NOISE FILTERING IN BANDLIMITED DIGITAL CHAOS-BASED COMMUNICATION SYSTEMS

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ABSTRACT

In recent years, many chaos-based communication schemes were proposed. However, their performance in non-ideal scenarios must be further investigated. In this work, the performance of a bandlimited binary communication system based on chaotic synchronization is evaluated considering a white Gaussian noise channel. As a way to improve the signal to noise ratio in the receiver, and thus the bit error rate, we propose to filter the out-of-band noise in the receiver. Numerical simulations show the advantages of using such a scheme.

Index Terms— Chaos-based communication; bandlimited channels; additive noise; synchronization.

1. INTRODUCTION

In digital chaos-based communications systems, each bit of information is transmitted using a different fragment of a chaotic signal [1, 2]. Thus, it differs fundamentally from the conventional digital communication systems, where each symbol is associated with a constant and predefined waveform. Although it may have interesting features, like an improvement in security [3, 4] and ultra-wideband possible applications [5], it also poses practical challenges because the conventional optimal receiver, the matched filters bank [6], is not directly available.

The chaos-based communication systems have been studied for at least 20 years now [7, 8]. Many interesting and innovative communication schemes based on chaos synchronization were proposed [1, 3]. However, they seldom surpassed the frontier between theoretical and laboratory setup to practical or commercial environments. One important reason for this fact is the sensibility of the chaotic synchronization to channel imperfections, like noise or distortion [9, 10]. The adaptation of the chaos-based digital communication systems, so that they can really work and compete with conventional ones in practical setups, is a relevant and active research field [10, 11].

One of the issues when it comes to practical channels is related to the chaotic signals bandwidth. Many works cite "large bandwidth" as a property of chaos [12]. However, this characterization is not enough in practical settings. As the physical channels are always bandlimited, it is necessary to know exactly what is the frequency range occupied by a transmitted chaotic signal and, preferably, it should be possible to control it. Although there are some recent results related to this [13, 14], there is still much to progress.

Recently, it was proposed a discrete-time chaos-based communication system capable of generating and transmitting bandlimited signals [15]. However, a performance analysis of such system in a noisy channel was not provided. Furthermore, only analogical messages were considered. In the current paper we apply this system to the transmission of binary data. This way, it is possible to evaluate its performance in terms of Bit Error Rate (BER), as usual in the digital communication literature. Furthermore, as the transmitted signal is essentially bandlimited, we are able to use a tuning filter in the receiver in order to improve this BER.

The paper is organized as following: in Section 2 we review the bandlimited chaos-based communication scheme proposed by [15] and describe how it can be adapted to transmit digital information. Next, in Section 3, the use of a tuning filter for noise filtering is detailed and in Section 4 numerical simulations are presented. Finally, in Section 5 we draft some conclusions.

2. BANDLIMITED DIGITAL CHAOS-BASED COMMUNICATIONS

A simple way to synchronize master-slave chaotic systems was proposed by Wu and Chua [16]. Their scheme is based in the separation of linear and non-linear components of the involved systems. As long as the linear part is stable and the non-linear component is transmitted from master to slave, both systems will identically synchronize [16]. This scheme was adapted for discrete-time systems in [15]. In this work the master equation is written as

$$\mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n) + \mathbf{b} + \mathbf{f}(\mathbf{x}(n)), \tag{1}$$

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and the slave counterpart, which depends on $\mathbf{x}(n)$, is given by

$$\mathbf{y}(n+1) = \mathbf{A}\mathbf{y}(n) + \mathbf{b} + \mathbf{f}(\mathbf{x}(n)) \tag{2}$$

where $n \in \mathbb{N}$ represents time instants, $\mathbf{A}$ and $\mathbf{b}$ are constants, $\{\mathbf{x}(n), \mathbf{y}(n)\} \subset \mathbb{R}^K$, $\mathbf{x}(n) = [x_1(n), x_2(n), \ldots, x_K(n)]^T$ and $\mathbf{y}(n) = [y_1(n), y_2(n), \ldots, y_K(n)]^T$. The function $\mathbf{f}(\cdot) : \mathbb{R}^K \to \mathbb{R}^K$ is non-linear in general.

