FRACTIONALLY SPACED NON-LINEAR EQUALIZATION OF FASTER THAN NYQUIST SIGNALS

Stefano Tomasin and Nevio Benvenuto

Department of Information Engineering, University of Padova, {tomasin, nb}@dei.unipd.it

ABSTRACT

Faster than Nyquist transmissions provide the opportunity of increasing data rate at the expenses of additional inter-symbol interference. The optimum receiver requires a maximum likelihood sequence detector, whose complexity grows exponentially with the number of filter taps and with the number of bits per symbol. In this paper we consider two suboptimal approaches based on non-linear equalization of the received signal. In order to further reduce the receiver complexity we consider an implementation of equalization filters in the frequency domain. The contributions of the paper are a) a receiver architecture for fractionally spaced non-linear equalizers, and b) efficient design methods of the equalization filters in the frequency domain. In particular, the derived optimal (in the mean square error sense) filters overcome approaches proposed in the literature.

1. INTRODUCTION

For a transmission over non dispersive channels, if the cascade of the transmit and receive filters satisfies the Nyquist criterion for the symbol period, an equivalent symbol-rate system with no intersymbol interference (ISI) is obtained. The request for a higher spectral efficiency in modern communication systems has led to the investigation of configurations in which the symbol rate is increased, while keeping unchanged the bandwidth of the transmitted pulse. Therefore, the Nyquist criterion is not anymore satisfied and we obtain a *faster than Nyquist (FTN)* transmission system [1], which is characterized by ISI [2] even for transmissions on flat channels.

When ISI is present, the minimization of the bit error rate (BER) is obtained by the maximum likelihood sequence detector (MLSD) receiver, whose complexity however grows exponentially with the number of both interferers and bits per transmitted symbol. Various suboptimal solutions have been proposed in the literature: equalization, channel shortening, simplified MLSDs, also in combination with error correcting coding schemes (see [2] for an extensive literature review). With the exception of the *linear equalizer* of [3], no much attention however has been devoted to the use of equalization structures operating in the frequency domain (FD): indeed, for generic systems affected by ISI, it has been shown that high performance with low complexity can be achieved by implementing filters in the FD by means of discrete Fourier transforms (DFTs) [4]. However, it has been shown that for FTN transmissions non-linear equalization structures outperform linear ones by partially canceling ISI [5].

In this paper we aim at merging the advantages of FD equalization and non-linear equalization for FTN signals. In [6] and [7] two receiver structures have been proposed for FD non-linear equalization: the hybrid DFE (H-DFE) and the iterative block DFE (IB-DFE). The H-DFE implements a feedforward (FF) filter in the FD. Next, sample by sample a hard decision is made which then feeds a feedback (FB) filter, implemented in the time domain (TD). The IB-DFE instead implements both FF and FB filters in the FD, and detection is performed on a block-basis by an iterative detection/equalization structure. The contributions of the paper are a) a receiver architecture for fractionally spaced non-linear equalizers, and b) efficient design methods of the equalization filters in the FD.

2. SYSTEM MODEL

We consider a transmission system where the cascade of the transmit, the channel and the receive filters has impulse response $h(t)$, while the convolution of the transmit and the channel filter is $h^{(\mathrm{eq})}(t)$. The receive filter has impulse response $g(t)$. Symbols a_m, $m = 0, 1, \ldots$, are transmitted at rate $1/T$, and the signal after the receive filter is sampled at rate $2/T$. Let s_i be the upsampled version of a_m, i.e., $s_{2m} = a_m$, and $s_{2m+1} = 0$, for $m = 0, 1, \ldots$. The signal after sampling at the receiver can be written as

$$x_i = v_i + w_i = \sum_{k=0}^{N_\mathrm{h}-1} h_k s_{i-k} + w_i\,, \qquad (1)$$

where, by a proper sampling phase, $h_i = h(iT/2)$, $i = 0, 1, \ldots, N_\mathrm{h} - 1$ and w_i is the noise term. In particular, w_i is obtained by filtering a complex value zero-mean additive white Gaussian noise (AWGN) $w_i^{(\mathrm{IN})}$, having power spectral density N_0, with a filter having impulse response $\{g_i\}$, with $i = 0, 1, \ldots, N_\mathrm{g} - 1$, a sampled version at $T/2$ of the receive filter impulse response. Assuming that the bandwidth of the transmit filter is lower than $2/T$, then the reference signal to noise ratio at the channel output is

$$SNR = \frac{\mathrm{M}_a \sum_{k=-\infty}^{\infty} |h^{(\mathrm{eq})}(kT/2)|^2}{\mathrm{M}_{w_i^{(\mathrm{IN})}}} , \quad (2)$$

with M_a the power of $\{a_m\}$ and $\mathrm{M}_{w_i^{(\mathrm{IN})}} = N_0 2/T$. In the equivalent discrete-time model of our system data sequences are assumed independent and identically distributed (i.i.d.), with zero-mean, and statistically independent of noise.

