ARRAY-BROADBAND EFFECTS ON DIRECT GEOLOCATION ALGORITHM

Cyrile Delestre[1,2], *Anne Ferréol*[1,2] *and Pascal Larzabal*[2]

[1]: Thales Communications & Security
4, Avenue des Louvresses 92230 Gennevilliers Cedex, France
[2]: Laboratoire SATIE - ENS Cachan - CNRS - UniverSud
61 av du President Wilson 94235 CACHAN Cedex, France

ABSTRACT

Recent works have introduced powerful 1-step geolocation methods in comparison with traditional, and suboptimal, 2-steps methods. As these 1-step methods directly and simultaneously work on the observations of the whole array, there is now an important issue concerning the possible array-broadband effect. To counteract that effect, the recent methods introduce an imperfect narrowband decomposition, by the way of a filter bank or, equivalently, by a structured multidimensional modelization. The purpose of this work is to study the residual array-broadband effect on the 1-step algorithms performances. The study will compare two 1-step methods by the way of the bias and the ambiguity problem, giving some tools for operational design.

***Index Terms*—** Geolocation - Narrowband - Broadband - Parameter bias - Error on covariance matrix - LOST - DPD

1. INTRODUCTION

The discussed problem concerns the geolocation of radiating sources using remote multi-sensors stations (each one satisfying the narrowband hypothesis, commonly exploited in array processing). The transmitters (or sources) locations are traditionally estimated in 2-steps with conventional algorithms. For instance, the sources Angles of Arrivals (AoAs) are estimated on each station in the first step and in a second step the location of the sources are computed from the AoAs (eg. triangulation) [1]. Recently, new 1-step methods [2], [3] have been introduced in order to improve the performances of conventional algorithms. These algorithms are based on a direct estimation of the geographical coordinates of the transmitters thanks to high resolution methods assuming the narrowband signals hypothesis on the array of all the stations. For this reason, A.Amar and A.J.Weiss introduced the DPD (Direct Position Determination) algorithm [2] which is based on a frequency decomposition in K narrow sub-band with a filter bank and J.Bosse *et al.* the LOST (LOcalization by Space-Time) algorithm [3] with an equivalent space-time observation with K shifts in order to exploit more accurately the space-frequency modeling. More precisely, a signal of bandwidth B is processed as multiple signals of bandwidth $\frac{B}{K}$. These algorithms (LOST and DPD) assume that the signal of bandwidth $\frac{B}{K}$ is narrowband. However, if the value K is not sufficiently large in order to reduce the computation cost, the signals of bandwidth $\frac{B}{K}$ do not verify the narrowband hypothesis. In [7], the localization performances of the DPD algorithm have been evaluated when the narrowband hypothesis is not verified in each sub-channel of bandwidth $\frac{B}{K}$.

The purpose of this paper is to evaluate the performances of the LOST algorithm when the narrowband hypothesis is not verified for the sub-band of bandwidth $\frac{B}{K}$. More precisely, the purpose is to give a closed form expression of the geolocation bias when the covariance matrix of the received signals is perfectly known. In order to do this, we give in a first time a closed form expression of the covariance matrix error due to narrowband hypothesis and in a second time we give a link between the bias and the error on the covariance matrix. According to [7], we are able to compare the theoretical performances of the LOST and DPD algorithms in a broadband environment.

Notations: $\mathbf{A}$ or $(a_{ij})_{1\leq i\leq I,1\leq j\leq J}\,\forall(I,J)\in\mathbb{N}_*^2$ is a matrix of dimension $I\times J$, $\mathbf{a}$ or $(a_i)_{1\leq i\leq I}\,\forall I\in\mathbb{N}_*$ is a column vector of dimension I, $\mathbf{I}_I$ is the identity matrix of dimension I, a or A is a scalar, $(\cdot)^H$ is the Hermitian of a matrix or a vector, $(\cdot)^T$ is the transpose of a matrix or vector, $(\cdot)^*$ is the conjugate of a scalar, $\mathbb{E}[\cdot]$ is mathematical expectation, $\otimes$ is the tensor product, $[\![a,b]\!]$ is the set defined by $\left\{x\in\mathbb{Z}|\,a\leq x\leq b,\forall(a,b)\in\mathbb{Z}^2\right\}$, for all commutative ring or semiring $\mathbb{K}$ we have $\mathbb{K}_*=\mathbb{K}\backslash\{0\}$ and $\mathbb{K}_+=\{x\in\mathbb{K}:0\leq x<+\infty\}$.

