A SPECTRUM SENSING ALGORITHM BASED ON STATISTIC TESTS FOR COGNITIVE NETWORKS SUBJECT TO FADING

Fabrício B. S. de Carvalho∗†§, *Jerônimo S. Rocha*§‡, *Waslon T. A. Lopes*∗§, *Marcelo S. Alencar*∗§

∗ Post-Graduate Program in Electrical Engineering – PPgEE – COPELE
DEE – UFCG – 58429-900 – Campina Grande – Brazil
† DEE – CEAR – UFPB – João Pessoa – Brazil
§ Iecom – UFCG – Campina Grande – Brazil
‡ IFPB – UAIC – Campina Grande – Brazil

ABSTRACT

Cognitive radio is a viable technology for the next generation of wireless communications. The ability to sense the electromagnetic spectrum and to enable vacant bands to other users has been investigated in the past years. One important issue is the use of an efficient spectrum sensing algorithm to monitor the frequency band occupancy. Usually, the effects of fading are overseen in the analysis of those algorithms. This paper aims to evaluate the performance of a spectrum sensing algorithm based on Jarque-Bera test. Rayleigh fading is considered in this paper. Preliminary simulation results are provided, to demonstrate the potential of the proposed strategy.

***Index Terms*—** Cognitive Radio, Signal Processing, Spectrum Sensing, Statistic Tests, Jarque-Bera Test, Rayleigh Fading Channel

1. INTRODUCTION

The electromagnetic spectrum is overloaded in some frequency bands and is sub-utilized in others. Government agencies stipulate technical criteria to authorize additional frequency bands for allocation of new services [1]. The objective is to reduce the interference among devices operating in near frequencies. As most of the spectrum is already allocated, it is complicated to grant new licenses or to increase the quality of the services in operation. On the opposite, some frequency bands face low spectrum usage [2].

Spectrum scarcity and the inefficient use of this important resource inspired the rising of new techniques to better exploit the electromagnetic spectrum. Cognitive Radio (CR) is the key to extend the spectral efficiency, with opportunistic access, for the available frequency bands [3].

Cognition refers to the process of knowing through perception, reasoning, knowledge and intuition by the observation of an environment. Cognitive radio is a wireless communication technique that monitors the spectrum and adapts its transceivers to occupy an available radio frequency channel (when temporarily not occupied by primary or licensed users) in that time [4].

Spectrum users are classified as Primary (or licensed) Users (PU) or Secondary (or cognitive) Users (SU). Primary users are those who have a licence to operate in a specific frequency band, while cognitive users do not hold authorization to transmit and receive signals in that channels. A cognitive user should be able to monitor the frequency spectrum and, based on its observations, to find out if there is any licensed user occupying the spectrum [5].

Cognitive radio is the technology that permits the verification of the availability of the electromagnetic spectrum. Cognitive radio and its dynamic access capacity can be employed in several wireless applications [6]. Spectrum sensing is the evaluation of the frequency bands that can be opportunistically occupied by the cognitive users. The sensing should be dynamic and should provide requirements to guarantee acceptable interference levels [7].

Traditionally, the spectrum sensing problem is formulated as in the following. The probability distribution of the detected signal under the two hypotheses can be compared to a specific distribution in the spectral band of interest. In this scenario, if samples of the received signal present a Gaussian distribution when transmitted through an additive white Gaussian noise (AWGN) channel, then the secondary user understands that there is a chance of transmission. Otherwise, if the probability distribution of the gathered samples is different from the Gaussian, the cognitive user considers that the channel is occupied.

However, the effects of fading represent an important issue regarding the detection of signals in cognitive communications. Researchers evaluate how distinct fading models affect the spectrum sensing in cognitive networks. Different fading models, which include Rayleigh, Nakagami, Rice, $\kappa-\mu$, among others, are subject to evaluation, and new detection probability expressions have been found [8–10].

