UNIFORMLY MOST POWERFUL DETECTION FOR INTEGRATE AND FIRE TIME ENCODING

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ABSTRACT

A time encoding of a random signal is a representation of this signal as a random sequence of strictly increasing times. The goal of this paper is the rule for testing the mean value of a Gaussian signal from asynchronous samples given by the Integrate and Fire (IF) time encoding. The optimal likelihood ratio test is calculated and its statistical performance is compared with a synchronous test which is based on regular samples of the Gaussian signal. Since the IF samples based detector takes a decision at a random time, the regular samples based test exploits a random number of samples. The time encoding significantly reduces the number of samples needed to satisfy a prescribed probability of detection.

Index Terms— Likelihood ratio test, Time encoding, Random sampling, Integrate and fire sampler.

1. INTRODUCTION

A time encoding [1] of a random signal $x(t)$, $t \in \mathbb{R}$, is a representation of $x(t)$ as a random sequence, called a pulse train, of strictly increasing times (t_k), $k \in \mathbb{N}$, where $\mathbb{R}$ and $\mathbb{N}$ denote the set of real numbers and non-negative integers. Time encoding is a very relevant technique for dealing with asynchronous transmission and processing of signals. The renewed interest in asynchronous processing of signals is due to applications where low energy consumption and continuous-time processing are essential: electronic circuits [1], biomedical implants [2], sensor networks [3] and asynchronous event-based cameras [4] among others. Asynchronous processing is also interesting in the continuous time signal processing [5,6].

Many models [7] exist for encoding a signal in the time domain: level crossing sampler [8], the Integrate and Fire (IF) sampler [9] and the sigma-delta converter [10] among others. The traditional motivation for using time-based encoding schemes relies on the simplicity of their hardware implementation which are promising [11, 12]. Despite this strong motivation, studying the benefits of time encoding is still in its infancy. Hence, this paper proposes to evaluate the interest of time encoding for deciding between two hypotheses. In such a context, the decision rule is typically based on asynchronous samples produced by a low energy sensor. This paper deals with the well known IF time encoding which remains one of the most used techniques. Its use as a sampling framework has only recently been investigated [1, 13–15]. Most efforts in the literature have been devoted towards the design of reconstruction algorithms [1, 13, 16, 17] for bandlimited signals. Some first attempts for classification of pulse trains based on electrophysiological recordings is made in [9, 14, 18]. The comparison between IF sampling and standard regular sampling for deciding between two hypotheses is challenging.

This paper proposes three contributions. First, the detection problem is stated as a statistical test between two hypotheses. Each hypothesis corresponds to a random signal $x(t)$, completely characterized by its mean m, which is sampled by the IF sampler. Second, the statistical performance of the proposed test is calculated. It is shown that the test uniformly maximizes the probability of detection whatever the mean m of the random signal $x(t)$, provided that this mean is increasing from the first hypothesis to the second one. Third, the proposed test, called the IF test, is theoretically compared to the Regular Samples (RS) test which is based on regular samples of the input signal. Since the decision time of the IF test is random (it depends on the moment when the pulses are collected), the RS test exploits a random number of regular samples, which requires a specific attention. It is shown that IF sampling can save energy within this ideal theoretical framework.

The paper is organized as follows. Section 2 describes the IF sampler and the statistical pulse trains. Section 3 derives the optimal test based on the pulse train under a constrained false alarm probability. Section 4 theoretically compares the IF test and the RS test. Section 5 proposes a numerical study. Finally, Section 6 concludes this paper.

2. INTEGRATE AND FIRE TIME ENCODING

The adaptive IF time encoding typically takes place within the sensor. The sampling information is sent to a decision center which takes the decision from the whole family of sensors.

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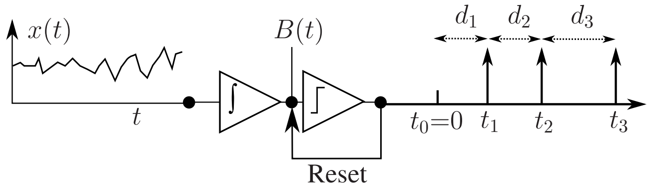

Fig. 1. Non-Leaky Integrate and Fire model within the sensor.

