other boundary information. Moreover, being a member of wavelet family, it offers multiresolution, space-frequency localization ability, very high directional sen-

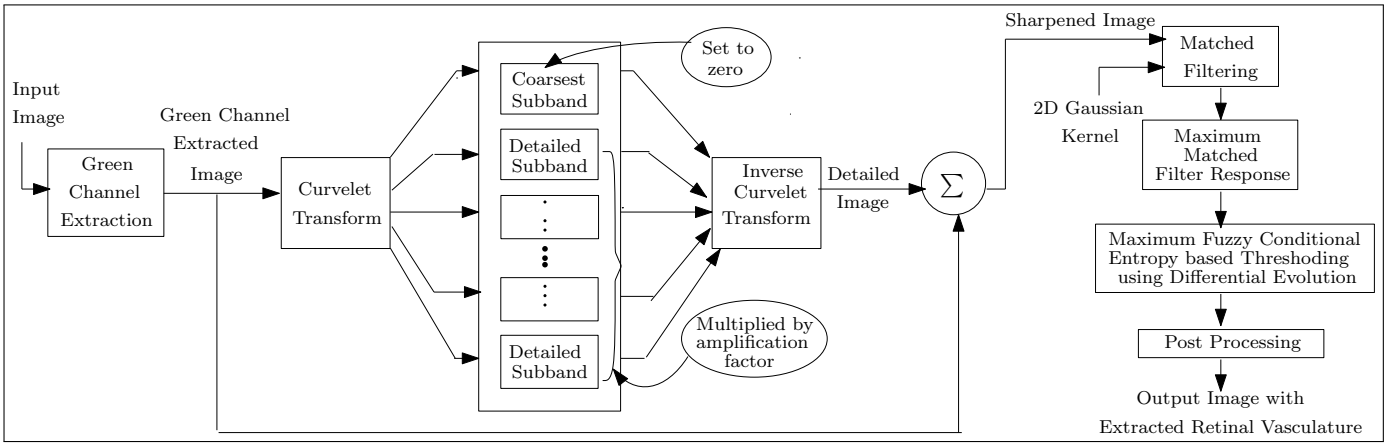


Fig. 1. Schematic Diagram of Proposed Method for Retinal Vessel Detection

sitivity and anisotropy in frequency domain. After enhancing the image in the curvelet domain, matched filter with proper Gaussian profile is applied in the spatial domain to intensify the response against the various types of blood vessels. The extraction of the different types of vessel silhouettes from the background basically involves automatic selection of a set of robust and optimal thresholds. To obtain the optimal threshold values, the conditional fuzzy entropy function corresponding to the maximum matched filter response (MFR) is maximized. This requires an optimal combination of a set of fuzzy parameters. Differential Evolution (DE), a population based global optimization algorithm, is used to obtain the same. Next the image is analyzed into the non-vessel, the thin, the medium and the thick vessels which are then logically OR-ed to obtain the entire vessel map.

The rest of the paper is organized as follows. Proposed method of vessel detection is described in Section 2. Section 3 reports the results of the experimental runs and discussions while conclusions and future works are mentioned in Section 4.

2. PROPOSED METHOD

The proposed method for retinal blood vessel extraction is diagrammatically represented in Fig 1. The entire methodology can be divided into the following six steps.

2.1. Green Channel Extraction

In this work, the green color plane is used for vessel extraction since the contrast between the blood vessels and the background is the maximum for this green channel [5].

2.2. Curvelet based Edge Enhancement

To enhance the edges of the blood vessels, the image is decomposed first into a number of subbands using curvelet transform with different scales and orientations. Curvelet coefficients contain the most important information like the missing and the broken boundary information, horizontal, vertical, diagonal edge details as well as the coarse approximation of the image. Next the approximate subband i.e. the coefficients corresponding to the coarse approximation of the image are set to zero and the detailed subbands are intensified multiplying by proper amplification factor. As a result, the background gets suppressed, while the detail edges are highlighted. After that the inverse curvelet transform is done to obtain the background suppressed image and is superimposed on the original image. This in turn increases the contrast, specially between the thin, the tiny, the faint vessels and the background. The edges of the narrow vessels which were hardly distinguishable from the background are also sharpened. Thus, using curvelet transform both the strongest and the finest edges in the retinal image can be enhanced.