This paper analyzes the spectrum sensing based on sta-

The authors would like to express their thanks to COPELE, CAPES and CNPq for the financial support of this work.

tistic tests when the wireless channel is subject to Rayleigh fading. The remaining of the paper is organized as follows: Section II details the concept of spectrum sensing and the hypothesis testing, as well as the most important spectrum sensing techniques; Section III presents the main statistic tests and the most relevant spectrum sensing algorithms based on these tests; Section IV highlights the proposed algorithm and Section V evaluates the preliminary results. Finally, Section VI presents the conclusions and the perspectives for the continuation of this work.

2. SPECTRUM SENSING IN COGNITIVE NETWORKS

Spectrum sensing is one of the main tasks in a cognitive radio network. Advantages of opportunistic spectrum allocation (such as higher bandwidth and lower error rates in the transmission) can be achieved by monitoring the occupation of a channel. If the channel is available, the cognitive user can opportunistically occupy the bandwidth; otherwise, when a primary user is transmitting in the channel, the frequency band is not available for the secondary user.

The detection problem is analyzed as a binary hypothesis model, defined as [3, 11]

$$y[n] = \begin{cases} w[n], & \text{if } H_0 \\ w[n] + h \cdot x[n], & \text{if } H_1 \end{cases} \tag{1}$$

in which $y[n]$ is the signal received by the CR during the observation time; $x[n]$ is the transmitted signal of the PU; $w[n]$ is additive white Gaussian noise (AWGN) with zero mean and variance σ^2, and h is the channel gain due to the fading over the signal [3, 11].

H_0 indicates the absence of primary signal in the channel, while H_1 indicates that the spectrum is occupied by a PU (this occupancy can refer to a PU or to a SU). Based on these hypotheses, one can define the probability of detection $P_d = \text{Prob}(signal\ detected|H_1)$ and the probability of false detection $P_f = \text{Prob}(signal\ detected|H_0)$. The objective is to maximize P_d while minimizing P_f [3, 12].

Another important parameter is the probability of missed detection P_m, which is the complement of P_d: $P_m = 1 - P_d = \text{Prob}(signal\ not\ detected|H_1)$. The probability of a wrong decision in the band occupancy is the weighted sum of P_f and P_m [3, 12].

The following spectrum sensing techniques are used in cognitive networks [3]: Energy Detection (ED); Matched Filtering detection (MF); Cyclostationary (or Feature) Detection (CD); Covariance based detection and other techniques can be employed to improve the cognitive radio network operation. Also, the combination of two or more spectrum sensing techniques can be investigated to obtain better results when compared to these techniques individually. This approach is known as hybrid sensing techniques [6, 13].

3. STATISTIC TESTS

Spectrum sensing algorithms can be divided in terms of the *a priori* knowledge of the transmitted signal that the cognitive user can obtain. Features such as the modulation technique, noise variance and spread spectrum sequence, among others, lead to an improved identification of the spectral occupation. Although in real scenarios such characteristics are normally unknown by the secondary users – spectrum sensing is generally blind (cognitive users must identify the spectrum holes with no details of the primary user).

In view of the difficulty to obtain *a priori* information, the usage of statistics from the primary signal is rising as an alternative to spectrum sensing. In a channel with additive white Gaussian noise, the transmitted signal has a specific probability distribution; in the absence of the primary user, only a Gaussian random variable with zero mean is detected. Thus, as the normal distribution and its statistic parameters are known, the cognitive user can decide on the availability of a band by using these statistics [14].

In general, spectrum sensing strategies based on statistic tests aim to determine the probability distribution from a group of samples and to compare it with the target distribution. This approach is named goodness of fit (GOF). According to the hypothetical distributions, the hypothesis test is defined as [15]:

- H_0, if the samples fit the distribution;
- H_1, if the samples do not fit the distribution.

Statistic tests can be classified as parametric or non-parametric. If the distributions of the random variables are previously known, the tests are named *parametric*. Otherwise, if no information from the signal is available *a priori*, the tests are classified as *non-parametric tests*. Non-parametric tests allow the assumption of different hypothesis regarding to the samples; although, these tests are less robust when compared to parametric test under the same conditions [16].