In a conventional transmission over a flat fading AWGN channel, by choosing the cascade of the transmit and receive filters as a Nyquist filter at rate $1/T$, oversampling at the receiver can be omitted as sampling at rate $1/T$ yields a sufficient statistic without ISI. In the following we consider FTN systems where a faster sampling rate than $1/T$ must be used at the receiver to obtain a sufficient statistics.

Once x_i has been computed, various approaches can be considered to detect a_m. The solution that minimizes the error probability is provided by a MLSD receiver, that first whitens the noise and then performs a Viterbi detection. If the receive filter is matched to the received pulse a simpler MLSD is given by the Ungerboek's formulation [8]. However, the complexity of MLSD grows exponentially with both the number of taps N_h and the constellation size of a_m. We focus here on equalization approaches, where x_i is processed by a non-linear structure. [1]

The equalization structures that will be presented in the following sections require a particular block transmission format, denoted *pseudo noise (PN)-extended transmission*. In particular, data symbols $\{d_m\}$ are organized into blocks of length M, and each block is extended with a fixed sequence of L symbols, for example a PN sequence $\{q_m\}$, $m = 0, 1, \ldots, L-1$, which is assumed to be known at the receiver. Therefore, signal $\{a_m\}$ can be written as

$$a_{m+\ell P} = \begin{cases} d_{m+M\ell} & m = 0, 1, \ldots, M-1 \\ q_{m-M} & m = M, \ldots, P-1 \, . \end{cases} \quad (4)$$

where the last L symbols are the PN sequence. An additional PN extension is required before the first data block. Note that if $1/T$ remains the transmission rate, the symbol rate of a_m will be now $1/T' = (M/P)1/T$, i.e., we are transmitting at a slower data rate.

In the following we will assume that $L \geq (N_\mathrm{h} - 1)/2$, so that by taking blocks of x_i of size $2P$, in each block the first $2M$ samples are not affected by interference due to adjacent blocks. Therefore, without restrictions, we focus on the transmission/reception of first block, i.e., for x_i with $i = 0, 1, \ldots, 2P$, and we set the block index to zero, $\ell = 0$.

[1] **Notation:** Signals in the TD are denoted by lowercase italic letters. The DFT over $2P$ samples of a signal in the TD u_i, $i = 0, \ldots, 2P-1$ is denoted by its corresponding upper case letter, and DFT and inverse DFT (IDFT) are defined, respectively, as

$$U_p = \sum_{k=0}^{2P-1} u_k e^{-j\frac{2\pi kp}{2P}} , \quad u_i = \frac{1}{2P} \sum_{k=0}^{2P-1} U_p e^{j\frac{2\pi ip}{2P}} , \quad (3)$$

where $j = \sqrt{-1}$, and $i, p = 0, 1, \ldots, 2P-1$. The real part of x is denoted as $\Re(x)$. The complex conjugate of x is denoted as x^*.

3. HYBRID DFE

The H-DFE is a non-linear equalizer with the FF filter implemented in the FD on blocks of size $2P = 2(M+L)$, by means of DFT of the received signal, and the FB filter implemented in the TD, [6]. P/S and S/P blocks are parallel to serial and serial to parallel converters, respectively.

