STOCHASTIC MODELING OF EEG RHYTHMS WITH FRACTIONAL GAUSSIAN NOISE

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ABSTRACT

This paper presents a novel approach to signal modeling for EEG signal rhythms. A new method of 3-stage DCT based multirate filterbank is proposed for the decomposition of EEG signals into brain rhythms: delta, theta, alpha, beta, and gamma rhythms. It is shown that theta, alpha, and gamma rhythms can be modeled as 1st order fractional Gaussian Noise (fGn), while the beta rhythms can be modeled as 2nd order fGn processes. These fGn processes are stationary random processes. Further, it is shown that the delta subband imbibes all the nonstationarity of EEG signals and can be modeled as a 1st order fractional Brownian motion (fBm) process. The modeling of subbands is characterized by Hurst exponent, estimated using maximum likelihood (ML) estimation method. The modeling approach has been tested on two public databases.

***Index Terms*—** Fractional Gaussian noise, EEG, DCT

1. INTRODUCTION

Epilepsy is one of the world's most common neurological diseases, affecting more than 40-50 million people worldwide [1]. Electroencephalograph (EEG), a record of the electrical activity generated by a large number of cortical neurons in the brain, is the main diagnostic tool for the determination and treatment of epilepsy [2]. An EEG signal is bandlimited in frequency to 60 Hz. It is studied via five frequency bands: delta 0.1-4 Hz, theta 4-8 Hz, alpha 8-12 Hz, beta 12-30 Hz, and gamma 30-60 Hz band [3]. These subband signals are also called as brain rhythms that capture different brain activities [3]. EEG signal analysis is helpful in predicting epileptic seizures [2], classifying sleep stages [4], detection and monitoring of brain injury [5], and detecting abnormal brain states [6].

Visual analysis of EEG signals in the time domain is an empirical science and requires a considerable amount of neurological knowledge [7]. In particular, the process of epilepsy detection by visual inspection is subjective [8] that may lead to incorrect diagnosis. This limitation calls for a need of fully or semi-automated analysis of these signals.

Fourier transform (FT) has been used extensively to analyze EEG signals assuming it to be a stationary random process, but EEG is a non-stationary random process [9]. For example, [10] uses the autoregressive model for EEG signals, [11] uses the Welch method to estimate the periodogram for the analysis of EEG signals. However, it is known that EEG signals are random processes [9] [12] that are non-stationary [9]. Although in [13–16] they have been modeled using fractal dimension assuming them to be self-similar in nature, there is no attempt to model them as discrete-time fractional Brownian motion (dfBm) or discrete-time fractional Gaussian noise (dfGn) random processes. Motivated with the above discussion, we attempt to model these processes as dfBm processes.

The contributions of this paper are as follows:

1. We present a new method of discrete cosine transform (DCT) based multirate filterbank on EEG signals to extract delta, theta, alpha, beta, and gamma brain rhythms;
2. We present a new random process modeling approach, where we have shown that delta rhythms imbibe the non-stationary property of EEG signals and can be modeled as 1^{st} order fBm processes;
3. Further, it has been shown that theta, alpha, gamma, and beta rhythms are stationary random processes and can be modeled as higher order fGn.

This modeling may prove to be helpful in the detection and classification problems in EEG signal processing. This paper is organised into five sections. Section 2 presents the theory of fBm and fGn processes in brief. Section 3 describes the data sets used. In section 4, we present method and results on subband modeling of EEG signals. In the end, conclusions are presented in section 5.

2. BRIEF REVIEW OF FBM AND FGN POCESSES

2.1. Fractional Brownian motion

A continuous-time random process is called self-similar if its statistical properties are scale invariant. Symbolically, it is represented as [17]

$$x(ct) \stackrel{d}{=} c^H x(t), \tag{1}$$

where the random process $x(t)$ is self similar with index H (Hurst exponent) for any scale parameter $c>0$. This is to note that equality in (1) holds in statistical sense for all finite order distributions [18]. An important class of these non-stationary self-similar processes is those with self-similarity index H and having stationary increments (H-sssi). An H-sssi Gaussian process with $0 < H < 1$ is known as fractional Brownian motion [17]. Although an fBm process is a non-stationary process, the averaged power spectral density (PSD) of these processes follows a power law and is directly proportional to $1/|f|^{\beta}$ with $\beta = 2H + 1$ where f is frequency in Hertz [19]. Corresponding to the discrete data set, discrete-time fractional Brownian motion [20] is defined as