Non-parametric tests are widely adopted for spectrum sensing as the sensing is generally blind (with no previous information from the primary user occupying the channel). These statistic tests can be used to evaluate if the samples fit the Gaussian distribution as well other distributions [14].

The main statistic tests described in the technical literature are [17–19]:

- Pearson Chi-square χ^2 test: this test verifies the adherence of a set of samples to a probability distribution; the statistic test converges to a Chi-square distribution.
- Skewness: third standard moment of a distribution. For a random variable X, the skewness S is given by

$$S = \frac{\mu_3}{\sigma^3} = \frac{E[(X - E[X])^3]}{(E[(X - E[X])^2])^{\frac{3}{2}}} \tag{2}$$

The skewness for a Gaussian distribution is zero [16].

- Kurtosis: fourth standard moment of a distribution. It is the degree of flatness of a probability function near its center. The kurtosis K of a random variable X is calculated as

$$K = \frac{\mu_4}{\sigma^4} = \frac{E[(X - E[X])^4]}{(E[(X - E[X])^2])^2} \tag{3}$$

The value of the kurtosis for a Gaussian random process tends to 3 as the sample size increases [16].

- Jarque-Bera (JB): the JB test is selected to verify the adherence to a Gaussian distribution. It is based on the skewness and on the kurtosis of the samples, which are used to be compared with the normal distribution. The Jarque-Bera test is defined as

$$JB = \frac{N_S}{6}\left(S^2 + \frac{(K-3)^2}{4}\right) \tag{4}$$

in which N_S is the number of analyzed samples. If the signal monitored is Gaussian, then the JB test results in a Chi-square distribution with two degrees of freedom [14].

- Kolmogorov-Smirnov (KS): This test is selected when the mean and the variance of the distribution is known. KS test can be employed for a Gaussian or other distributions [16].
- Lilliefors: it is a modification of the Kolmogorov-Smirnov test; the mean and the variance of the signal are unknown. It is used only to test the fit to a Gaussian distribution.
- Anderson-Darling (AD): an adaptation from Lilliefors, the adherence to a specific distribution probability is evaluated.
- Shapiro-Wilk (SW): another goodness of fit test for normal distributions. The SW test performs better for a small set of samples if compared to KS or Lilliefors tests [16].

3.1. Spectrum Sensing based on Statistic Tests

Different statistic tests are adopted for spectrum sensing and to detect spectral occupancy opportunities. A detected signal in an AWGN channel follows a given probability distribution. When no signal is transmitted the cognitive user only detects the additive white Gaussian noise. Because this distribution and its statistical parameters are known, the secondary user can decide on the availability of a spectrum hole based on known tests. Under the hypothesis H_0 only AWGN noise is detected in the channel and the samples fit the Gaussian distribution. Otherwise, the signal does not fit the Gaussian distribution and the H_1 hypothesis is selected (which means that the band is occupied by a primary user).

Some spectrum sensing algorithms based on statistical tests are detailed in the following:

- Kurtosis test: after the samples are obtained, a Fast Fourier Transform (FFT) processing is performed. The kurtosis is then calculated from the absolute values of the FFT. The value of the kurtosis test is then compared to a predefined threshold λ; if the value of K is higher than λ, the detector understands that the channel is occupied (H_1 hypothesis); otherwise the cognitive user selects H_0 and considers that there is a spectrum occupancy opportunity [17].
- Skewness and Kurtosis test: a spectrum sensing algorithm named GHOST (Goodness of fit HOS Testing) was proposed by [18]. It is based on the kurtosis and skewness computed from the detected signal.
- Jarque-Bera test: the algorithms presented in [19] implement the JB test over the transmitted samples to compare the statistical test with the predefined threshold λ. The JB test presents the best performance, when compared to the kurtosis or the skewness tests [15].