The received signal x_i at $T/2$ is split into blocks of size $2P$, whose DFT is [6]

$$X_p = H_p S_p + W_p \, . \quad (5)$$

Then FF filtering is performed by multiplying X_p by the FF filter coefficients $\{C_p\}$, yielding

$$Y_p = X_p C_p \, , \quad p = 0, 1, \ldots, 2P-1 . \quad (6)$$

Through IDFT of $\{Y_p\}$ we obtain $\{y_i\}$ in the TD. Decimation by 2 follows to obtain signal $\bar{y}_m$, which is summed to the FB signal to obtain $\bar{u}_m$, the signal at the detection point. Next, detection of $\bar{u}_m$ gives $\hat{a}_m$. If $\{\bar{b}_k\}$, $k = 1, 2, \ldots, N_\mathrm{b}$, are the coefficients of the FB filter, it is

$$\bar{u}_m = \bar{y}_m + \sum_{k=1}^{N_\mathrm{b}} \bar{b}_k \hat{a}_{m-k} \, , \quad m = 0, 1, \ldots, M-1 . \quad (7)$$

In (7), $\hat{a}_m$ for $m < 0$ corresponds to the inserted PN sequence.

3.1. H-DFE design

When H-DFE was first introduced [6, 9, 10], and also in more recent contributions [4, 11], its design is performed by a) expressing the optimal FF filter *as a function the FB filter*, b) obtaining the mean square error (MSE) as a function of the FB filter coefficients only; c) minimizing the resulting MSE expression to obtain the FB coefficients; d) inserting the FB coefficients into the first FF filter expression and evaluate. However, this procedure does not yield the global minimum MSE. We derive here the optimal solution, that holds for both fractionally-space and non-fractionally-spaced equalizers.

Let M_S and $\mathrm{M}_{W^{(\mathrm{IN})}}$ be the power of S_p and $W_p^{(\mathrm{IN})}$, respectively. It is $\mathrm{M}_S = \frac{P}{2}\mathrm{M}_a$ and $\mathrm{M}_{W^{(\mathrm{IN})}} = P\mathrm{M}_{w^{(\mathrm{IN})}}$. Moreover, $W_p = W_p^{(\mathrm{IN})} G_p$, where G_p is the $2P$-size DFT of the sampled impulse response g_i. The MSE can be written as

$$J = \frac{1}{P} \mathrm{E}\left[\sum_{m=0}^{P-1} |\bar{u}_m - a_m|^2\right] \quad (8)$$

$$\frac{\partial J}{\partial C_k} = \frac{1}{P^2}\frac{\mathrm{M}_{W^{(\mathrm{IN})}}|G_k|^2 C_k}{2} - \frac{\mathrm{M}_S}{P^2} H_k^* \sum_{p=0}^{P-1}\left\{1-\frac{C_pH_p+C_{p+P}H_{p+P}}{2} - \right.$$
$$\left. - \sum_{q=0}^{P-1}\left(1-\frac{C_qH_q+C_{q+P}H_{q+P}}{2}\right)\frac{1}{P}\sum_{k'=1}^{N_\mathrm{b}} e^{j\frac{2\pi k'(q-p)}{P}}\right\}\left\{(\frac{N_\mathrm{b}}{P}-1)\delta(p-k) - (1-\delta(p-k))\frac{1}{P}\sum_{k'=1}^{N_\mathrm{b}} e^{-j\frac{2\pi k'(k-p)}{P}}\right\} = 0\,, \tag{15}$$

By using the Parseval's theorem we obtain

$$J = \frac{1}{P^2}\sum_{p=0}^{P-1} \mathrm{E}[|\bar{U}_p - \bar{S}_p|^2]\,, \tag{9}$$

where the FD properties of upsampling yield

$$\bar{U}_p = \frac{1}{2}[Y_p + Y_{p+P}], \quad \bar{S}_p = \frac{1}{2}[S_p + S_{p+P}] = S_p\,. \tag{10}$$

Now we can rewrite the MSE as

$$J = \frac{1}{P^2}\sum_{p=0}^{P-1}\left[\mathrm{M}_{W^{(\mathrm{IN})}}\frac{|G_pC_p|^2+|G_{p+P}C_{p+P}|^2}{4} + \right.$$
$$\left. \mathrm{M}_S\left|1-\frac{C_pH_p+B_p+C_{p+P}H_{p+P}+B_{p+P}}{2}\right|^2\right]\,, \tag{11}$$

where

$$B_p = \sum_{k=1}^{N_\mathrm{b}} \bar{b}_k e^{-j\frac{2\pi kp}{P}} \tag{12}$$

is the $2P$-size DFT of the upsampled version of the FB filter $\{\bar{b}_k\}$, hence $B_p = B_{p+P}$, $p = 0, 1, \ldots, P-1$. Note that it must be $\bar{b}_0 = 0$. Assuming correct detections (i.e., $\hat{a}_m = a_m$), for a given FF filter, the optimal FB filter removes all residual interference. Hence, from (11) we have