Using (1)-(2), the synchronization error, $\mathbf{e}(n) \triangleq \mathbf{x}(n) - \mathbf{y}(n)$, can be written as

$$\mathbf{e}(n+1) = \mathbf{A}\mathbf{e}(n), \tag{3}$$

so master and slave identically synchronize if the eigenvalues λ_i of $\mathbf{A}$ satisfy

$$|\lambda_i| < 1,\ 1 \leq i \leq K. \tag{4}$$

Using the master-slave structure of (1)-(2), it is straightforward to propose a chaos-based communication system that presents zero error in ideal channel conditions [15, 16]. For this, we assume that $\mathbf{f}(\cdot)$ depends uniquely on the first component $x_1(n)$ of $\mathbf{x}(n)$ and can be written as $\mathbf{f}(\mathbf{x}(n)) = [f(x_1(n)), 0, \ldots, 0]^T$.

The message $m(n)$ is encoded by the chaotic signal $x_1(n)$ through the revertible function $c(\cdot,\cdot)$, resulting in the transmitted signal $s(n) = c(x_1(n), m(n))$. This way, $m(n)$ can be decoded by

$$m(n) = c^{-1}(x_1(n), s(n)). \tag{5}$$

The master-slave equations (1)-(2) are rewritten as

$$\mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n) + \mathbf{b} + \mathbf{f}(s(n)) \tag{6}$$

$$\mathbf{y}(n+1) = \mathbf{A}\mathbf{y}(n) + \mathbf{b} + \mathbf{f}(r(n)) \tag{7}$$

where $r(n)$ is the signal received from the channel. Note that in the ideal channel case, $s(n) = r(n)$ and the synchronization error is still given by (3). This way, if the eigenvalues of $\mathbf{A}$ satisfies (4), $y_1(n) \to x_1(n)$ and if we define the recovered message as $\hat{m}(n) = c^{-1}(y_1(n), r(n))$, using (5), $\hat{m}(n) \to m(n)$. Clearly this may not be the case for a non-ideal channel where $r(n) \neq s(n)$.

This scheme is represented in Figure 1. Here the channel is modeled as an additive white Gaussian noise (AWGN), $w(n)$, with zero mean and power σ_w^2.

As chaotic signals are broadband in general, $s(n)$ will be broadband. In [15] it was proposed to adjust the spectrum of $s(n)$ using a Finite Impulse Response (FIR) filter in the feedback loop. The spectrum of $x_1(n)$ is limited using a low-pass filter, $H_S(\omega)$, with cut-off frequency ω_S. This way, for each input $x_1(n)$, the output $x_{K+1}(n)$ is written as

$$x_{K+1}(n) = \sum_{j=0}^{N} c_j x_1(n-j) \tag{8}$$

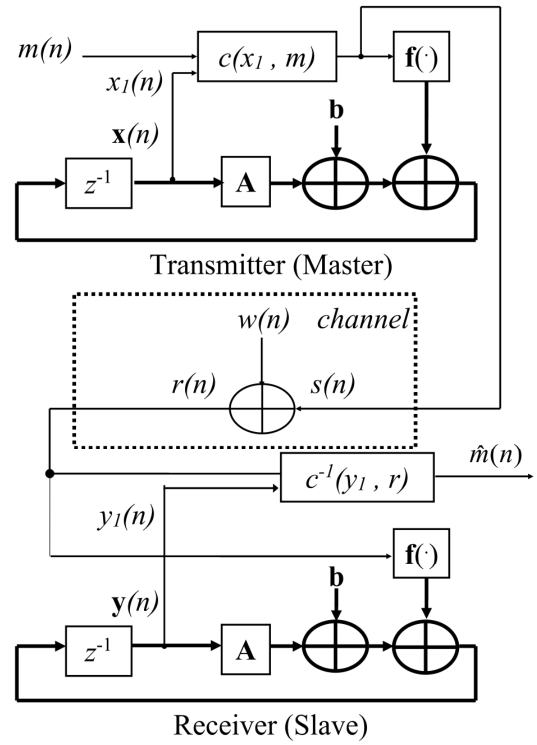

Fig. 1. Communication system proposed in [15] considering an AWGN channel.

where c_j, $0 \leq j \leq N$, are the $H_S(\omega)$ filter coefficients. Now, $s(n)$ is rewritten as $s(n) = c(x_{K+1}(n), m(n))$.

An identical filter is also placed in the receiver, so that master and slave are modified in the same way. Notice that the filters can be aggregated to the linear part of the original systems [15]. Nevertheless, as long as the matrix $\mathbf{A}'$, corresponding to this new system, satisfies the condition (4), the identical synchronization and chaos-based communication system still work in the same way as before. Figure 2 shows a block diagram of this scheme, for $H_T(\omega) = 1$ and $r'(n) = r(n)$.