4. ALGORITHM FOR FADING COGNITIVE NETWORKS BASED ON STATISTIC TESTS

Fading is an important effect to be considered in wireless communications. This phenomenon induces random fluctuations in the amplitude and phase of signals transmitted in wireless channels. These effects degrade the performance of the communication systems due to an increase in the error rates [20]. Nonetheless, the effects of the fading on the phase of the signals are usually disregarded in several analyzes. In this paper, the authors consider that fading affects only the amplitude of the signals in AWGN channels.

Several authors evaluate how the fading corrupts the spectrum sensing. However, to the best of the authors' knowledge, the effects of fading on the spectrum sensing based on statistic tests have not been explored. The objective of this paper is to propose a novel algorithm that considers the effects of the fading over the spectrum sensing based on statistical tests.

Figure 1 presents the performance of a Jarque-Bera detector for a BPSK signal transmission in AWGN channel and when affected by Rayleigh fading. 500 Monte Carlo simulations are performed.

It can be seen that the performance of the detector is penalized by the effects of the fading. The probability of detection for the JB test needs a better signal to noise ratio to obtain higher P_d values. Also, a higher number of simulations would lead to flatter curves for JB without fading and JB over Rayleigh fading, as the detection becomes more precise.

4.1. New Algorithm

The proposed algorithm is based on the statistical moments computed for a signal affected by fading. All the moments are multiplied with a degrading factor, which models the effects of the fading on the envelope of the signal that is detected. When considering that the signal and the fading are independent of each other, the fractional order moments of the received signal *y[n]* can be written as [21]:

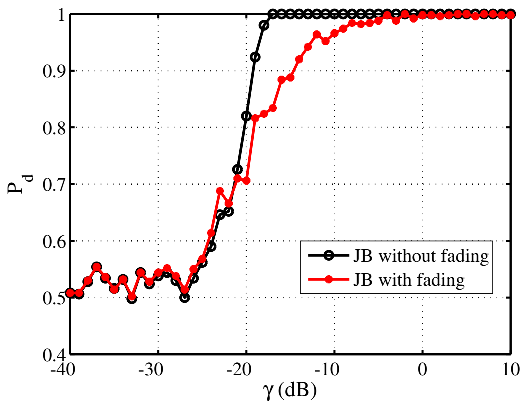

Fig. 1. Probability of detection (P_d) as a function of the signal-to-noise ratio (γ) for the Jarque-Bera test considering AWGN channel and the Rayleigh fading channel.

$$E(|y[n]|^k) = E(|x[n] \cdot R|^k) = E(|x[n]|^k) \cdot E(|R|^k), \quad (5)$$

in which $E(|R|^k)$ is the k-th order moment of the fading model that affects the channel. As the Jarque-Bera test is based on the third and fourth moments S and K, then one can consider that the fading affects the expressions (2) and (3) according to the relation (5) (if the independency between the signal and the fading is observed).

Then, a new spectrum sensing algorithm based on statistic test is proposed to deal with the effects of the fading in the performance of the signal detection. Jarque-Bera test was chosen as its performance was the better compared to other statistic sensing methods (kurtosis and skewness). In terms of the moments calculated in function of the k-th order of the distribution of the fading, the skewness and the kurtosis of the signal can be calculated by:

$$S_y = S_x \cdot E[|R|^3] \quad (6)$$

$$K_y = K_x \cdot E[|R|^4] \quad (7)$$

And the weighted skewness and kurtosis of the signal transmitted via the fading channel can then by written as [21]:

$$S_x = \frac{S}{E[|R|^3]} \quad (8)$$

$$K_x = \frac{K}{E[|R|^4]} \quad (9)$$

Finally, the new statistic test for Jarque-Bera under the above assumptions is:

$$JB_{modified} = \frac{N_S}{6}\left[\left(\frac{S}{E[|R|^3]}\right)^2 + \frac{(\frac{K}{E[|R|^4]} - 3)^2}{4}\right] \quad (10)$$