2.3. Matched Filtering using 2D kernel

The grey-level profiles of the cross-section of blood vessels in retinal images are Gaussian in nature and the intensity profile is symmetric about the straight line that passes through the center of the vessel [4]. A two dimensional prototype matched filter kernel may be mathematically expressed as: $K(x,y) = -exp(x^2/2\sigma^2)$ for $|x| \leq (3\sigma)$ and $|y| \leq (L/2)$, where L is the length of the blood vessel segment considered to have fixed orientation and σ denotes the scale of the Gaussian function. Here the negative sign implies the fact that the background is brighter than the vessels.

In an image, a particular vessel may be oriented at any angle between 0 and π and if the vessel is aligned at an angle $\pi/2$, then only the matched filter will have its peak response. This needs rotation of a Gaussian kernel at an incremental angle and to observe the MFR value each time. Implementation is done here by rotating the filter at an increment of 12^0 and convolving with the image under consideration. From the set of 15 MFR values, for each pixel, only the maximum one is retained.

2.4. Fuzzy Conditional Entropy Maximization

Fuzzy conditional entropy [6] describes the fuzziness in the amount of information needed to describe the outcome of a particular random variable, for example, may be non-vessel (thin

vessel) type in this problem, given that the value of the other one, thin vessel (non-vessel) class is known. The entropy of a random variable Y conditioned on a random variable X is denoted by $H(Y|X)$ and is called conditional entropy (or equivocation). Since the cross-sections of the retinal vessels are Gaussian in nature, the MFR are the highest for the vessels and the lowest for the non-vessels. Let the maximum MFR of each pixel forms a $M \times N$ matrix and $G = \{m, m+1, \ldots\ldots, q-1, q\}$ is a set of integers where m and q represent the nearest integer values of the lowest and the highest maximum MFR, respectively.

A 4-level simple trapezoidal membership function shown in Fig. 2 is used to classify the maximum MFR into 4 classes, namely the non-vessel, the thin, the medium and the thick vessels. The symbols a, b, c indicate the fuzzy function parameters. The membership functions μ_{nv}, μ_{thin}, μ_{med} and μ_{thick} for the non-vessel, the thin, the medium and the thick vessels, respectively can be calculated from the following equations.

$$\mu_{nv}(k) = \begin{cases} 1 & k \leq a_1 \\ \dfrac{k - b_1}{a_1 - b_1} & a_1 \leq k \leq b_1 \\ 0 & k > b_1 \end{cases} \tag{1}$$

$$\mu^{'}(k) = \begin{cases} 0 & k \leq a_{n-1} \\ \dfrac{k - a_{n-1}}{b_{n-1} - a_{n-1}} & a_{n-1} < k \leq b_{n-1} \\ 1 & b_{n-1} < k \leq a_n \\ \dfrac{k - b_n}{a_n - b_n} & a_n < k \leq b_n \\ 0 & k > b_n \end{cases} \tag{2}$$

$\mu^{'}(k) = \mu_{thin}(k)$ for $n = 2$ and $\mu^{'}(k) = \mu_{med}(k)$ for $n = 3$

$$\mu_{thick}(k) = \begin{cases} 0 & k \leq a_3 \\ \dfrac{k - a_3}{b_3 - a_3} & a_3 < k \leq b_3 \\ 1 & k > b_3 \end{cases} \tag{3}$$

where $m < a_1 \leq b_1 \leq a_2 \leq b_2 \leq a_3 \leq b_3 < q$.
The conditional entropy for the area overlapped between the non-vessel and the thin vessel class is given by

$$H_A = H(nv|thin) + H(thin|nv) \tag{4}$$

where

$$H(nv|thin) = -\frac{1}{n} \sum_{k \in K^+} \{\mu^{'}_{nv} log(\mu^{'}_{nv}) - \mu^{'}_{thin} log(\mu^{'}_{thin}) + [1 - \mu^{'}_{nv}] log\,[1 - \mu^{'}_{nv}] - [1 - \mu^{'}_{thin}] log\,[1 - \mu^{'}_{thin}]\} \tag{5}$$

$$H(thin|nv) = -\frac{1}{n} \sum_{k \in K^-} \{\mu^{'}_{thin} log(\mu^{'}_{thin}) - \mu^{'}_{nv} log(\mu^{'}_{thin}) + [1 - \mu^{'}_{thin}] log\,[1 - \mu^{'}_{thin}] - [1 - \mu^{'}_{thin}] log\,[1 - \mu^{'}_{thin}]\} \tag{6}$$