$$B_H(n) = B_H(nT_s), \tag{2}$$

where n is an integer and T_s is the sampling period. Because the process is self-similar for any value of $c > 0$, T_s can be taken to be equal to one without any loss of generality. A dfBm process is a zero mean, self similar, non-stationary random process with the auto-covariance sequence given as below [20]:

$$R_B^H(n_1, n_2) = \frac{\sigma_H^2}{2}(|n_1|^{2H} - |n_1 - n_2|^{2H} + |n_2|^{2H}), \tag{3}$$

where $\sigma_H^2 = \text{var}(B_H(1)) = \dfrac{1}{\Gamma(2H+1)|sin(\pi H)|}$.

2.2. Fractional Gaussian Noise

It is observed that the normalized incremental process of an fBm is a self-similar stationary process and is termed as fractional Gaussian noise (fGn) [20]. For $H = 1/2$, fractional Brownian motion is a well known Wiener process and the resulting fGn is a zero-mean stationary white Gaussian noise. Thus, corresponding to dfBm process, a 1^{st} order discrete time fGn (1-dfGn) process is defined as the 1st order difference of dfBm as below [20]:

$$X_1(k) \triangleq B_H(k+1) - B_H(k). \tag{4}$$

On using (3) and (4), the covariance of 1-dfGn process is:

$$R_{X_1}(k) = \frac{\sigma_H^2}{2}(|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}). \tag{5}$$

It is evident from (6) that 1-dfGn process is a wide-sense stationary random process. It can be shown that the power spectral density of 1-dfGn falls as $1/|f|^{2H-1}$ where $0<H<1$ [20]. Or, we can say that the spectrum of 1-dfGn is proportional to $1/|f|^{2H+1}$ where $-1<H<0$. Likewise, a 2^{nd} order dfGn (2-dfGn) is defined as the 2^{nd} order difference of dfBm as below [20]:

$$X_2(k) \triangleq B_H(k+2) - 2B_H(k+1) + B_H(k). \tag{6}$$

Correspondingly, the covariance of 2-dfGn process is:

$$\begin{aligned} R_{X_2}(k) =& \frac{\sigma_H^2}{2}(-|k+2|^{2H} + 4|k+1|^{2H} \\ & - 6|k|^{2H} + 4|k-1|^{2H} - |k-2|^{2H}). \end{aligned} \tag{7}$$

The spectrum of 2-fGn is directly proportional to $1/|f|^{2H-2}$ where $0<H<1$ or can be said to be directly proportional to $1/|f|^{2H+1}$ where $-2<H<-1$. In general, an n^{th} order dfGn can be defined as

$$X_n(k) \triangleq \sum_{j=0}^{n} (-1)^{n-j} \begin{pmatrix} n \\ j \end{pmatrix} B_H(k+j) \tag{8}$$

with the covariance given by

$$R_{X_n}(k) = \frac{\sigma_H^2}{2}(-1)^n \sum_{j=-n}^{n} (-1)^j \begin{pmatrix} 2n \\ n+j \end{pmatrix} |k+j|^{2H}. \tag{9}$$

In order to distinguish dfBm, 1-dfGn, and 2-dfGn based on the value of Hurst exponent H, we consider $0<H<1$ for dfBm, $-1<H<0$ for 1-dfGn, and $-2<H<-1$ for 2-dfGn, i.e., we assume that the spectrum of all these random processes is proportional to $1/|f|^{2H+1}$, irrespective of whether they are dfBm or dfGn processes of any order. Using this convention, we note that a dfBm process with $0<H<1$ on first difference (equation (5)) leads to a 1-dfGn with

$$H_{1-dfGn} = H_{1-dfBm} - 1, \tag{10}$$

and a dfBm process on second difference (in equation (7))leads to a 2-dfGn with

$$H_{2-dfGn} = H_{1-dfBm} - 2. \tag{11}$$

2.3. Hurst Exponent Estimation of Higher Order fGn and fBm Processes

In this paper, the Hurst exponent H is estimated using the maximum likelihood (ML) method. The ML estimation method is applicable on stationary random processes. A 1^{st} order dfBm process is non-stationary. However, the 1^{st} order incremental process, 1-dfGn, of a dfBm process is a stationary process. Using this fact, ML estimation is employed in [21] for the estimation of H of a dfBm process. We extend this method to estimate H for higher order (n^{th} order with $n \geq 1$) dfGn processes.