An input signal $x(t)$ for $t \geq 0$ is integrated until a positive threshold $\delta > 0$ is reached, at which point an output pulse is generated and the integrator is reset to zero. This process is illustrated in Fig. 1. The output signal $B(t)$ of the IF integrator satisfies the linear stochastic differential equation

$$dB(t) = m\,dt + \sigma\,dW(t) \tag{1}$$

where $W(t)$ is a standard Brownian motion. Hence, $B(t)$ is a Brownian motion with the drift m and the diffusion coefficient σ. The drift m represents the mean intensity of the input signal $x(t)$. Without loss of generality, it is assumed that $m > 0$. If not, a positive bias is added to $dW(t)$ in order to obtain a positive drift m. Hence, the delay for generating a pulse corresponds to the first passage time of $B(t)$ for the level δ:

$$D = \inf\{t > 0 \ : \ B(t) \geq \delta\}. \tag{2}$$

It follows that the IF sampler generates a sequence of pulses whose interarrival delays $D_1, \ldots, D_k, \ldots$ are distributed according to the probability density function (pdf) of D. The pdf of D is the well-known [19] Inverse Gaussian (IG) distribution $IG(\mu, \lambda)$ defined by

$$f(d; \mu, \lambda) = \left(\frac{\lambda}{2\pi d^3}\right)^{1/2} \exp\left(\frac{-\lambda(d-\mu)^2}{2\mu^2 d}\right) \tag{3}$$

for $d > 0$, where $\mu > 0$ is the mean and $\lambda > 0$ is the shape parameter. The variable d represents the delay between two pulses; it corresponds to the realizations of random variables $\{D_i\}_{i\geq 1}$. The time encoding of the input signal corresponds to the random times $T_0, T_1, \ldots$ where $T_0 = 0$ is the time starting point and $T_i = T_{i-1} + D_i$. The parameters of the IG distribution are $\mu = \mu(m) = \delta/m$ and $\lambda = \delta^2/\sigma^2$.

3. OPTIMAL TEST BASED ON THE PULSE TRAIN

Let us assume that the input signal $x(t)$ has two possible mean values m_1 and m_2 satisfying $m_2 > m_1$. The goal is to test the mean value of the signal, either m_1 or m_2, when σ is known. Let us consider a sequence of n encoded times $t_1^n = (t_1, \ldots, t_n)$ where t_i is a realization of the random variable T_i. This sequence is statistically equivalent to the interarrival delays $d_k = t_k - t_{k-1}$ where $t_0 = 0$. From Section 2, it follows that the detection problem can be formulated as the choice between the two hypotheses

$$\begin{aligned} \mathcal{H}_1 &: \big\{D_k \sim IG(\mu_1, \lambda)\,,\ k = 1\,\ldots, n\big\}, \\ \mathcal{H}_2 &: \big\{D_k \sim IG(\mu_2, \lambda)\,,\ k = 1\,\ldots, n\big\}, \end{aligned} \tag{4}$$

where $\mu_i = \mu_i(\delta) = \delta/m_i$ and $\mu_1 > \mu_2$. The parameter λ is known since δ and σ are known. A detailed introduction to statistical hypotheses testing theory is given in [20, 21]. Let

$$\mathcal{K}_\alpha = \{\phi : \Pr_1(\phi(D_1^n) = \mathcal{H}_2) \leq \alpha\} \tag{5}$$

be the class of tests of level α with an upper-bounded false alarm probability $0 < \alpha < 1$, where $\Pr_i(\cdot)$ stands for D_1^n being generated by the distribution $IG(\mu_i, \lambda)$. The probability of correct detection, also called the power, is

$$\beta_\phi = \Pr_2(\phi(D_1^n) = \mathcal{H}_2).$$

The optimal test for testing $\mathcal{H}_1$ and $\mathcal{H}_2$ in the class $\mathcal{K}_\alpha$, i.e. the test which maximizes the power under a constrained false alarm probability, is given by the well known Likelihood Ration Test (LRT). The LRT, which is called the IF test in the following, is given by [22, 23]