$$\bar{b}_k = \frac{1}{P}\sum_{p=0}^{P-1}\left(1-\frac{C_pH_p+C_{p+P}H_{p+P}}{2}\right)e^{j\frac{2\pi kp}{P}}\,, \tag{13}$$

for $k = 1, \ldots, N_\mathrm{b}$. Now, substituting B_p, the MSE can then be written as a function of the FF filter only as

$$J = \frac{1}{P^2}\sum_{p=0}^{P-1}\left\{\mathrm{M}_{W^{(\mathrm{IN})}}\frac{|G_pC_p|^2+|G_{p+P}C_{p+P}|^2}{4} + \right.$$
$$\mathrm{M}_S\left|1-\frac{C_pH_p+C_{p+P}H_{p+P}}{2} + \right.$$
$$\left.\left. - \sum_{q=0}^{P-1}\left(1-\frac{C_qH_q+C_{q+P}H_{q+P}}{2}\right)\frac{1}{P}\sum_{k=1}^{N_\mathrm{b}} e^{j\frac{2\pi k(q-p)}{P}}\right|^2\right\}. \tag{14}$$

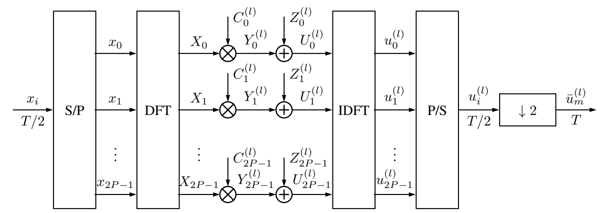

Fig. 1. feedforward part of the IB-DFE.

By setting to zero the gradient of J with respect to C_k we obtain the linear equation system (15) at the top of the page where $\delta(0) = \delta(-P) = 1$ and $\delta(k) = 0$, for $k \neq 0, -P$. Its solution yields the optimal FF coefficients. Next by (13) we derive the FB coefficients.

4. ITERATIVE BLOCK DFE

The fractionally spaced IB-DFE scheme, operating on blocks of the received signal, is shown in Figs. 1 and 2. The scheme performs iterated operations on each $2P$-size block of the received signal [7]. We denote with $l = 0, 1, \ldots, N_\mathrm{I} - 1$, the iteration number. The equalizer includes two parts: 1) the FF filter with coefficients $\{C_p^{(l)}\}$, $p = 0, 1, \ldots, 2P-1$, in the FD, (see Fig. 1), which partially equalizes the interference, and 2) the FB filter with coefficients $\{B_p^{(l)}\}$, $p = 0, 1, \ldots, 2P-1$, and output $\{Z_p^{(l)}\}$ in the FD (see Fig. 2), which removes part of the residual interference.

In details, similarly to the H-DFE, also for IB-DFE $2P$-size blocks of x_i are transformed by DFT and then multiplied by the FF filter coefficients in the FD obtaining signal

$$Y_p^{(l)} = C_p^{(l)} X_p\,, \quad p = 0, 1, \ldots, 2P-1. \tag{16}$$

Then the output of the FB filter is summed to the FF output to obtain

$$U_p^{(l)} = Y_p^{(l)} + Z_p^{(l)}\,. \tag{17}$$

Lastly, $U_p^{(l)}$ is transformed by IDFT into $\{u_i^{(l)}\}$, which is downsampled to $\{\bar{u}_m^{(l)}\}$.

With regard to the FB part of Fig. 2, detection is performed on $\bar{u}_m^{(l-1)}$ to obtain $\hat{a}_m^{(l-1)}$, $m = 0, 1, \ldots, M-1$. Next, $\{\hat{a}_i^{(l-1)}\}$ is extended to size P by PN insertion. By

$$J^{(l)} = \frac{1}{P^2} \sum_{p=0}^{P-1} \left\{ \left| \frac{C_p^{(l)} H_p + C_{p+P}^{(l)} H_{p+P}}{2} - 1 \right|^2 \mathrm{M}_S + \left| \frac{B_p^{(l)} + B_{p+P}^{(l)}}{2} \right|^2 \mathrm{M}_{\hat{S}}^{(l)} + \right.$$
$$\left. + 2\Re \left[\left(\frac{C_p^{(l)} H_p + C_{p+P}^{(l)} H_{p+P}}{2} - 1 \right) \left(\frac{B_p^{(l)} + B_{p+P}^{(l)}}{2} \right)^* \mathrm{r}_{S\hat{S}}^{(l)} \right] + \mathrm{M}_{W^{(\mathrm{IN})}} \frac{|C_p^{(l)} G_p|^2 + |C_{p+P}^{(l)} G_{p+P}|^2}{4} \right\} . \quad (21)$$

