

# Decentralized Multi-cell Beamforming via Large System Analysis in Correlated Channels

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**Abstract**—The optimal decentralization of multi-cell minimum power beamforming requires exchange of terms related to instantaneous inter-cell interference (ICI) values or channel state information (CSI) via a backhaul link. This limits the achievable performance in the limited backhaul capacity scenarios, especially when dealing with a fast fading scenario or a large number of users and antennas. In this work, we utilize the results from random matrix theory for developing two algorithms based on uplink-downlink duality and optimization decomposition relying on limited cooperation between nodes to share knowledge about channel statistics. As a result, approximately optimal power allocations are achieved based on statistics of the channels with greatly reduced backhaul information exchange rate. The simulations show that the performance gap due to the approximations is small even when the problem dimensions are relatively small.

## I. INTRODUCTION

In a cellular system, serving nodes share limited resources to serve users across the coverage area and the individual decisions of all nodes affect others. Thus, cooperation among nodes is required for optimal utilization of resources. Coordinated multi-point transmission (CoMP) allows cooperation and coordination between nodes for delivering services to users which improves the resource utilization and service quality [1]. However, the coordination requires sharing some information between nodes which makes the practical implementation difficult specially when the dimensions of the problem (the number of users and antennas) grow large or when dealing with a fast fading scenario.

In general, the coordinated resource allocation problems can be formulated as optimization problems. Maximizing a desired utility in the network, subject to some constraints can be solved iteratively along with exchange of some information between nodes at each iteration [2]–[7]. Coordinated multi-cell minimum power beamforming approach, which is the focus of this paper, satisfies a given signal-to-interference-plus-noise ratio (SINR) for all users while minimizing the total transmitted power. In [6], the problem was solved iteratively relying on the uplink-downlink duality and exchange of dual uplink powers and channel state informations (CSI) between serving nodes. An alternative approach based on optimization decomposition provides the locally feasible beamformers at each node relying on backhaul information exchange between BSs [7]–[9].

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Assuming the size of the problem becomes large, the result of random matrix theory (RMT) can be utilized for providing simpler approximations for the available algorithms. The work in [10] has considered such approximations for achieving the optimal regularization parameter for regularized zero forcing (RZF). A large dimension analysis for the network sum-rate maximization in a simple channel model with two cells has been provided in [11].

The authors in [12] extended the uplink (UL) - downlink (DL) duality approach from [6] for a large dimension system under i.i.d assumption on channel entries. They show that an approximately optimal beamformer can be achieved at each BS relying only on the local CSI and exchanged pathloss information from the other BS channels. However, the error in approximations causes variations in the resulting SINR values which violate the target SINR feasibility. In our earlier work [13], which is based on the work in [7], we have proposed another approach for decoupling the subproblems at BSs by considering the inter-cell interference (ICI) as the principal coupling parameter among BSs. The large dimension approximation for ICI terms provides an approximately optimal distributed algorithm that gives the locally feasible beamformers based on the exchanged pathlosses with a specific assumption that the channels follow i.i.d. statistics. The approximate algorithm has lower processing load and backhaul exchange rate and it guarantees the SINR constraints with slightly higher transmit power compared to the optimal method. In this work, we develop both of these approximated algorithms [12], [13] to a generalized channel model with arbitrary correlation characteristics. The generalized algorithm gives the approximately optimal beamformers based on statistics of the interfering channels and local instantaneous CSI. Depending on the assumptions about the propagation environment and antenna array, the generalized algorithm can be further simplified and as special cases with diagonal correlation matrices, the results from [13] and [12] can be reproduced.

## II. SYSTEM MODEL

A cellular system is considered which consists of  $N_B$  BSs, each BS has  $N_a$  transmit antennas and each user has a single receive antenna. Users allocated to the  $b^{\text{th}}$  base station are in set  $\mathcal{U}_b$ . Each user is served by a single base station and the BS that serves user  $k$  is denoted by  $b_k$ . Sets of all users and all BSs are represented by  $\mathcal{U}$  and  $\mathcal{B}$  respectively. The signal for user  $k$  consists of the desired signal, the intracell and the

intercell interference which can be presented as follows

$$\mathbf{y}_k = \mathbf{h}_{b_k,k}^H \mathbf{x}_{b,k} + \mathbf{h}_{b_k,k}^H \sum_{l \neq k \in \mathcal{U}_{b_k}} \mathbf{x}_{b,l} + \sum_{b \neq b_k} \mathbf{h}_{b,k}^H \sum_{l \in \mathcal{U}_b} \mathbf{x}_{b,l} + \mathbf{n}_k \quad (1)$$

where  $\mathbf{n}_k \sim \mathcal{CN}(0, N_0)$  is the noise with power density  $N_0$ .  $\mathbf{x}_{b,k} = \mathbf{w}_k d_k$  is the transmitted vector from the  $b^{\text{th}}$  BS to  $k^{\text{th}}$  user, in which  $d_k$  is the normalized complex data symbol ( $E[|d_k|^2] = 1$ ) and  $\mathbf{w}_k \in \mathbb{C}^{N_a}$  is the downlink beamforming vector of the  $k^{\text{th}}$  user.