$$\phi^*(d_1^n) = \begin{cases} \mathcal{H}_1 \text{ if } \Lambda(d_1^n) = \sum_{k=1}^n \log \frac{f(d_k;\mu_2,\lambda)}{f(d_k;\mu_1,\lambda)} \leq h, \\ \mathcal{H}_2 \text{ else,} \end{cases}$$

where h is a threshold. A short calculation shows that the LRT can be rewritten as

$$\phi^*(d_1^n) = \phi^*(t_1^n) = \begin{cases} \mathcal{H}_1 \text{ if } \Lambda^*(d_1^n) = \sum_{k=1}^n d_k = t_n \geq h^*, \\ \mathcal{H}_2 \text{ else,} \end{cases} \tag{6}$$

where h^* is a threshold such that $\phi^* \in \mathcal{K}_\alpha$. From the basic properties of the IG distribution [24], $\Lambda^*(D_1^n)$ is distributed as $IG(n\mu_i, n^2\lambda)$ under $\mathcal{H}_i$. Let $F_{i,n}(\cdot)$ be the Cumulative Distribution Function (cdf) of $IG(n\mu_i, n^2\lambda)$ and $F_{i,n}^{-1}(\cdot)$ its inverse function. Then, the threshold h^* satisfies

$$h^* = F_{1,n}^{-1}(\alpha)$$

and the power $\beta_{\phi^*} = \Pr_2(\Lambda^*(D_1^n) > h^*)$ of the test ϕ^* is

$$\beta_{\phi^*} = \beta_{\phi^*}(\delta) = F_{2,n}(h^*) = F_{2,n}(F_{1,n}^{-1}(\alpha)) \tag{7}$$

where $\beta_{\phi^*}(\delta)$ is a function of δ through the parameters

$$n\mu_i = n\frac{\delta}{m_i} = \frac{\tilde{\delta}}{m_i} \quad \text{and} \quad n^2\lambda = n^2\frac{\delta^2}{m_i^2} = \frac{\tilde{\delta}^2}{m_i^2}. \tag{8}$$

It is straightforward to verify that the power function $\beta_{\phi^*}(\delta)$ is an increasing function of δ and/or n. Hence, changing the number of IF pulses n used by the test or the threshold δ has the same effect. For this reason, for the numerical experiments, it is assumed that $n = 1$ and only the value of δ varies.

From (6), it is interesting to note that the LRT takes its decision by comparing the last encoded time t_n to a threshold. Since the decision function t_n does not depend on $\mathcal{H}_2$ and h^* depends only on $\mathcal{H}_1$ and α, the test is Uniformly Most Powerful (UMP), i.e. it maximizes the probability to detect the value m_2 provided that $m_2 > m_1$ [20].

4. COMPARISON WITH REGULAR SAMPLING

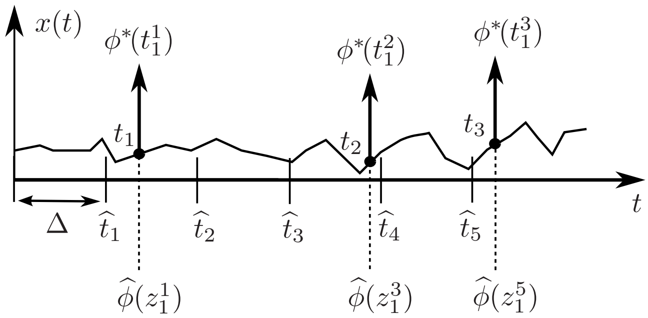

Fig. 2. Random decision times t_1, t_2 and t_3 for both the IF and RS tests. The regular sampling times are $\widehat{t}_1$, $\widehat{t}_2$, $\widehat{t}_3$, $\widehat{t}_4$ and $\widehat{t}_5$. At time t_i, the two tests do not necessarily exploit the same number of samples.