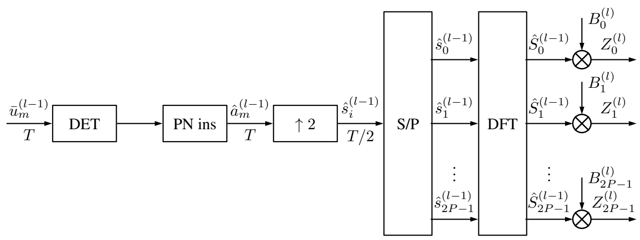

Fig. 2. feedback part of the IB-DFE.

upsampling, $\{\hat{s}_i^{(l-1)}\}$ is then obtained. DFT follows, whose output is multiplied by the FB filter coefficients to yield

$$Z_p^{(l)} = B_p^{(l)} \hat{S}_p^{(l-1)} , \quad p = 0, 1, \ldots, 2P-1. \quad (18)$$

Since $Z_p^{(l)}$ depends upon the detected data at iteration $(l-1)$, for $l = 1$, when no detected data is available, we set

$$\hat{a}_m^{(0)} = 0 , \quad m = 0, 1, \ldots, M-1 , \quad (19)$$

while for $m = M, \ldots, P-1$ we have the PN sequence. Note that when IB-DFE is used, since detection is performed on blocks of signal, we can perform decoding and re-encoding of signal (with respect to error correcting codes used at the transmitter) in order to improve the reliability of the signal used in the FB.

4.1. Design method

Also for IB-DFE we consider the minimization of the MSE at the detection point as a design criterion. From Fig. 1, the MSE $J^{(l)}$ at the detection point for iteration l is given by (9) where now $\bar{U}_p = (U_p^{(l)} + U_{p+P}^{(l)})/2$.

We also impose the constraint that the FB filter does not remove the desired component, i.e.,

$$\sum_{p=0}^{P-1} B_p^{(l)} = 0 . \quad (20)$$

Let $\mathrm{M}_{\hat{S}}^{(l)}$ be the power of $\hat{S}_p^{(l)}$ and let us define the average correlation between S_p and $\hat{S}_p^{(l)}$ as $\mathrm{r}_{S\hat{S}}^{(l)} = \mathrm{E}[S_p \hat{S}_p^{(l)*}]$. Methods to estimate $\mathrm{r}_{S\hat{S}}^{(l)}$ are given in [7]. Then $J^{(l)}$ can be rewritten as in (21) at the top of the page. By using the Lagrange multiplier method, the FB filter coefficients minimizing $J^{(l)}$ under constraint (20) are

$$B_p^{(l)} = -\frac{\mathrm{r}_{S\hat{S}}^{(l-1)}}{\mathrm{M}_{\hat{S}}} \left[\frac{C_p^{(l)} H_p + C_{p+P}^{(l)} H_{p+P}}{2} - \gamma^{(l)} \right] , \quad (22)$$

and $B_{p+P}^{(l)} = B_p^{(l)}$, $p = 0, 1, \ldots, P-1$

$$\gamma^{(l)} = \frac{1}{2P} \sum_{p=0}^{2P-1} C_p^{(l)} H_p . \quad (23)$$

Defining $\kappa^{(l)} = 1 - \frac{|\mathrm{r}_{S\hat{S}}^{(l)}|^2}{\mathrm{M}_S \mathrm{M}_{\hat{S}}}$, apart from an irrelevant constant factor, the optimal FF filter coefficients are

$$C_p^{(l)} = H_p^* |G_{p+P}|^2 \times \left[\kappa^{(l-1)} \frac{|H_p|^2 |G_{p+P}|^2 + |H_{p+P}|^2 |G_p|^2}{2} + \frac{\mathrm{M}_{W^{(\mathrm{IN})}}}{2\mathrm{M}_S} |G_p|^2 |G_{p+P}|^2 \right]^{-1} , \quad p = 0, 1, \ldots, 2P-1 \quad (24)$$

where we consider a periodic extension of both H_p and G_p with period $2P$, i.e., $H_{p+P} = H_{p-P}$ for $p = P, \ldots, 2P-1$, and similarly for G_p.