$\mathbf{h}_{b,k} \in \mathbb{C}^{N_a \times 1}$  represents the channel from the  $b^{\text{th}}$  BS to  $k^{\text{th}}$  user. The per-user channel correlation model is used for representing channel vectors, i.e.,  $\mathbf{h}_{b,k} = \boldsymbol{\theta}_{b,k}^{\frac{1}{2}} \mathbf{z}_{b,k}$ , where  $\boldsymbol{\theta}_{b,k}$  is the channel correlation matrix and entries of  $\mathbf{z}_{b,k}$  are i.i.d of zero mean and variance 1. This channel model allows different correlation matrices for distinct users which results a generalized channel model applicable to various propagation environments.

### III. PROBLEM FORMULATION

The optimization problem for achieving the optimal downlink beamformers as proposed by [7] can be stated as

$$\begin{aligned} & \underset{\mathbf{w}_k, \epsilon_{b,k}}{\text{minimize}} && \sum_{b \in \mathcal{B}} \sum_{k \in \mathcal{U}_b} \|\mathbf{w}_k\|^2 \\ & \text{subject to} && \Gamma_k \geq \gamma_k \quad \forall k \in \mathcal{U}_b, \forall b, \\ & && \sum_{l \in \mathcal{U}_b} |\mathbf{h}_{b,k}^H \mathbf{w}_l|^2 \leq \epsilon_{b,k}^2, \forall k \notin \mathcal{U}_b, \forall b, \end{aligned} \quad (2)$$

where,  $\gamma_k$  represents target SINR for user  $k$ . The intercell interference from  $b^{\text{th}}$  base station to user  $k$  is denoted by  $\epsilon_{b,k}^2$  and the SINR of user  $k$  is given as

$$\Gamma_k = \frac{|\mathbf{h}_{b_k,k}^H \mathbf{w}_k|^2}{N_0 + \sum_{l \in \mathcal{U}_{b_k} \setminus k} |\mathbf{h}_{b_k,k}^H \mathbf{w}_l|^2 + \sum_{b \neq b_k} \epsilon_{b,k}^2} \quad (3)$$

This problem can be rewritten in a convex form, for example, as a second order cone program (SOCP) [7], and it can be solved in a centralized manner by using convex optimization tools.

#### A. Solution via uplink-downlink duality

Another approach for solving the optimization problem defined by (2) is based on UL-DL duality. Authors in [6] have shown that the problem dual to (2) which gives the optimal uplink power allocation and detection vectors is defined as follows

$$\begin{aligned} & \underset{\hat{\mathbf{w}}, \lambda}{\text{minimize}} && \sum_{b \in \mathcal{B}} \sum_{k \in \mathcal{U}_b} \lambda_k N_0 \\ & \text{subject to} && \frac{\lambda_k |\hat{\mathbf{w}}_k^H \mathbf{h}_{b_k,k}|^2}{\sum_{l \neq k} \lambda_l |\hat{\mathbf{w}}_l^H \mathbf{h}_{b_k,l}|^2 + \|\hat{\mathbf{w}}_k\|^2} \geq \gamma_k \quad \forall k \in \mathcal{U}. \end{aligned} \quad (4)$$

The dual uplink power of user  $k$  is denoted by  $\lambda_k$  and its optimal value can be calculated by a fixed point iteration [6]

$$\lambda_k = \frac{1}{(1 + \frac{1}{\gamma_k}) \mathbf{h}_{b_k,k}^H (\boldsymbol{\Sigma}_{b_k} + \mathbf{I})^{-1} \mathbf{h}_{b_k,k}} \quad (5)$$

where  $\boldsymbol{\Sigma}_b = \sum_{l \in \mathcal{U}} \lambda_l \mathbf{h}_{b,l} \mathbf{h}_{b,l}^H$ . The dual UL detection vector  $\hat{\mathbf{w}}_k$  is given by the minimum mean square error receiver at the optimal point [6]

$$\hat{\mathbf{w}}_k = (\boldsymbol{\Sigma}_{b_k} + \mathbf{I})^{-1} \mathbf{h}_{b_k,k}^H. \quad (6)$$