The IF test $\phi^*(t_1^n)$ takes a decision at random time t_n by using n encoded times t_1^n. Let us compare the performances of this test at a time t_n with respect to an optimal test based on regular samples, called the RS test. Let $\Delta > 0$ be the regular sampling period and $\widehat{t}_k = k\Delta$ be the sampling times. At time t_n, the regular sampling provides $\widehat{n} = \lfloor t_n/\Delta \rfloor \in \mathbb{N}$ samples where $\lfloor a \rfloor$ denotes the greatest integer less than or equal to a (see Fig. 2). This is the realization of the random variable $\widehat{N} = \lfloor T_n/\Delta \rfloor$ which admits the distribution:

$$p_{i,n}(\widehat{n}) = \Pr_i(\widehat{N} = \widehat{n}) = \int_{\widehat{n}\Delta}^{(\widehat{n}+1)\Delta} f(u; n\mu_i, n^2\lambda)du \quad (9)$$

for $\widehat{n} \in \mathbb{N}$. Since $\widehat{N}$ may be equal to 0 if $T_n < \Delta$, the RS test for $\widehat{n} = 0$ samples corresponds to a coin flip with the probability α to decide $\mathcal{H}_2$.

Let us assume that $\widehat{n}$ regular samples, denoted $z_1, \ldots, z_{\widehat{n}}$, are measured at time t_n. The regular samples are obtained by using a first-order integration sampler which integrates the signal (1) over a time interval of length Δ and normalizes the sample by $\sqrt{\Delta}$. Hence, the regular samples represent the realizations of the Gaussian random variables $Z_1, \ldots, Z_{\widehat{n}}$, where $Z_k \sim \mathcal{N}(m_i\sqrt{\Delta}, \sigma^2)$. In the field of decentralized sequential detection, such a sampling model has been studied in [25]. The RS test has to decide between

$$\begin{aligned}
&\mathcal{H}_1(\widehat{N} = \widehat{n})\colon \left\{Z_k \sim \mathcal{N}(m_1\sqrt{\Delta}, \sigma^2)\,,\ k = 1 \ldots, \widehat{n}\right\},\\
&\mathcal{H}_2(\widehat{N} = \widehat{n})\colon \left\{Z_k \sim \mathcal{N}(m_2\sqrt{\Delta}, \sigma^2)\,,\ k = 1 \ldots, \widehat{n}\right\}, \quad (10)
\end{aligned}$$

given the number $\widehat{N} = \widehat{n}$ of samples, at time t_n. The LRT between $\mathcal{H}_1(\widehat{N} = \widehat{n})$ and $\mathcal{H}_2(\widehat{N} = \widehat{n})$ is [20]:

$$\widehat{\phi}(z_1^{\widehat{n}}) = \begin{cases} \mathcal{H}_1(\widehat{N} = \widehat{n}) \text{ if } \widehat{\Lambda}(z_1^j) = \frac{1}{\sqrt{\widehat{n}\Delta}} \sum_{k=1}^{\widehat{n}} z_k \le \widehat{h}_{\widehat{n}}, \\ \mathcal{H}_2(\widehat{N} = \widehat{n}) \text{ else}, \end{cases} \quad (11)$$

where $\widehat{h}_{\widehat{n}}$ is given by $\widehat{h}_{\widehat{n}} = m_1\sqrt{\widehat{n}} + \sigma\Phi^{-1}(1-\alpha)$ and the conditional power, given that $\widehat{N} = \widehat{n}$, is

$$\begin{aligned}
\beta_{\widehat{\phi}}(\widehat{N} = \widehat{n}) &= \Pr_2\left(\widehat{\Lambda}(Z_1^{\widehat{n}}) > \widehat{h}_{\widehat{n}} | \widehat{N} = \widehat{n}\right) \\
&= 1 - \Phi\big((m_1 - m_2)\sqrt{\widehat{n}\Delta} + \sigma\Phi^{-1}(1-\alpha)\big) \quad (12)
\end{aligned}$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution. The power of the test $\widehat{\phi}$ obviously depends on the realization $\widehat{N} = \widehat{n}$ of the number of samples at time t_n. This test is UMP for all $m_2 > m_1$ [20]. In order to compare the IF test and the RS test, it is necessary to define the mean power of the RS test with respect to the distribution of $\widehat{N}$. Hence, let $\alpha_{\widehat{\phi}}(n, \delta)$ and $\beta_{\widehat{\phi}}(n, \delta)$ be the mean probability of false alarm and the mean power, respectively, of the RS test when the IF test deals with n samples at time t_n:

$$\alpha_{\widehat{\phi}}(n, \delta) = \sum_{\widehat{n}=0}^{\infty} p_{1,n}(\widehat{n})\alpha = \alpha, \quad (13)$$

$$\beta_{\widehat{\phi}}(n, \delta) = \sum_{\widehat{n}=0}^{\infty} p_{2,n}(\widehat{n})\beta_{\widehat{\phi}}(\widehat{N} = \widehat{n}), \quad (14)$$

where $p_{i,n}(\widehat{n})$, given in (9), depends both on n and δ.

5. NUMERICAL EXPERIMENTS

This section proposes a numerical comparison of the IF test and the RS test when $m_1 = 1$, $m_2 = 4$, $\sigma = 2$ and the prescribed false probability is $\alpha = 10^{-3}$.

Fig. 3.(a) gives the power function $\beta_{\phi^*}(\delta)$ and $\beta_{\widehat{\phi}}(\delta)$ with respect to the IF threshold δ when $\Delta = 1$ ms. As underlined in Section 3, the IF test deals with only $n = 1$ sample obtained at time t_1. The IF test is more powerful than the RS test when δ is large, which corresponds to a high probability of detection. Otherwise, the RS test is more powerful. Consequently, the IF test seems more interesting in practice since a high power is often targeted. Fig. 3.(b) gives the mean waiting delay of decision (in milliseconds) for each test. For the IF test, the mean waiting delay of decision corresponds to the mean time necessary to obtain $n = 1$ sample from the IF sampler. The RS test exploits a random number of samples $\widehat{N} = \widehat{n}$, hence its mean waiting delay of decision is $\Delta\,\overline{N}_i(\delta)$ where the mean number of samples $\overline{N}_i(\delta)$, depending on the true hypothesis $\mathcal{H}_i$, is given by

$$\overline{N}_i(\delta) = \sum_{\widehat{n}=0}^{\infty} \widehat{n}\, p_{i,n=1}(\widehat{n}) \approx \mu_i(\delta)/\Delta. \quad (15)$$

The approximation $\Delta\overline{N}_i(\delta) \approx \mu_i(\delta)$ comes from the fact that $p_{i,n}$ is obtained by quantizing the IG distribution. When Δ is small, the mean waiting delays of the two tests are almost the same.

Fig. 4.(a) gives the power function $\beta_{\phi^*}(\delta)$ and $\beta_{\widehat{\phi}}(\delta)$ with respect to Δ for three values of δ: 10, 20 and 30. Since δ

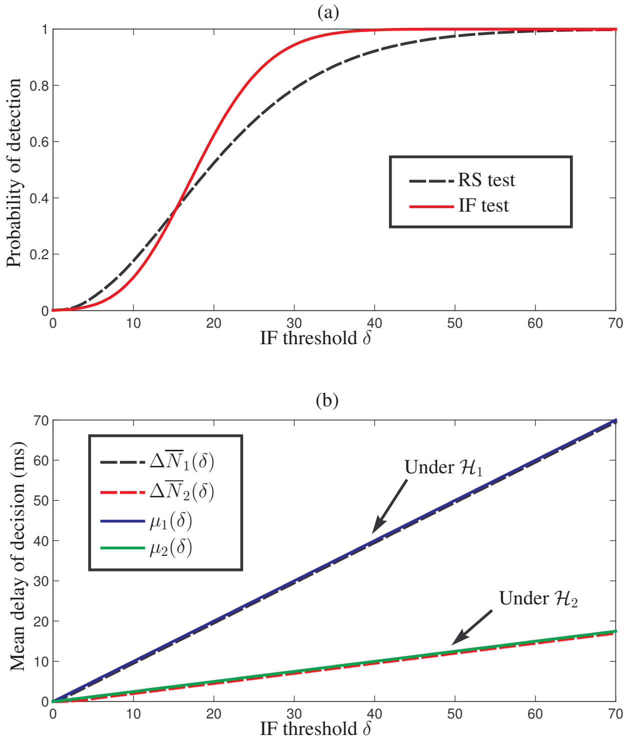

Fig. 3. The power function (a) and the mean delay (b) of the IF test and the RS test with respect to the IF threshold δ.