We just point out that the design computational complexity of the IB-DFE is much lower than that of H-DFE if N_{I} is small. Conversely, with regard to processing, the complexity of IB-DFE is much greater than that of H-DFE, especially when coding is used.

5. NUMERICAL RESULTS

In order to assess the performance of fractionally spaced H-DFE and IB-DFE in the context of satellite transmissions, we assume that the transmitter uses a square-root raised cosine filter with bandwidth $\beta \frac{1+\alpha}{T}$, where α is the roll-off factor and $\beta \leq 1$ is the FTN ratio between the data rate $1/T$ and the transmit filter Nyquist rate. Here we consider $\alpha = 0.2$. We also assume that at the receiver a matched filter to the transmit filter is used, and the channel is flat. Therefore $\{g_i\}$ is obtained from a square-root raised cosine filter which is sampled and properly delayed such that $|g_{(N_{\mathrm{g}}-1)/2}| = \max_i |g_i|$,

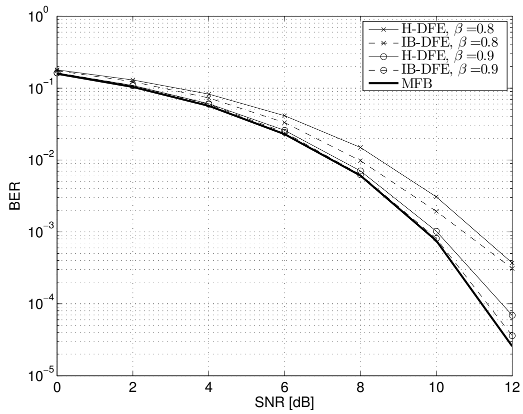

Fig. 3. BER vs SNR for H-DFE and IB-DFE without coding.

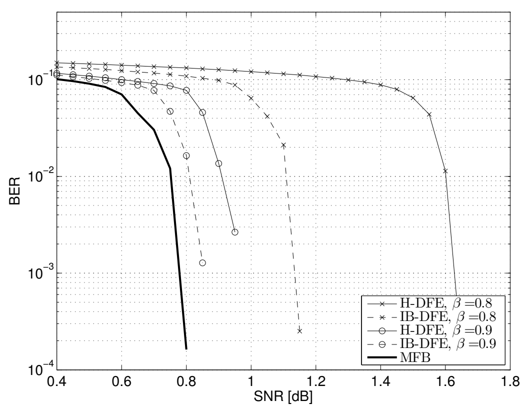

Fig. 4. BER vs SNR for H-DFE and IB-DFE with coding.

and we set $N_{\rm g} = 81$. Similarly, filter $\{h_i\}$ is a raised cosine filter, with bandwidth $\beta(1+\alpha)/T$, which is sampled at $T/2$ and properly delayed such that $|h_{(N_{\rm h}-1)/2}| = \max_i |h_i|$, and we set $N_h = 81$. Therefore we consider $L = 40$. The block size is $P = 256$. For H-DFE we set $N_{\rm b} = 40$, while for IB-DFE we set the maximum number of iterations $N_{\rm I} = 2$.

For coding purposes, we have considered the low density parity check (LDPC) code of the digital video broadcasting satellite standard DVB-S2, with rate 1/2 and block size of 64800 bit. The mapped symbols are quadrature phase shift keying (QPSK) modulated.

Fig. 3 shows the BER of uncoded systems vs signal to noise ratio (SNR) for $\beta = 0.8$ and $\beta = 0.9$. We also show the matched filter bound (MFB) as a reference performance. We note that for the considered range of SNR values, both IB-DFE and H-DFE perform quite well even for $\beta = 0.8$, which is the most adverse situation with respect to ISI.

When coding is considered, we obtain the performance reported in Fig. 4. Note the different range (both in size and in starting value) of SNR values with respect to Fig. 3. Again we observe that performance of the two equalization systems are close to that of the MFB.

6. CONCLUSIONS

In this paper we have proposed two FD equalization techniques for FTN transmissions. The H-DFE and the IB-DFE structures have been revisited for the special scenario, deriving receiver architectures and design procedures for an oversampled input.

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