2. SIGNAL MODEL AND PROBLEM FORMULATION

2.1. Assumptions about the system

The global geolocation system is composed of L remote stations (or bases). Each of these bases are composed of M_l sensors for $l\in[\![1,L]\!]$. Thus, the system has M sensors ($\sum_{l=1}^{L}M_l=M$). In this paper, we consider Q uncorrelated transmitters denoted $s_q(t)$ for all $q\in[\![1,Q]\!]$ at location $\mathbf{p}_q$. These sources stem from the stationary and spectrally white signal $e_q(t)$ where the q-th signal is filtered by a shaping filter named $h_q(t)$ (*e.g.* Nyquist):

$$s_q(t) = (e_q * h_q)(t) \times e^{2i\pi f_{0_q} t} \quad (1)$$

Then, the auto-correlation function is:

$$r_q(\tau) = \mathbb{E}\left[s_q(t) s_q^*(t-\tau)\right] \quad (2)$$

$$= \mathbb{E}\left[|e_q(t)|^2\right] e^{2i\pi f_{0_q}\tau} \int_{\mathbb{R}} h_q(t) h_q^*(t-\tau)\, dt \quad (3)$$

where $\mathbb{E}\left[|e_q(t)|^2\right] = \sigma_{s_q}^2$. The narrowband hypothesis is verified on the global system when:

$$\max_{q \in [\![1,Q]\!],(l,j)\in[\![1,L]\!]^2} |\tau_l(\mathbf{p}_q) - \tau_j(\mathbf{p}_q)| \times B_q \ll 1 \quad (4)$$

where $\tau_l(\mathbf{p}_q)$ is the time of arrival of the q-th source to the l-th base and B_q the bandwidth of the q-th source. The receiver bandwidth of each station is $B = F_e$ where F_e is the sampling frequency of all the bases (afterwards we note $T_e = \frac{1}{F_e}$ the sampling time). Moreover, all the bases are perfectly synchronous with each other.

Finally, the noise at the output of each base station is stationary, white, zero mean and with a variance σ^2. In Fig.1 the propagation of one source to the remote stations is represented.

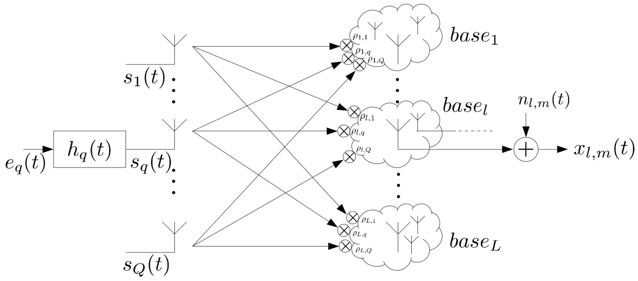

Fig. 1. System diagram

2.2. Signal modeling

The new 1-step algorithms [2], [3] use a global observation composed by the signals at the output of each base. The associated concatenated vector is:

$$\mathbf{y}(t) = \left[\mathbf{x}_1^T(t), ..., \mathbf{x}_l^T(t), ..., \mathbf{x}_L^T(t)\right]^T \quad (5)$$

In the Line of Sight (LoS) assumption, the M sensors of the global system only observe the direct paths of each source. Then, the output of the l-th station is:

$$\mathbf{x}_l(t) = \sum_{q=1}^{Q} \rho_{l,q} \mathbf{a}_l(\theta_l(\mathbf{p}_q)) s_q(t - \tau_l(\mathbf{p}_q)) + \mathbf{n}_l(t) \quad (6)$$

where $\rho_{l,q}$, $\mathbf{a}_l(\mathbf{p}_q)$, $\theta_l(\mathbf{p}_q)$ and $\tau_l(\mathbf{p}_q)$ are the complex attenuation, the steering vector, the Angle of Arrival (AoA) and the Time of Arrival (ToA) respectively associated to the q-th source and the l-th base. In the following of the paper, the complex attenuations $\rho_{l,q}$ are *a priori* know.