In this work, Rayleigh fading was considered and the wireless channel was simulated as operating over its effects. An approximation for the k-th moment for the Rayleigh distribution is available at [22]:

$$E[|R|^k] = (2\sigma^2)^{\frac{k}{2}} \cdot \Gamma\left(\frac{k}{2} + 1\right). \quad (11)$$

5. SIMULATION AND RESULTS

First simulation efforts considered the transmission of a digital television signal over a AWGN channel. The effect of the Rayleigh fading under the detection is compared when the fading is not present in the channel. Each sample image was simulated with 25 frames and submitted to 2048 FFT processing. A Binary Phase Shift Keying (BPSK) signal with 5.000 symbols was generated and transmitted through the channel under 500 Monte Carlo simulations. False alarm probability P_{fa} was fixed in 0.1. Figure 2 presents the preliminary results obtained via simulation.

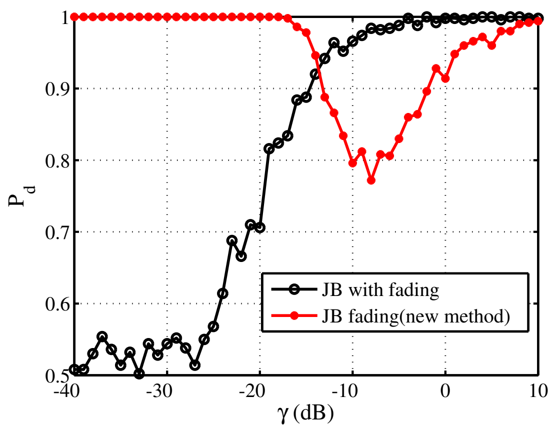

Fig. 2. Comparison of the modified JB algorithm with JB under Rayleigh fading.

One can verify that the proposed method performed better when compared to the JB detector over fading. The modified Jarque-Bera algorithm achieved a high detection probability (P_d = 1) even for a small values of signal to noise ratio (SNR). As the SNR is improved, the performance of the modified JB test presents a degradation in its performance and finally the probability of detection converges to the unity.

The preliminary results still need some analysis to provide a better insight of the behavior of the modified JB algorithm. A more precise threshold can improve the performance of the detector in order to not degrade the performance when the SNR increases. More simulations must be performed to improve the spectrum sensing. In addition, more simulations can lead to flatter curves for the JB test and the new JB test for the Rayleigh fading.

6. CONCLUSIONS

A new spectrum sensing based on statistic tests is proposed. Specifically, the new approach deals with the fading that can corrupt a cognitive transmission in a AWGN channel. In this paper, the Rayleigh fading is analyzed and preliminary results demonstrate the potential of the strategy. The setting of the Jarque-Bera statistic test threshold can be determinant to achieve a better performance in the spectrum sensing. Further simulation efforts will be executed to achieve better performance results for the proposed method. Additionally, the modified algorithm will be extended to other fading models aiming to compare its performance with the new strategy presented in this paper.

REFERENCES

[1] D. Raychaudhuri and N. B. Mandayam. Frontiers of Wireless and Mobile Communications. *Proceedings of the IEEE*, Vol. 100, No. 4, pp. 824-840, 2012.

[2] T. M. Taher et al. Long-term Spectral Occupancy Findings in Chicago. *IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks*, pp. 100 - 107, 2011.

[3] R. Umar and A. U. H. Sheikh. A Comparative Study of Spectrum Awareness Techniques for Cognitive Radio Oriented Wireless Networks. *Physical Communication*, 23 pp., 2012.

[4] S. Haykin. Cognitive Radio: Brain-Empowered Wireless Communications. *IEEE Journal on Selected Areas in Communications*. Vol. 23, No 2, pp. 201-220, 2005.

[5] I. F. Akyildiz et al. NeXt generation/dynamic spectrum access/cognitive radio wireless networks: A survey. *Computer Networks*. Vol. 50, pp. 2127-2159, 2006.