Previous works [15, 17] used analog sine messages $m(n)$, which are naturally bandlimited. As we are considering digital messages, represented by an equiprobable sequence of bits $b_i = \pm 1$, we have the additional problem of making $m(n)$ essentially bandlimited. The message is generated as $m(n) = b_i p(n), iM \leq n < (i+1)M$, where $p(n) = 1, 0 \leq n < M$, is a M-width rectangular unitary pulse. The Power Spectral Density (PSD) of $m(n)$ is then given by [6]

$$\mathcal{M}(\omega) = \frac{1}{M}\left[\frac{\sin\left(\frac{\omega M}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right]^2. \tag{9}$$

Considering the first positive null as a criteria for the bandwidth ω_S of $m(n)$, M must be chosen as $M = \frac{2\pi}{\omega_S}$.

For retrieving the binary sequence, we use the optimum receiver [6], that in this case consists of attributing $\hat{b}_i = 1$ if

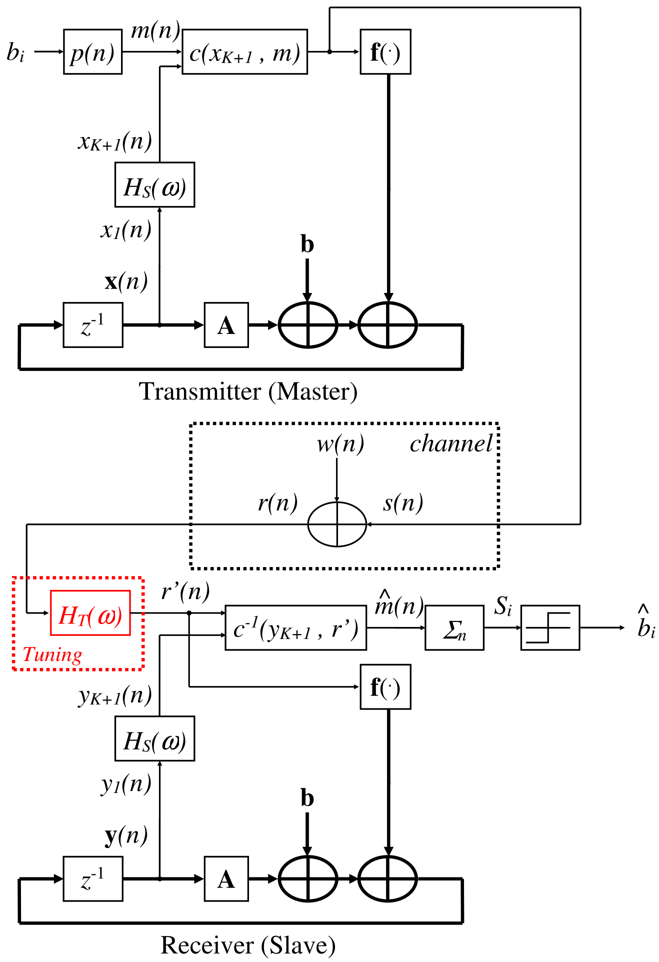

Fig. 2. Bandlimited digital chaos-based communication scheme.

the summation

$$S_i = \sum_{n=iM}^{(i+1)M-1} \hat{m}(n) \tag{10}$$

is positive and $\hat{b}_i = -1$ otherwise.

As chaos generator we consider the Hénon map [18], as in [19, 20], given by

$$\mathbf{x}(n+1) = \underbrace{\begin{bmatrix} 0 & 1 \\ \beta & 0 \end{bmatrix}}_{\mathbf{A}} \mathbf{x}(n) + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\mathbf{b}} + \underbrace{\begin{bmatrix} -\alpha x_1^2(n) \\ 0 \end{bmatrix}}_{\mathbf{f}(x_1(n))}. \tag{11}$$

In the numerical simulations, we consider $\alpha = 0.9$ and $\beta = 0.3$, as in [19]. In this reference it is shown that these choices are convenient for obtaining chaotic signals for a large range of cut-off frequencies and filter orders for $H_S(\omega)$.

Figure 3 shows an example of $m(n)$, the corresponding transmitted signal $s(n)$ and recovered message $\hat{m}(n)$ for $M = 10$, in the noiseless case, in time and frequency domain. Clearly, the message is recovered in this case.