3. CENTRALIZED METHODS

3.1. DPD algorithm

In the DPD (Direct Position Determination) algorithm [2], the signals at the output of the bases are filtered by a filter bank composed of K filters. Then, the signal is decomposed in K sub-signals of bandwidth $\frac{B}{K}$ where the narrowband assumption is assumed on all the stations. If we note $\mathbf{x}_{l,k}(t)$ and $s_{q,k}(t)$ the received signal and the transmitted signal at the output of the k-th filter of the filter bank respectively, the model of the received signals is:

$$\mathbf{y}_k(t) = \begin{bmatrix} \mathbf{x}_{1,k}(t) \\ \vdots \\ \mathbf{x}_{L,k}(t) \end{bmatrix} \approx \sum_{q=1}^{Q} \mathbf{u}(\mathbf{p}_q, f_k) s_{q,k}(t) + \mathbf{n}_k(t) \quad (7)$$

where the vector $\mathbf{n}_k(t)$ is the noise filtered by the k-th filter of the filter bank and

$$\mathbf{u}(\mathbf{p}_q, f_k) = \left(\rho_{l,q} \mathbf{a}_l(\mathbf{p}_q) e^{-2i\pi f_k \tau_l(\mathbf{p}_q)}\right)_{1 \le l \le L} \quad (8)$$

The signal $s_{q,k}(t)$ of bandwidth $\frac{B}{K}$ is assumed to be narrowband, consequently:

$$s_{q,k}(t-\tau) \approx s_{q,k}(t) e^{-2i\pi f_k \tau} \quad (9)$$

Then, the covariance matrices $\hat{\mathbf{R}}_k$ $\forall k \in [\![1, K]\!]$ of the observations $\mathbf{y}_k(t)$ are estimated at the output of each filter of the filter bank. Thus, the DPD algorithm estimates the sources location $\hat{\mathbf{p}}_q$ by searching the zeros of a criterion which is an incoherent sum of the MUSIC criteria in each sub-band of covariance matrix $\hat{\mathbf{R}}_k$ [2].

We studied in [7] the influence of the approximation of the narrowband assumption on the signals $s_{q,k}(t)$ for the DPD algorithm in order to estimate the sources position. In the remainder of this paper, we focus on a second 1-step approach recently introduced: the LOST algorithm [3].

3.2. LOST algorithm

The LOST (LOcalization by Space-Time) algorithm [3] uses the following space-time observation:

$$\mathbf{y}_{LOST}^T(t) = [\mathbf{y}(t), \mathbf{y}(t - T_{shift}), ..., \mathbf{y}(t - (K-1)T_{shift})] \quad (10)$$

where $\mathbf{y}(t)$ is, according to Eq.(5), the observation at the output of the global array and T_{shift} is the time delay (usually $T_{shift} = T_e$). According to Eq.(7), the relation between $\mathbf{y}(t)$ and $\mathbf{y}_k(t)$ is $\mathbf{y}(t) = \sum_{k=1}^{K} \mathbf{y}_k(t)$. We will now consider that the decomposition into narrowband signals adapts to each signals. In presence of K temporal delay, the q-th source can be decomposed into K_q narrowband sub-signals with $1 \le K_q \le K$ $\forall q \in [\![1, Q]\!]$. From the DPD model (*e.g.* Eq.(7)) we have:

$$\mathbf{y}(t) \approx \sum_{q=1}^{Q} \sum_{k=1}^{K_q} \mathbf{u}(\mathbf{p}_q, f_{q,k}) s_{q,k}(t) + \mathbf{n}(t) \quad (11)$$