[6] O. B. Akan, O. B. Karli, and O. Ergul. Cognitive Radio Sensor Networks. *IEEE Network*. Vol. 23, No. 4, pp. 34-40, 2009.

[7] R. Tandra, S. M. Mishra, and A. Sahai. What is a Spectrum Hole and What Does It Take to Recognize One? *Proceedings of the IEEE*. Vol. 97, No. 5, pp. 824 - 848, 2009.

[8] P. C. Sofotasios et al. Energy Detection Based Spectrum Sensing Over $\kappa-\mu$ and $\kappa-\mu$ Extreme Fading Channels. *IEEE Transactions on Vehicular Technology*, Vol. 62, No. 3, pp. 1031 - 1040, 2013.

[9] F.F. Digham, M.S. Alouini, and M.K. Simon. On the Energy Detection of Unknown Signals Over Fading Channels. *IEEE Transactions on Communications*, Vol. 55, No. 1, pp. 21-24, 2007.

[10] O. Altrad and S. Muhaidat. A New Mathematical Analysis of the Probability of Detection in Cognitive Radio over Fading Channels. *Eurasip Journal on Wireless Communications and Networking*, 2013.

[11] W. Ejaz et al. I3S: Intelligent Spectrum Sensing Scheme for Cognitive Radio Networks. *EURASIP Journal on Wireless Communications and Networking*, doi:10.1186/1687-1499-2013-26. 2013.

[12] T. Yücek and H. Arslan. A Survey of Spectrum Sensing Algorithms for Cognitive Radio Applications. *IEEE Communications Surveys & Tutorials*, Vol. 11 , No. 1, pp. 116 - 130, 2009.

[13] A. Ghasemi and E. S. Sousa. Spectrum Sensing in Cognitive Radio Networks: Requirements, Challenges and Design Trade-offs. *IEEE Communications Magazine*. No 4, pp. 32-39, 2008.

[14] J. M. Tarongi and A. Camps. Normality Analysis for RFI Detection in Microwave Radiometry. *Remote Sensing*, No. 2, pp. 191 - 210, 2010.

[15] J. S. Rocha, J. E. P. Farias, and M. S. Alencar. Spectrum Sensing Based on Statistic Tests: A Bibliographic Review. *Revista de Tecnologia da Informação e Comunicação*, Vol. 3, No. 2, pp. 20 - 27, 2013.

[16] J. P. M. de Sá. Applied Statistics Using SPSS, STATISTICA, MATLAB and R. *Springer-Verlag Berlin Heidelberg*, 2007.

[17] A. Subekti et al. A Blind Spectrum Sensing Method for DTV Signal Detection. *International Conference of Information and Communication Technology (ICoICT)*, pp. 269-272, 2013.

[18] D. Denkovski, V. Atanasovski, and L. Gavrilovska. GHOST: Efficient Goodness-Of-Fit HOS Testing Signal Detector for Cognitive Radio Networks. *IEEE ICC - Cognitive Radio and Networks Simposium*, 2012.

[19] L. Lu, H. C. Wu, and S. S. Iyengar. A Novel Robust Detection Algorithm for Spectrum Sensing. *IEEE Journal on Selected Areas in Communications*, Vol. 29, No. 2, pp. 305-315, 2011.

[20] W. J. L. Queiroz et al. An Alternative Method to Compute the Bit Error Probability of Modulation Schemes Subject to Nakagami-m Fading. *EURASIP Journal on Advances in Signal Processing*, Article ID 574109, doi:10.1155/2010/574109, 2010.

[21] N. Madhavan et al. Spectrum Sensing and Modulation Classification for Cognitive Radios using Cumulants based on Fractional Lower Order Statistics. *International Journal of Electronics and Communications*, No. 67, pp. 479 - 490, 2013.

[22] J. G. Proakis and M. Salehi. Digital Communications. *The McGraw-Hill Companies*, 5th Ed., 2008.