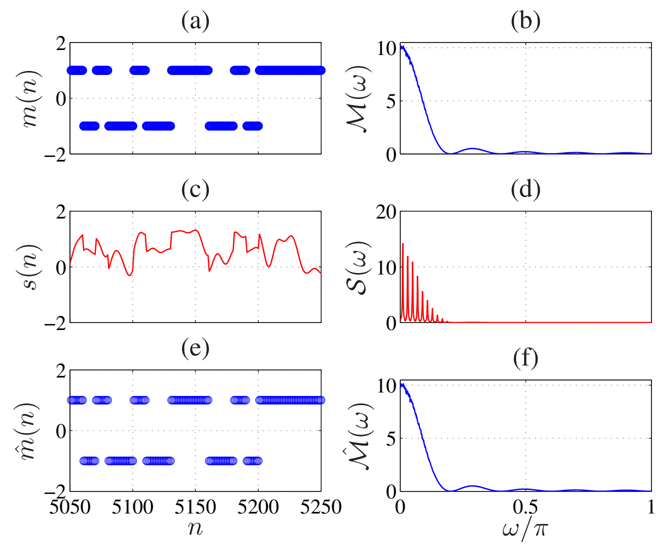

Fig. 3. Digital chaos-based communication signals: (a) $m(n)$, (c) $s(n)$, (e) $\hat{m}(n)$ and PSD functions: (b) $\mathcal{M}(\omega)$, (d) $\mathcal{S}(\omega)$ and (f) $\hat{\mathcal{M}}(\omega)$.

3. TUNING AND NOISE FILTERING

The transmitted chaotic signal $s(n)$ of the presented system is bandlimited. Since AWGN has power equally distributed in all frequencies, including the ones where $s(n)$ has no significant components, a low-pass FIR filter, $H_T(\omega)$, with cut-off frequency ω_T, is placed at the entrance of the receiver to eliminate the out-of-band noise interference, as shown in Figure 2. The resulting filtered signal, $r'(n)$, is used in the message recovering as

$$\hat{m}(n) = c^{-1}\left(y_{K+1}(n), r'(n)\right). \tag{12}$$

To illustrate the tuning process, Figure 4 presents the frequency response of a tuning filter with cut-off frequency $\omega_T = 0.38\pi$, the noise and $s(n)$ PSDs $\mathcal{W}(\omega)$, $\mathcal{S}(\omega)$ namely, for $M = 20$, $\omega_S = 0.2\pi$ and $\sigma_w^2 = 0.05$. The filter is clearly eliminating out-of-band noise.

4. NUMERICAL SIMULATIONS

In this section, we access the proposed digital communication systems performance, in terms of BER, through numerical simulations. As coding function we consider

$$s(n) = (1-\gamma)\, x_1(n) + \gamma m(n), \tag{13}$$

with $\gamma = 0.3$. The lowpass filters in the transmitter and receiver systems were designed using a Blackman windowing of order 200 and cut-off frequency $\omega_S = 0.2\pi$.

Initially, Figure 5 shows the BER as a function of the channel SNR for different time symbol duration M and no

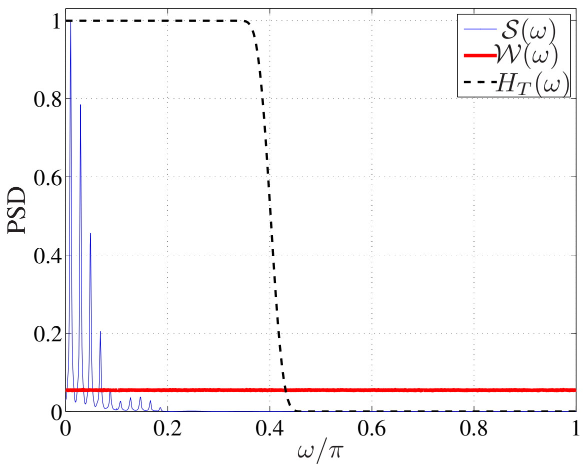

Fig. 4. Noise and $s(n)$ bandwidth and $H_T(\omega)$ frequency response.

tuning, considering the transmission of 10^6 bits. As expected, for a given SNR, BER decreases with M. The choice of M presents a compromise. Increasing M implies in spending more energy and less bandwidth per symbol, as in conventional modulations. However, the quantitative properties of this exchange is not straightforward because of the nonlinearity characteristics of transmitter and receiver: the BER for a given energy-per-bit to noise-power-spectral-density ratio (E_b/N_0) is not constant. Bearing this figure in mind, we have chosen $M = 20$ for the remaining simulations, considering that this time symbol provides a BER of approximately 10^{-3} at SNR $= 12$ dB.