Here n is the total number of points between a_1 and b_1, $\mu^{'}_{nv}$ and $\mu^{'}_{thin}$ represent the probability of membership for the non-vessel and the thin vessel, i.e. μ_{nv} and μ_{thin}, respectively and

$$K^+ = \{\, k \,|k \in G, \mu^{'}_{nv} \geq \mu^{'}_{thin}\} \tag{7}$$

$$K^- = \{\, k \,|k \in G, \mu^{'}_{nv} < \mu^{'}_{thin}\} \tag{8}$$

The conditional fuzzy entropy functions for the remaining classes can also be expressed in the similar ways. The total conditional entropy $H_{nonvess|vess}$ can be expressed as:

$$H_{nonvess|vess} = H_A + H_B + H_C \tag{9}$$

where

$$H_A = H(nv|thin) + H(thin|nv) \tag{10}$$

$$H_B = H(thin|med) + H(med|thin) \tag{11}$$

$$H_C = H(med|thick) + H(thick|med) \tag{12}$$

The total fuzzy conditional entropy is a function of 6 parameters. In order to extract the retinal vasculatures from the background, proper values for these 6 parameters to be determined that maximize $H_{nonvess|vess}$. The thresholds that partition the maximum MFR into 4 classes, namely the non-vessel, the thin, the medium and the thick vessels are T_1, T_2 and T_3, respectively and can be calculated from the following equation.

$$T_n = \frac{a_n + b_n}{2} \quad for \quad n = 1, 2, 3 \tag{13}$$

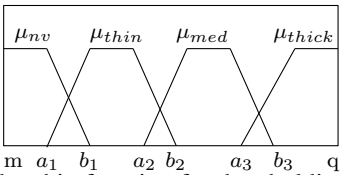


Fig. 2. Membership function for thresholding in 4 classes

2.5. DE for Maximization of Conditional Fuzzy Entropy

DE is an efficient and powerful stochastic search technique for solving population based global optimization algorithms. In this work, DE is used to obtain the optimal combination of all the fuzzy parameters. Maximization of fuzzy conditional entropy function is used to find the parameters followed by the respective threshold values. The fitness function used is given by the following equation:

$$f = \max H_{nonvess|vess}(a_1, b_1, a_2, b_2, a_3, b_3) \tag{14}$$

Algorithm 1 describes DE based fuzzy conditional entropy maximization. Once the thresholds T_1, T_2, T_3 are obtained, the image is partitioned into the non-vessel, the thick, the medium and the thin vessels which are then fusioned by logical OR operator to obtain the entire vascular pattern.

2.6. Post Processing

The MFR of the thin vessels are very low. Therefore, during thresholding using multiple values noise and other spurious components may also be detected as thin vessels as observed in Fig. 4(c). This needs removal of these spurious components through some kind of post processing operation. This work uses connected component analysis to remove the isolated small pixel blocks. After detection, the connected components above a specific area are labeled using 8 connected neighborhood and considered as thin vessel pixels. Thus the false edges are separated from the original vessel edges efficiently.

Table 1. Performance Comparison Results

Method	TPR	FPR	Average ACC
Chaudhuri et al [4]	0.6168	0.0259	0.9284
Mendonca et al [2]	0.7344	0.0236	0.9452
Miri et al [7]	0.7352	0.0205	0.9458
Proposed Method	**0.7608**	**0.0189**	**0.9552**

3. RESULTS

This section presents the performance of the proposed method tested on DRIVE database [8]. It consists of 40 images divided into a training set and a test set, each of them contains 20 images. Both the test and the training set provide manually segmented images. For edge enhancement purpose, the source code available in [10] is used for curvelet transform. This paper uses an amplification factor=2 to intensify the detailed subbands. The value is obtained through experimental runs over the available images in the database. Experimental results shown here use initial population of 100 chromosomes.