In the case of the k-th delay of the space-time process, the Eq.(11) becomes:

$$\mathbf{y}(t-(k-1)T_{shift}) \approx \sum_{q=1}^{Q}\sum_{k=1}^{K_q}\mathbf{u}(\mathbf{p}_q, f_{q,k})c(f_{q,k})^{k-1} \times s_{q,k}(t)+\mathbf{n}(t) \quad (12)$$

with $c(f) = e^{-2i\pi f T_{shift}}$. Then the space-time observation is, according to Eq.(7) and Eq.(12), given by:

$$\mathbf{y}_{LOST}(t) \approx \mathbf{y}_{LOST}^{NB}(t) = \sum_{q=1}^{Q}\sum_{k=1}^{K_q}\mathbf{v}(\mathbf{p}_q, f_{q,k})s_{q,k}(t) + \mathbf{n}(t) \quad (13)$$

where the superscript or subscript NB means narrowband signals and

$$\mathbf{v}(\mathbf{p}_q, f) = \mathbf{c}(f) \otimes \mathbf{u}(\mathbf{p}_q, f_{q,k}) \quad (14)$$

with $\mathbf{c}(f) = \Big(c(f)^{k-1}\Big)_{1\leq k\leq K}$. We decomposed the Q signals in $\sum_{q=1}^{Q} K_q$ sub-signals respecting the narrowband assumption around the frequencies $f_{q,k}$ $\forall k \in [\![1, K]\!]$. According to Eq.(13), with the narrowband hypothesis, the covariance matrix of the space-time observations is:

$$\begin{aligned}\mathbf{R}_{NB} &= \mathbb{E}\left[\mathbf{y}_{LOST}^{NB}(t)\left(\mathbf{y}_{LOST}^{NB}\right)^{H}(t)\right] \quad (15)\\ &= \sum_{q=1}^{Q}\sum_{k=1}^{K_q}\mathbb{E}\left[|s_{q,k}(t)|^2\right]\mathbf{v}(\mathbf{p}_q, f_{q,k})\,\mathbf{v}^H(\mathbf{p}_q, f_{q,k}) + \sigma^2\mathbf{I}_{M\times K} \quad (16)\end{aligned}$$

where $\mathbb{E}\left[|s_{q,k}(t)|^2\right]$ is the power of the k-th sub-signal.

Thus, the signal subspace of $\mathbf{R}_{NB}$ is of rank $\sum_{q=1}^{Q} K_q$ and is spanned by the vectors $\mathbf{v}(\mathbf{p}_q, f_{q,k})$. The LOST algorithm exploits this property and extracts the projector $\mathbf{\Pi}_{NB}^{\perp}$ onto the noise subspace from the covariance matrix $\mathbf{R}_{NB}$ [3], as in high resolution methods like MUSIC [4]. The couples of parameters $(\mathbf{p}_q, f_{q,k})$ can then be estimated by searching the zeros of the criterion:

$$J_{LOST}(\mathbf{p}, f) = \frac{\mathbf{v}^H(\mathbf{p}, f)\,\mathbf{\Pi}_{NB}^{\perp}\mathbf{v}(\mathbf{p}, f)}{\mathbf{v}^H(\mathbf{p}, f)\,\mathbf{v}(\mathbf{p}, f)} \quad (17)$$

Moreover, in a narrowband context, the estimated q-th source location is asymptotically equal to the true source location.

In the next section we analyze the case of small values of K where the sub-signals $s_{q,k}(t)$ potentially do not respect the narrowband assumption.

4. ANALYSIS IN BROADBAND CONTEXT

In this section, we will analyze the performances of LOST when the narrowband hypothesis is not verified for the sub-signal $s_{q,k}(t)$. For that, it is necessary to give the expression of the theoretical covariance matrix, named $\mathbf{R}$, when the narrowband hypothesis is not assumed.

4.1. Problem formulation

This part will allow us to have the theoretical criterion of LOST (in the broadband context) and to observe the bias. Therefore, the approximation made in Eq.(12) is no longer verified. Consequently, according to Eq.(6), the LOST observation is:

$$\mathbf{y}_{LOST}(t) = \sum_{q=1}^{Q}\mathbf{V}(\mathbf{p}_q)\mathbf{s}_{q,K}(t) + \mathbf{n}(t) \quad (18)$$