A link between the DL and UL beamformers is provided by the equation [6]

$$\mathbf{w}_k = \sqrt{\delta_k} \hat{\mathbf{w}}_k \quad (7)$$

where,  $\delta_k$  can be found by using the matrix inversion [6]

$$\boldsymbol{\delta} = \mathbf{G}^{-1} \mathbf{1}_{Nu} \quad (8)$$

where  $\boldsymbol{\delta}$  contains all  $\delta_k$  values,  $\mathbf{1}_{Nu}$  is a  $Nu \times 1$  vector with all elements equal to one. The elements of  $\mathbf{G}$  are defined as

$$[\mathbf{G}]_{i,j} = \begin{cases} \frac{1}{\gamma_i} |\hat{\mathbf{w}}_i^H \mathbf{h}_{b_i,i}|^2 & i = j \\ -|\hat{\mathbf{w}}_j^H \mathbf{h}_{b_j,i}|^2 & i \neq j. \end{cases} \quad (9)$$

The above set of equations defines an algorithm which gives the optimal power allocation and beamformers for the DL (2) and UL (4) problems. However, the final step of this algorithm in (8) requires a global knowledge about the CSI which makes its distributed implementation difficult, especially when dealing with a large number of users and antennas.

#### B. Decentralized solutions via optimization decomposition

The centralized problem in (2) is decoupled among BSs as soon as the ICI terms  $\epsilon_{b,k}$  are set to fixed values. In [7], the coupling is handled by taking the local copies of the interference terms at each BS and enforcing consistency between them. Then, the consistency constraints become decoupled by applying a standard dual decomposition approach that results in a distributed algorithm. The decentralized algorithm can follow the optimal solution in a time correlated scenario by exchanging the ICI terms while the channel realizations change. There are also alternative decentralized solutions based on primal decomposition [8], [9] and alternating direction method of multipliers (ADMM) [14].

### IV. LARGE SYSTEM ANALYSIS

It is known that the growing dimensions of a random matrix results in some deterministic behaviors about the distribution of its eigenvalues that can be utilized for approximations and processing simplification purposes [15]. The results of the system with large dimensions (the number of users and antennas) can be used as an approximation for the system with practically limited dimensions. In this section we use this approach for developing two approximated algorithms based on UL-DL duality and ICI decoupling methods introduced in previous sections.

#### A. Generalized approach based on UL-DL duality

This section introduces deterministic approximations for the solution based on UL-DL duality. In order to get these deterministic equivalents, the following assumptions on the correlation matrices are required.

*Assumption1:* The spectral norm of  $\boldsymbol{\theta}_{b,i}$  on  $N_a$  is uniformly bounded:

$$\limsup_{N_a, N_u \rightarrow \infty} \sup_{\forall b, i} \|\boldsymbol{\theta}_{b,i}\| \leq \infty \quad (10)$$

*Assumption2:* The set of all correlation matrices belongs to a finite family (a set with bounded cardinality) [10].

*Assumption3<sup>1</sup>:* The variances of entries of  $\mathbf{z}_{b,k}$  are scaled by the number of antennas.

Under these assumptions, we can derive approximations for the power allocation equations (5) and (9), the results of which are summarized in Theorem 1 and 2.

*Theorem 1:* If the assumptions 1,2 and 3 on correlation matrices hold true, then

$$\lambda_k - \lambda_k^o \xrightarrow{N_a \rightarrow \infty, \frac{N_a}{N_u} = cte.} 0 \quad (11)$$

almost surely, where  $\lambda_k$  is the optimal uplink power and the approximated power  $\lambda_k^o$  is given by

$$\lambda_k^o = \left( (1 + \frac{1}{\gamma_k}) \frac{m_{\Sigma_{b_k}, \boldsymbol{\theta}_{b_k, k}}(-1)}{1 + \lambda_k^o m_{\Sigma_{b_k}, \boldsymbol{\theta}_{b_k, k}}(-1)} \right)^{-1} \quad (12)$$

where

$$m_{\Sigma_{b_k}, \boldsymbol{\theta}_{b_k, k}}(z) = \frac{1}{N_a} \text{tr} \boldsymbol{\theta}_{b_k, k} \left( \frac{1}{N_a} \sum_{l \in \mathcal{U}} \frac{\lambda_l^o \boldsymbol{\theta}_{b_k, l}}{1 + e_{N_a, l}(z)} - z \mathbf{I}_{N_a} \right)^{-1}. \quad (13)$$

The term  $m_{\Sigma_{b_k}, \boldsymbol{\theta}_{b_k, k}}(-1)$  in (12) is the Stieltjes transform of a measure given by (13). The functions  $e_{N_a, 1}(z), \dots, e_{N_a, n}(z)$  are given as the unique solution of the following system of equations,

$$e_{N_a, i}(z) = \frac{1}{N_a} \text{tr} \lambda_i^o \boldsymbol{\theta}_{b_k, i} \left( \frac{1}{N_a} \sum_{l \in \mathcal{U}} \frac{\lambda_l^o \boldsymbol{\theta}_{b_k, l}}{1 + e_{N_a, l}(z)} - z \mathbf{I}_{N_a} \right)^{-1} \quad (14)$$

Iterations of (12) converges to the deterministic approximation  $\lambda_k^o$  when the functions  $e_{N_a, l}(z)$  are properly initialized with  $e_{N_a, l}(z) = \frac{-1}{z}$ .