3. DESCRIPTION OF DATASETS

3.1. Description of dataset-1

The data set consists of five Sets (SET A-E) of EEG signals containing 100 EEG segments each [22]. SETA and B consist of data collected from five healthy volunteers using scalp electrodes. SET C and D segments have been measured in seizure-free intervals from five patients in the epileptogenic zone (D) and from the hippocampal formation of the opposite hemisphere of the brain (C). SET E consists of data recorded during a seizure. Sets C, D, E data have been acquired intracranially. Each of these sets has 100 single-channel recordings with the sampling rate of signals as 173.6 Hz resulting in

86.8 Hz bandwidth. The duration of each segment is 23.6 s that leads to 4097 samples in each segment.

3.2. Description of dataset-2

In order to test the robustness of our classifier, we tested our method as outlined in Sections 5 and 6 on one more dataset taken from another source. This data is extracted from the MIT online database [23] corresponding to 16 different subjects. This online database has long recordings of epileptic patients that contain inter-ictal and ictal periods. Data is stored into two sets of 100 segments each labelled as SET Y and Z. SET Y consists of segments during interictal (in between seizure) periods and SET Z consists of data of same patients during seizure. Data is sampled at 256 Hz. We considered 10 second segments consisting of 2560 samples each. In addition, we considered data of only one channel, i.e., CZ-PZ electrode because the signal recorded from this region of the brain has comparatively less artefact [24].

3.3. Band-limiting and artifact removal

EEG signals are limited to 60 Hz. Thus, we use a lowpass butterworth filter with cut-off frequency of 60Hz of order 6 in order to band limit EEG segments of both the datasets. The first dataset is provided for researchers after removing artifacts. On dataset-2 (SET Y and Z), we performed operations to remove artifacts due to eye blinks and eye movements, and sweat artifacts.

4. MODELING OF EEG SIGNALS AND THE CONSTITUENT BRAIN RHYTHMS

EEG signals are considered to be made up of two components: 1) the ongoing brain activity and 2) the stimulus related response. The ongoing brain activity has a mean of zero [25]. Thus, in the absence of stimulus, these signals will form zero-mean random processes. An EEG signal is a record of the overall activity of millions of neurons in the brain. Thus, using the central limit theorem, it will be appropriate to assume these to be Gaussian random processes. Our two datasets have recordings in the absence of any stimulus. Thus, using the above argument, we attempt to model these processes as dfBm processes. First, we estimate the H value of all the 100 segments of SET C, E, Y, and Z using the ML method presented in [21] (Refer to Table-I). It is seen from Table 1 that the value of H lies in the range $0<H<1$ that corresponds to a dfBm process. In [13], a dfBm process is passed through a multirate filterbank withfilters that are eigenvectors of its covariance matrix. Further, it has been shown that the subband outputs of such a filterbank are statistically uncorrelated with respect to each other at all time instants [26]. In [27], it is shown that DCT basis vectors form the eigenvectors of these covariance matrices in the asymptotic sense. Because we would like brain rhythms extracted from different subbands to be uncorrelated with respect to each other, first we propose to use a DCT based multirate filterbank on input EEG signals to extract brain rhythms.

Table 1. Average Hurst exponent H of data

Dataset	SET	Hurst Exponent H
1	C	0.8883
1	E	0.9610
2	Y	0.8981
2	Z	0.8816

4.1. DCT based subband decomposition of EEG signals

As mentioned earlier, there are five major brain waves: delta 0.1-4Hz, theta 4-8Hz, alpha 8-12Hz, beta 12-30Hz and gamma 30-60Hz.We use a DCT based 3-stage multirate filterbank to extract these brain rhythms. Figure 1 shows the block diagram of this system.

The first stage input signal is passed through a 2-channel DCT analysis filterbank. The highpass filter branch provides us gamma band. The lowpass output of first stage is passed through a 5-channel DCT analysis filterbank. The higher three frequency bands are combined using the corresponding synthesis branches to obtain beta band. The last stage is a 3-channel DCT analysis filterbank that provides us the last three subbands.

The filters used at each stage are the DCT basis vectors given as below:

$$\begin{aligned} h_{j,1}(n) &= \frac{1}{\sqrt{M}}; j=1,2,3;\ 0 \le n \le M-1 \\ h_{j,k}(n) &= \sqrt{\frac{2}{M}} \cos\frac{\pi(2n+1)(k-1)}{2M}; \\ j &= 1,2,3; 2 \le k \le M; 0 \le n \le M-1 \end{aligned} \quad (12)$$

where j is the stage number (1, 2, or 3), M is the downsampling factor (or the length of filters in each stage), and k denotes the analysis (or synthesis) filter number at each stage with $k=1$ as the lowpass filter, $k=M$ denotes the highpass filter, and $2<k<M-1$ denote the bandpass filters.