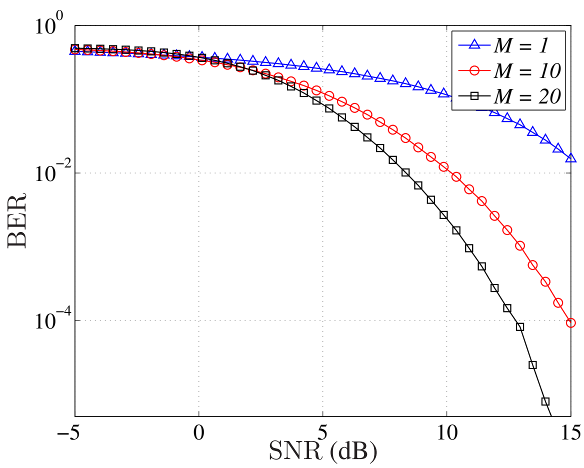

Fig. 5. BER for different time symbol duration M without tuning.

Figure 6 shows the obtained BER as a function of the tuning filter cut-off frequency for SNR $= 10$ dB. By one hand, as the cut-off frequency of the systems filters are $\omega_s = 0.2\pi$, we clearly see that if we choose $\omega_T < 0.2\pi$, BER rises abruptly, as we are cutting relevant components of the signal. By the other hand, the curve smoothly increases as we approach $\omega_T \approx \pi$, that represents the situation without tuning at all. We numerically found that

$$\omega_{T_o} \approx 0.38\pi \tag{14}$$

presents the lower BER in the considered setting.

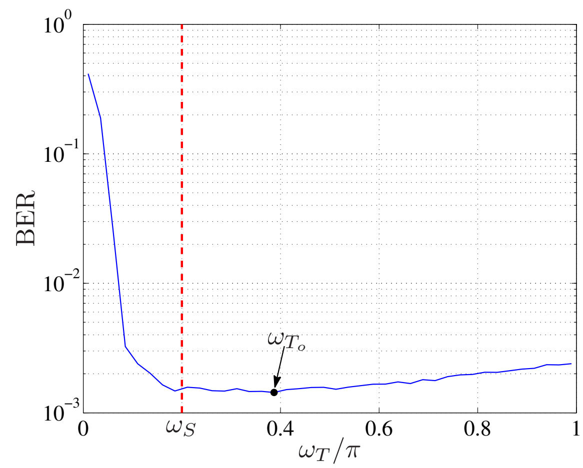

Fig. 6. BER as a function of the cut-off frequency of the tuning filter ω_T for SNR $= 10$ dB.

In Figure 7 we compare the BER performance with and without the tuning filter using ω_T determined in (14) and $M = 20$. There is advantage in using the tuning filter for an SNR lower than approximately 12 dB. Thus, the tuning filter is cutting out-of-band noise and the chaotic signal is in fact bandlimited, presenting therefore this advantage compared to previously proposed chaos-based communication systems.

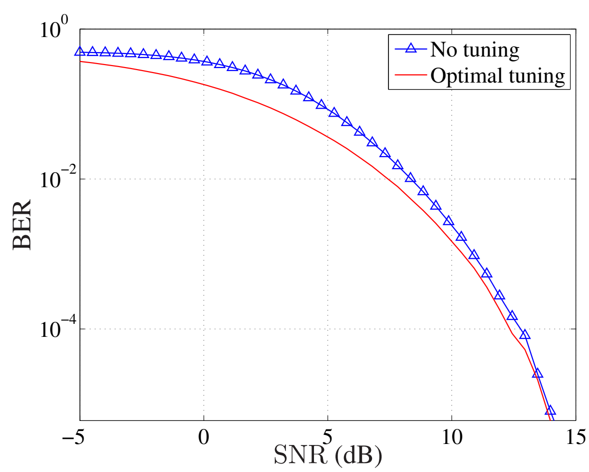

Fig. 7. BER as a function of the SNR for the systems with and without tuning.

Using a noise filtering at the receiver, besides the BER improvement described, allows the communication system proposed to be selective in frequency, a characteristic useful in traditional systems encountered in literature [6].

5. CONCLUSIONS

In this paper we adapted a bandlimited chaos-based communication system for binary data transmission. We access its performance considering AWGN channel and the possibility of using a tuning filter at the receiver to eliminate the out-of-band noise. Numerical simulations showed that the tuning filter can reduce BER in moderate SNRs and the feasibility of transmitting digital data using chaotic signals through a bandlimited and noisy channel. The frequency multiplexing of chaotic signal, how to analytically access the effect of linear filters on the proprieties of chaotic signals and the BER performance under more realistic scenarios are under research.

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