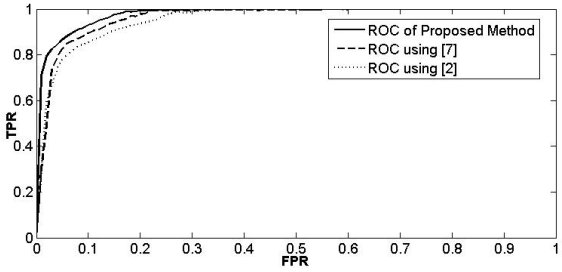


Fig. 3. Comparative study of the ROC curves

The metrices used to measure quantitatively the performance of the proposed algorithm include (1) True Positive Rate (TPR), i.e. the fraction of correctly classified vessel pixels, (2) False Positive Rate (FPR), the fraction of pixels erroneously detected as vessel pixels and (3) Detection Accuracy (ACC), given by the ratio $\frac{TP+TN}{TP+FN+TN+FP}$ where TP, TN, FN, FP represent true positive, true negative, false negative and false positive, respectively. The average receiver operating characteristics (ROC) obtained through studies of all the available images has been depicted in Fig. 3. The area under the curve is 0.9503 for the proposed method. The values of TPR, FPR and average ACC are shown collectively in Table 1. Results show that both TPR and average ACC values for the proposed method are the highest and the value of FPR is the lowest compared to other methods.

The extracted vasculatures are shown in Fig. 4 only for one image due to space limitation . The original retinal image and the curvelet based edge enhanced image are shown in Fig. 4(a) and Fig. 4(b), respectively. The extracted thin, medium and thick vessels are presented in Fig. 4(c), Fig. 4(d) and Fig. 4(e), respectively. Fig. 4(f) shows the entire vascular pattern extracted by the proposed method. The vessel structure extracted by the method proposed in [9] is shown in Fig. 4(g) and the hand labeled ground-truth is presented in Fig. 4(h). For the image in Fig. 4(f), 38.98% of the detected vessels are thin, 49.47% are medium and 11.55% are thick. Comparing Fig. 4(f) and Fig. 4(g), it is observed that some of the tiny vessels which are not detected in Fig. 4(g), are detected by the proposed method as shown in Fig. 4(f). Therefore, the proposed method outperforms the existing methods for detecting the finest details of the long and thick as well as short and narrow vessels.

DE based Fuzzy Conditional Entropy Maximization

Input : Randomly initialized population of N individuals $\overrightarrow{P_t} = \{\overrightarrow{X}_1, \overrightarrow{X}_2, \ldots \overrightarrow{X}_N\}$ where the i^{th} chromosome contains a set of 6 optimization parameters and can be expressed as:
$\overrightarrow{X}_i(t) = [a_{i,1}, b_{i,1}, a_{i,2}, b_{i,2}, a_{i,3}, b_{i,3},], i = 1, 2, \ldots, N$
uniformly distributed in the range $[m, q]$

Output: Optimal Thresholds that maximize the total fuzzy entropy function

begin

  repeat

    Step 1: Mutation: Generate a mutated vector $\overrightarrow{Y}_i(t)$ corresponding to the target vector $\overrightarrow{X}_i(t)$ for every i.

    for $i = 1$ *to* N **do**

      for $j = 1$ *to* 6 **do**

$$Y_{i,j}(t) = X_{r_1,j}(t) + F(X_{r_2,j}(t) - X_{r_3,j}(t))$$

        where r_1, r_2, and r_3 are three randomly selected parameter vectors and the scaling factor $F = 0.5$

      end

    end

    Step 2: Crossover: For every target vector $\overrightarrow{X}_i(t)$, create a trial vector $\overrightarrow{Z}_i(t)$ when a randomly generated number between 0 and 1 is less than crossover rate (CR).

    for $i = 1$ *to* N **do**

      for $j = 1$ *to* 6 **do**

$$\overrightarrow{Z}_{i,j}(t) = \begin{cases} \overrightarrow{Y}_{i,j}(t), & \text{if } rand_j(0,1) < CR(0.9) \\ \overrightarrow{X}_{i,j}(t), & \text{otherwise} \end{cases}$$

      end

    end

    Step 3: Selection: Evaluate the trial vector.

    for $i = 1$ *to* N **do**

$$\overrightarrow{X}_i(t+1) = \begin{cases} \overrightarrow{Z}_i(t), & \text{if } f(\overrightarrow{Z}_i(t)) \geq f(\overrightarrow{Y}_i(t)) \\ \overrightarrow{X}_i(t), & \text{otherwise} \end{cases}$$

      where $f(.)$ is the function to be maximized.

    end

  until *maximum iteration count is reached*;