4.2. Modeling of subband signals

After extracting brain rhythms, we would like to model them based on their statistical properties. In [26], it has been shown that a dfBm process when passed through a multirate filterbank with eigenvectors of its covariance matrix as filters leads to stationary processes in all the subbands except for the lowpass branch. This implies that except for the delta band, other brain rhythms should comprise zero-mean, Gaussian, stationary random processes, i.e., the fGn processes. Table 2 shows the mean Hurst exponent value for each subband. It is clearly seen that the estimated H value is positive for the delta band,

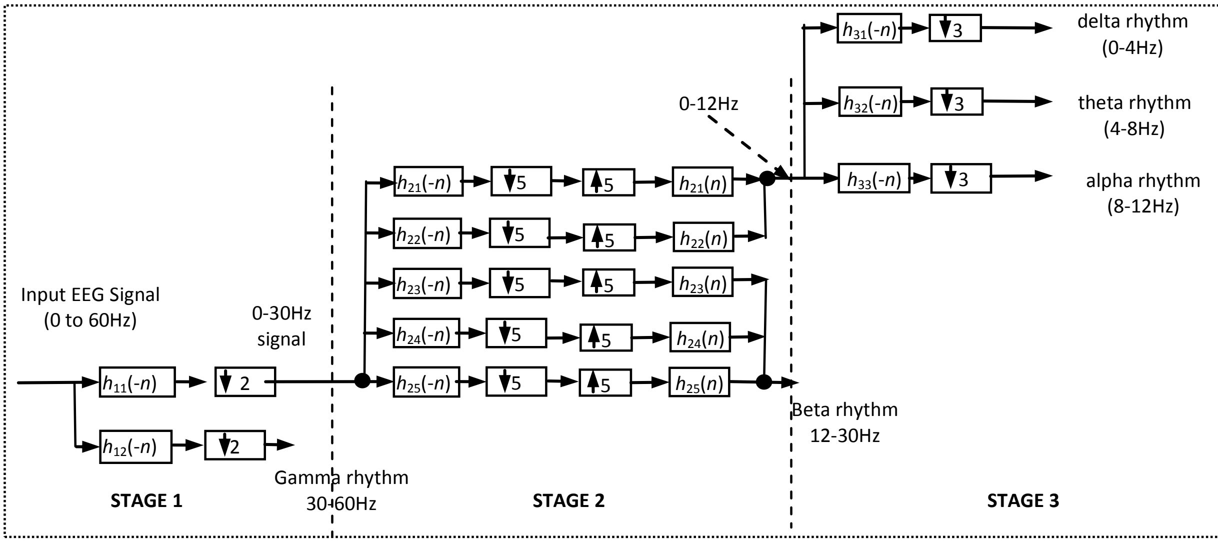

Fig. 1. DCT based 3-stage multirate filterbank for the extraction of brain rhythms

TABLE II: Hurst exponent H of all brain rhythms

Brain Rhythms	*Dataset* 1		*Dataset* 2	
	SET C	SET E	SET Y	SET Z
Delta	0.533	0.340	0.6372	0.8052
Theta	-0.545	-0.693	-0.4794	-0.2869
Alpha	-0.503	-0.515	-0.498	-0.4959
Beta	-1.42	-1.40	-1.4302	-1.4714
Gamma	-0.231	-0.119	-0.3593	-0.1901

while it is negative for the other subbands. Further, we note that beta band can be modeled as 2-dfGn random process. These observations match with the theoretical results of [26]. This provides validation to our modeling in this paper.

5. CONCLUSIONS

This paper proposes novel methods for the decomposition of EEG signals into brain rhythms and the subsequent stochastic modeling of these rhythms. Brain rhythms are extracted using 3-stage DCT based multirate filterbank. It has been shown that theta, alpha, and gamma rhythms can be modeled as 1^{st} order fGn processes, beta rhythms as 2^{nd} order fGn processes, and the delta rhythm as the fBm random process. The modeling of these processes (hence, brain rhythms) is characterized by the Hurst exponent H which is estimated by maximum likelihood (ML) estimation method.

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