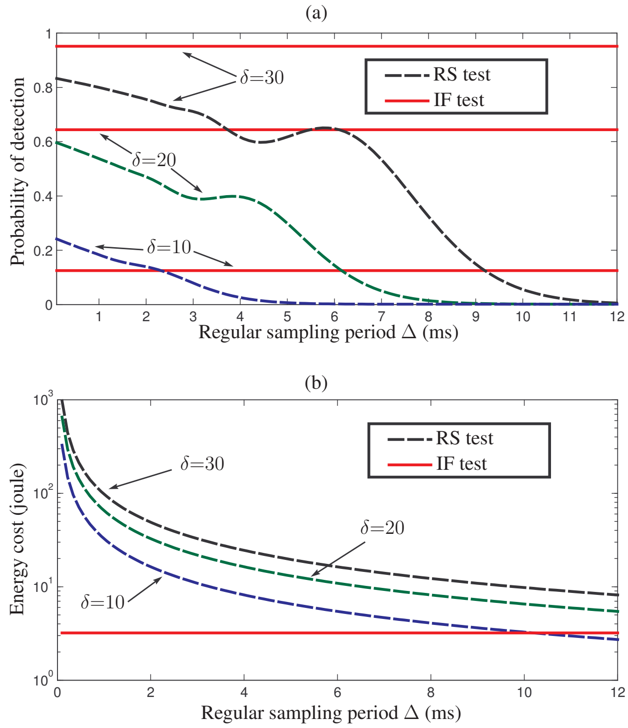

Fig. 4. Theoretical (a) power and (b) energy cost of the IF test and the RS test as a function of Δ for $\delta = 10, 20, 30$.

is fixed for each curve, the power of the IF test is constant. From (12), the power of the RS test clearly depends on Δ. If Δ is too large, there is no regular sample at time t_1, hence the power of the RS test is α. As Δ is decreasing, the power of the RS test increases (as the number of regular samples becomes larger) but the number of transmission always increases since the sampling rate is proportional to $1/\Delta$.

Finally, let us compare the energy consumption of each test. It is assumed that each sample is coded with q bits and each bit is transmitted with a constant energy-per-bit E_b (in joule). This energy consumption model is simple but the main goal is overall to show the advantages of the IF sampling within an ideal framework. The IF test uses only one IF sample to take a decision. Hence, the energy consumption of the IF test is

$$C_{\mathrm{IF}} = q\, E_b$$

which corresponds to the transmission of q bits. Contrary to the IF test, the RS test exploits a mean number of samples $\overline{N}_i(\delta)$ given by (15). Hence the mean energy consumption of the RS test is given by

$$C_{\mathrm{RS}} = q\, E_b\, \overline{N}_i(\delta) \approx q\, E_b\, \frac{\mu_i(\delta)}{\Delta} = q\, E_b\, \frac{\delta}{m_i \Delta} \qquad (16)$$

and it depends on the true hypothesis $\mathcal{H}_i$. The difference between C_{IF} and C_{RS} is due to the ratio $\delta/(m_i\Delta)$. Fig. 4.(b) shows the energy costs C_{IF} and C_{RS} as a function of Δ by assuming that $E_b = 0.1$ and $q = 32$. The IF test has the advantage of a constant cost whatever δ and the true hypothesis $\mathcal{H}_i$. Increasing the value of Δ reduces the energy cost C_{RS} but it involves a loss of detection performance as underlined in Fig. 4.(a).

6. CONCLUSION

This paper studies the optimal UMP test, called the IF test, to detect the mean level of a Gaussian signal from non-leaky Integrate and Fire (IF) samples. This test is compared to the UMP test, called the RS test, based on regular samples regularly spaced in time. The statistical performances of both the tests are explicitly calculated under a constrained prescribed false alarm probability. It is shown that the IF test can have a lower energy cost than the RS test while preserving the detection power. The proposed results are derived within a theoretical framework. Future work will concern the extension of this approach to real pulse-based systems.

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