with

$$\mathbf{V}(\mathbf{p}_q) = \mathbf{I}_K \otimes \begin{pmatrix}\rho_{1,q}\mathbf{a}_1(\mathbf{p}_q) & \cdots & \mathbf{0}_{M_1\times 1}\\ \vdots & \ddots & \vdots\\ \mathbf{0}_{M_L\times 1} & \cdots & \rho_{L,q}\mathbf{a}_L(\mathbf{p}_q)\end{pmatrix} \quad (19)$$

and $\mathbf{s}_{q,K}(t) = \Big(\mathbf{s}_q(t-(k-1)T_{shift})\Big)_{1\leq k\leq K}$ where $\mathbf{s}_q(t) = \Big(s_q(t-\tau_l(\mathbf{p}_q))\Big)_{1\leq l\leq L}$. Then, the theoretical broadband covariance matrix can be written as follows:

$$\begin{aligned}\mathbf{R} &= \mathbb{E}\left[\mathbf{y}_{LOST}(t)\mathbf{y}_{LOST}^{H}(t)\right] \quad (20)\\ &= \sum_{q=1}^{Q}\mathbf{V}(\mathbf{p}_q)\mathbf{R}_{s_q}\mathbf{V}^H(\mathbf{p}_q) + \sigma^2\mathbf{I}_{M\times K} \quad (21)\end{aligned}$$

where the matrix $\mathbf{R}_{s_q}$ is a Toeplitz block matrix defined by:

$$\begin{aligned}\mathbf{R}_{s_q} &= \mathbb{E}\left[\mathbf{s}_{q,K}(t)\mathbf{s}_{q,K}^{H}(t)\right] \quad (22)\\ &= \begin{pmatrix}\mathbf{S}_q(0) & \cdots & \mathbf{S}_q^H((K-1)T_{shift})\\ \vdots & \ddots & \vdots\\ \mathbf{S}_q((K-1)T_{shift}) & \cdots & \mathbf{S}_q(0)\end{pmatrix} \quad (23)\end{aligned}$$

According to Eq.(3), the matrix $\mathbf{S}_q(\mu)$ is expressed as follows:

$$\mathbf{S}_q(\mu) = \begin{pmatrix}r_q(\mu) & \cdots & r_q(\mu+\Delta\tau_{1,L}(\mathbf{p}_q))\\ \vdots & \ddots & \vdots\\ r_q(\mu-\Delta\tau_{1,L}(\mathbf{p}_q)) & \cdots & r_q(\mu)\end{pmatrix} \quad (24)$$

with $\Delta\tau_{l,v}$ is the Time Differential of Arrival (TDoA) between the l-th and v-th bases and the q-th source. In the remainder of this paper the expression of the covariance matrix in broadband assumption is used to establish the geolocation bias.

4.2. Bias closed form expression

A closed form expression of the geolocation bias $\Delta\mathbf{p}_q = \mathbf{p}_q - \mathbf{p}_q^{BB}$ is given in this section where $\mathbf{p}_q^{BB}$ (BB means broadband) is the estimation of the location of the q-th source when the narrowband assumption is assumed in the LOST algorithm. More precisely, the narrowband hypothesis assumes that the covariance matrix is $\mathbf{R} = \mathbf{R}_{NB}$ (*e.g.* Eq.(21) and Eq.(16)) and, thus, the associated signal subspace is spanned by the steering vectors $\mathbf{v}(\mathbf{p}_q, f_{q,k})$. The LOST algorithm is then disturb by the covariance matrix error $\Delta\mathbf{R} = \mathbf{R} - \mathbf{R}_{NB}$.

According to [5], it is shown that the first-order relationship between $\mathbf{\Pi}^{\perp}$ and $\Delta\mathbf{R}$ is the following:

$$\mathbf{\Pi}^{\perp} = \mathbf{\Pi}^{\perp}_{NB} - \Delta\mathbf{\Pi}^{\perp} \quad (25)$$

with

$$\mathbf{\Pi}^{\perp} \approx \mathbf{\Pi}^{\perp}_{NB}\Delta\mathbf{R}\mathbf{R}^{+}_{NB} + \mathbf{R}^{+}_{NB}\Delta\mathbf{R}\mathbf{\Pi}^{\perp}_{NB} \quad (26)$$

where $(\cdot)^{+}$ is the Moore-Penrose pseudoinverse. The expression of the bias on the parameters is given by the 2^{nd} order approximation:

$$\Delta\mathbf{p}_q \approx -\tilde{\mathbf{H}}^{-1}(J_{LOST}(\mathbf{p}_q|f_{q,k}))\nabla J_{LOST}(\mathbf{p}_q|f_{q,k}) \quad (27)$$

where ∇ is the gradient and $\tilde{\mathbf{H}}$ is the approximation of the Hessian matrix. If we define the gradient by $\nabla(\mathbf{p}) = \left(\frac{\partial}{\partial p_x}, \frac{\partial}{\partial p_x}\right)^T$, we have for each element of the gradient in the LOST algorithm:

$$\frac{\partial J_{LOST}(\mathbf{p}_q|f_{q,k})}{\partial p_i} = 2\Re\left\{\frac{\partial \mathbf{v}^H(\mathbf{p}_q, f_{q,k})}{\partial p_i}\mathbf{\Pi}^{\perp}\mathbf{v}(\mathbf{p}_q, f_{q,k})\right\} \quad (28)$$

with $i \in \{x, y\}$ and the approximation of the Hessian is:

$$\tilde{\mathbf{H}}(\mathbf{p}) = \begin{pmatrix} \tilde{H}_{11}(p_x) & \tilde{H}_{12}(p_x, p_y) \\ \tilde{H}_{21}(p_x, p_y) & \tilde{H}_{22}(p_x, p_y) \end{pmatrix} \quad (29)$$

where:

$$\tilde{H}_{ij}\left(J_{LOST}(\mathbf{p}_q|f_{q,k})\right) = 2\Re\left\{\frac{\partial \mathbf{v}^H(\mathbf{p}_q, f_{q,k})}{\partial p_i}\mathbf{\Pi}^{\perp}\times \frac{\partial \mathbf{v}(\mathbf{p}_q, f_{q,k})}{\partial p_j}\right\} \quad (30)$$

with $(i, j) \in \{x, y\}^2$. Using the equation (25) of the projector and performing a first order Taylor expansion of the bias $\Delta\mathbf{p}_q$ with respect to the matrix $\Delta\mathbf{R}$ [6], the expressions of $\frac{\partial J_{LOST}(\mathbf{p}_q|f_{q,k})}{\partial p_i}$ and $\tilde{H}_{ij}(J_{LOST}(\mathbf{p}_q|f_k))$ $\forall(i, j) \in \{x, y\}^2$ are:

$$\begin{cases} \frac{\partial J_{LOST}(\mathbf{p}_q|f_{q,k})}{\partial p_i} \approx -2\Re\left\{\frac{\partial \mathbf{v}^H(\mathbf{p}_q, f_{q,k})}{\partial p_i}\mathbf{\Pi}^{\perp}_{NB}\times \Delta\mathbf{R}\mathbf{R}^{+}_{NB}\mathbf{v}(\mathbf{p}_q, f_{q,k})\right\} \\ \tilde{H}_{ij}(J_{LOST}(\mathbf{p}|f_{q,k})) \approx 2\Re\left\{\frac{\partial \mathbf{v}^H(\mathbf{p}_q, f_{q,k})}{\partial p_i}\mathbf{\Pi}^{\perp}_{NB}\frac{\partial \mathbf{v}(\mathbf{p}_q, f_{q,k})}{\partial p_j}\right\} \end{cases} \quad (31)$$

5. SIMULATIONS

In this section, we will first analyze the influence of the broadband effect on the bias of the LOST algorithm and observe the influence of the number of spatio-temporal delays (K). Then, in a second step, we will compare the LOST and DPD algorithms and analyze the different behaviors of these two algorithms.

5.1. Visualization of the bias

We here consider two cases: the first case permits to observe the bias of the estimated position of a single source ($Q = 1$), and the second considers the bi-source ($Q = 2$) case.

In both cases we will consider a zero noise ($\sigma^2 = 0$), the goal being to eliminate any disturbance other than the broadband effect. We have two bases ($L = 2$). In a cartesian coordinate system, we place the first base at (-400m,-400m), and the second at (400m,-400m). These bases are composed of four sensors where three are in a circular formation around a fourth in the center. The bases radius is 0.5m. The two bases are perfectly synchronized with the sampling frequency $F_e = 500$kHz.