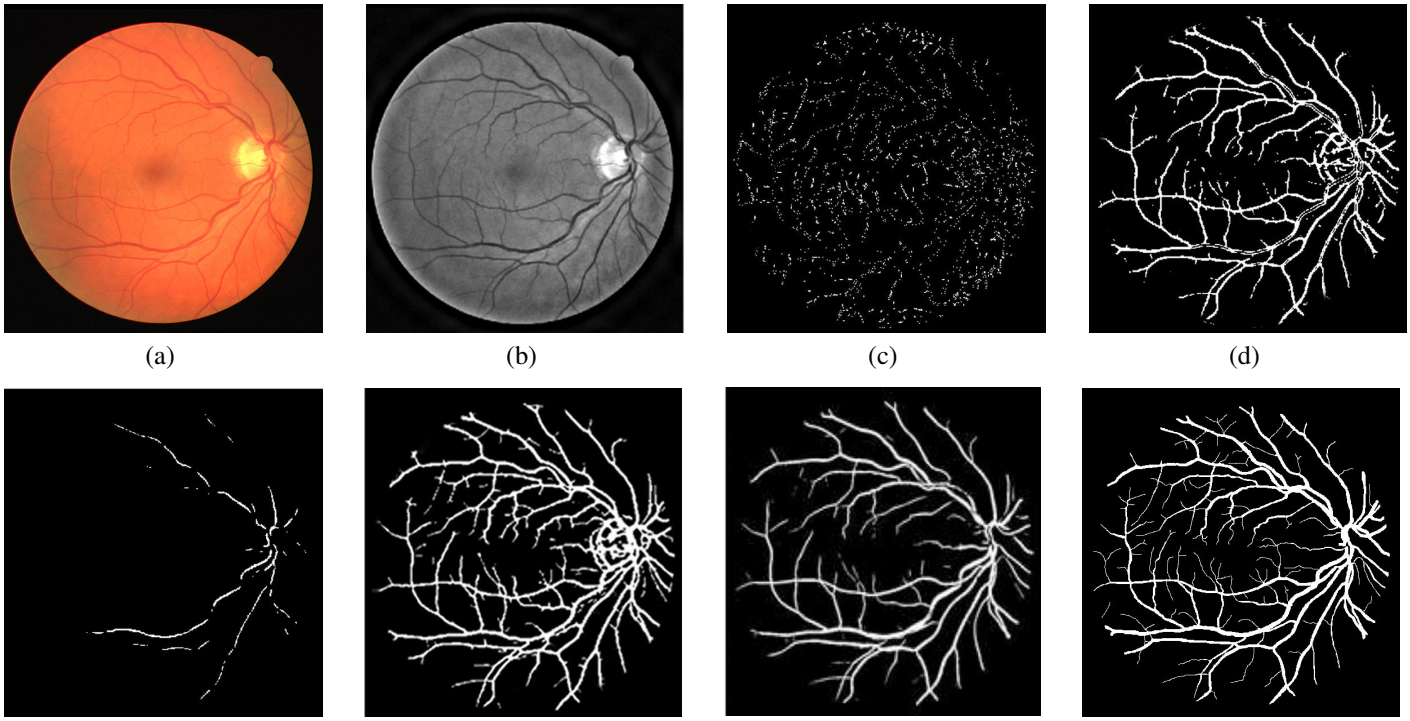
  The best chromosome in the population contains the optimal combination of fuzzy parameters. Calculate T_1,T_2,T_3 from equation (13).

end

Algorithm 1: Fuzzy Conditional Entropy Maximization

4. CONCLUSIONS AND FUTURE WORKS

An integrated system design for retinal blood vessel extraction is proposed where curvelet transform enhances the curvatures, the contours, the missing and the imprecise edge boundaries followed by intensifying the response using matched filter. The optimal threshold values that maximize the fuzzy conditional entropy function of the maximum MFR classify accurately the different types of vessels and the non-vessels. DE is found efficient to determine the different parameters for the fuzzy function. As observed from the ROC curve, even for the low values of FPR, the values of TPR are very high which in turn increases



(a) (b) (c) (d)

(e) (f) (g) (h)

Fig. 4. Results of Experimental runs (a) Original Retinal Image. (b) Curvelet based edge enhanced image (c), (d), (e) Extracted thin, medium, thick vessels, respectively by proposed method. (f) Entire retinal vasculature obtained by superimposing Fig. 4(c), Fig. 4(d) and Fig. 4(e). (g) Vessel Extraction by [9]. (h) Manually segmented image.

the average ACC of the proposed method. Classified vessels may now be used for detecting the stages of PDR for further medical investigation.

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