In the first case, we will move the source along the abscissa axis with the position (d,0) with $d \in \mathbb{R}_+$. We consider a source with a carrier frequency $f_0 = 900$MHz, with a Nyquist shaping filter and a bandwidth $B_1 = 426$KHz. To observe the source lobe relatively to the secondary lobes, we plot the bias and the lobes level with respect to the Time-Bandwidth (TB) product defined as $TB = \frac{B_1 \times \Delta D}{c}$, where c is the light speed in vacuum and ΔD is the differential distance between the bases and the source. In order to guaranty the narrowband effect, one must have $TB \ll 1$ and, thanks to the triangular inequality, we have $\Delta D \leq a$, with a the distance between the two bases. We note that, in this context, for $TB \to \frac{B_1 \times a}{c} \approx 1.14$, we have $d \to \infty$.

We note that for the single-source case we have no bias, which can be observed in Fig2 (blue square).

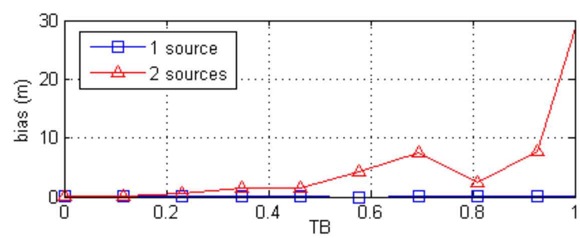

Fig. 2. Comparison 1 source / 2 sources

In the second case, we add a second source position (d,-100m). We will always consider the first source as the reference for the computation of the position. One sees very clearly that the more d increases the more the resolution limits on the bases are reached. This is a second effect of broadband. Here we see (always on Fig.2) that we have an emergence of the bias (red triangle).

We will now observe the influence of the number of spatio-temporal delays (K) on the bias and the robustness through the difference between the source lobe and the lowest secondary lobe. This value is negative or null if there is an ambiguity. It may be observed from Fig.3 that more K is greater more the bias due to the broadband effect is low. We see that for 7 temporal shifts (red triangle), the bias is very small compared to 2 temporal shifts (blue square). However it is interesting to note that the number of spatio-temporal delays does not affect the robustness of the system to ambiguities.

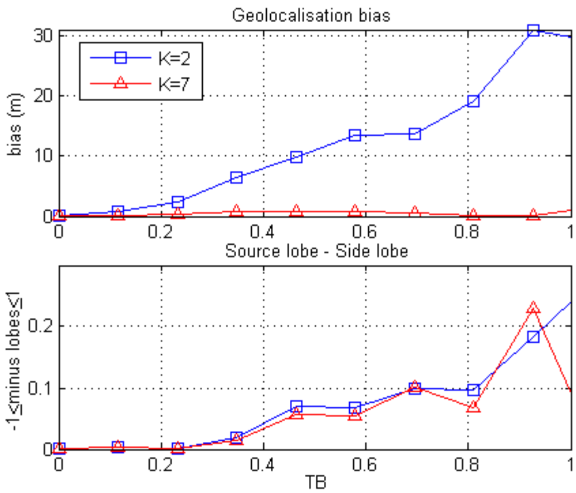

Fig. 3. Visualization of the bias and lobes

Finally, we check the validity of the closed form expression of the bias with the true bias. We can see in Fig.4 that the closed form expression of the bias (blue square) is very close to the real bias (red triangle). The more TB increases, the more a gap increases between these two curves. This is mainly due to the approximations made in Sec.4.2.

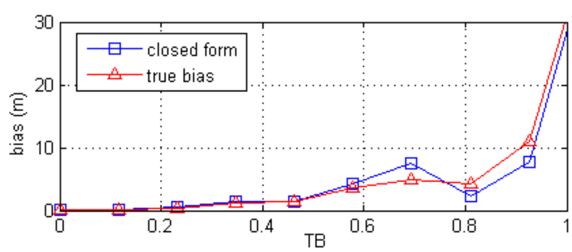

Fig. 4. Theoretical bias for $K = 3$

5.2. Comparison between the DPD and LOST bias

In this final section we will compare the performances of the DPD and LOST algorithms using the closed form expression of the bias and the robustness to ambiguities. The sensitivity of the DPD algorithm to the broadband effect has been studied in [7]. We take a number of spatio-temporal delays ($K = 3$) equal to the number of filters in the filter bank of the DPD algorithm (also called K). Similarly, the sources context is identical to the second case previously discussed (one source at the position (d,0) and the second at (d,−100m)).

Looking at Fig.5, we immediately observe that the DPD algorithm (red triangle) has a source position error higher than in the LOST algorithm (blue square). The parametric bias due to the broadband effect is consequently more sensitive to the DPD than to the LOST algorithm. However, we can see that the robustness to ambiguities is much better for the DPD algorithm than for the LOST algorithm.

6. CONCLUSION

In this paper a closed form expression of the geolocation bias of LOST is establish when the narrowband hypothesis is not

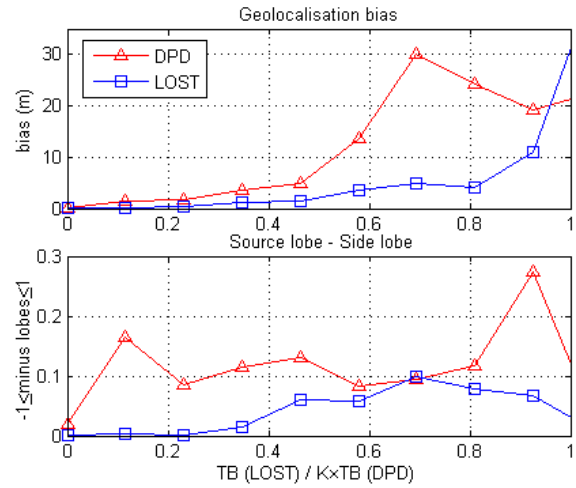

Fig. 5. Visualization of the bias and lobes, LOST vs. DPD

verified in each sub-band of bandwidth $\frac{B}{K}$. The simulations show the validity of this bias expression. As a byproduct we show how the geolocation bias is depending on the signal auto-correlation function and then on the signals modulation. In addition, we show through simulations that the bias of LOST is decreasing when K increases and that K does not noticeably impact on the robustness to ambiguities. Thus, the influence of the number K (spatio-temporal delays) on the performances of the LOST algorithm is then established. As the bias decreases and the computation cost increases with respect to K, this result gives us a tool to optimize the computation cost with respect to the LOST accuracy.

Although the DPD algorithm is more robust to ambiguities than LOST in a narrowband context, we show that the bias of LOST is less significant than the DPD algorithm.

7. REFERENCES

[1] Marco Porreta, Paolo Nepa, Giuliano Manara & Filippo Giannetti, *Location, Location, Location*, IEEE Vehicular Technology Magazine, vol. 3, # 2, 2008

[2] Anthony J. Weiss, *Direct Position Determination of Narrowband Radio Frequency Transmitters*, IEEE Signal Processing Letters, vol. 11, # 5, 2004

[3] Jonathan Bosse, Anne Ferréol & Pascal Larzabal, *A Spatio-Temporal Array Processing for Passive Localization of Radio Transmitters*, IEEE Transactions on Signal Processing, vol.61, # 22, 2013.

[4] Schmidt R.O, *Multiple Emitter Location and Signal Parameter Estimation*, IEEE Transactions Antennas Propagation, vol. 34, p.276-280, 1986.

[5] Hamid Krim, Philippe Forster & John G. Proakis, *Operator Approach to Performance Analysis of Root-MUSIC and Root-Min-Norm*, IEEE Transactions on Signal Processing, vol. 40, # 7, 1992.

[6] Anne Ferréol, Pascal Larzabal & Mats Viberg, *Performance Prediction of Maximum-Likelihood Direction-of-Arrival Estimation in the Presence of Modeling Errors*, IEEE Transactions on Signal Processing, vol. 56, # 10, 2008.

[7] Cyrile Delestre, Anne Ferréol & Pascal Larzabal, *On the Broadband Effect of Remote Stations in the DPD Algorithm*, Technical Report, 2014