*Proof:* The proof is similar to the one in [10, Theorem 1]. Due to the lack of space the proof of Theorem 1 is given in a supporting document [16].

The results of Theorem 1 are very general and can be applied to various propagation environments. However, under some assumptions about the correlation properties of the channels, the results can be further simplified. Diagonal correlation matrices or the case with the same correlation properties for all users are some examples. Also, in the single cell case, a closed form solution can be derived that gives the approximated uplink powers with a single matrix inversion [16]. However, these results are neglected due to the lack of space.

*Theorem 2:* If the assumption 1,2 and 3 hold true, then, for a set of approximated uplink powers given by Theorem 1, the deterministic equivalents for the elements of (9) are given by

$$[\mathbf{G}]_{l, k} = \begin{cases} \frac{\gamma_k}{((1 + \gamma_k) \lambda_k^o)^2} & l = k \\ -\frac{1}{N_a} \frac{m'_{\mathbf{B}_N(x=0), \boldsymbol{\theta}_{b_k, k}}(z=-1)}{(\chi_{b_k, k})^2 (\chi_{b_k, l})^2} & l \neq k \end{cases} \quad (15)$$

<sup>1</sup>The channel scaling by  $N_a$  in Assumption 3 does not change the optimal beamformer structure. It would just results in scaling the transmit powers by  $N_a$ , while the gap between transmit powers of various methods remain the same. Therefore, the results of Theorem 1 and 2 can be applied to the original channel model without scaling [16].

where

$$\chi_{b, l} = 1 + \lambda_l^o m_{\Sigma_b, \boldsymbol{\theta}_{b, l}}(-1). \quad (16)$$

The Stieltjes transform  $m_{\Sigma_{b_k}, \boldsymbol{\theta}_{b_k, k}}(-1)$  is the same as defined in Theorem 1.  $m'_{\mathbf{B}_N(x=0), \boldsymbol{\theta}_{b_k, k}}(z = -1)$  is the derivative of the Stieltjes transform of a measure with respect to an auxiliary variable  $x$  at point  $x = 0, z = -1$  defined as

$$m'_{\mathbf{B}_N(x=0), \boldsymbol{\theta}_{b_k, k}}(z = -1) = \frac{1}{N_a} \text{tr}(\boldsymbol{\theta}_{b_k, k} \mathbf{T}_{b_k} \left( \frac{1}{N_a} \sum_{i \in \mathcal{U}} \frac{\lambda_i^o \boldsymbol{\theta}_{b_k, i} e'_{N_a, i}(-1, 0)}{(1 + e_{N_a, i}(-1, 0))^2} + \boldsymbol{\theta}_{b_k, l} \right) \mathbf{T}_{b_k}) \quad (17)$$

where

$$\mathbf{T}_{b_k} = \left( \frac{1}{N_a} \sum_{i \in \mathcal{U}} \frac{\lambda_i^o \boldsymbol{\theta}_{b_k, i}}{1 + e_{N_a, i}(-1, 0)} + \mathbf{I}_{N_a} \right)^{-1} \quad (18)$$

The functions  $e_{N_a, i}(z, x = 0)$  are the same as  $e_{N_a, i}(z)$  in Theorem 1. Denoting  $e'_{N_a, i}(-1, 0) = e'_i$ , the scalars  $e'_i$  are given by

$$[e'_1, \dots, e'_n]^T = (\mathbf{I} - \mathbf{L})^{-1} \mathbf{v} \quad (19)$$

where

$$[\mathbf{L}]_{j, i} = \frac{1}{N_a^2} \frac{\lambda_j^o \lambda_i^o \text{tr}(\boldsymbol{\theta}_{b_k, j} \mathbf{T}_{b_k} \boldsymbol{\theta}_{b_k, i} \mathbf{T}_{b_k})}{(1 + e_i)^2} \quad (20)$$

$$\mathbf{v} = \left[ \frac{1}{N_a} \text{tr}(\lambda_1^o \boldsymbol{\theta}_{b_k, 1} \mathbf{T}_{b_k} \boldsymbol{\theta}_{b_k, 1} \mathbf{T}_{b_k}), \dots, \frac{1}{N_a} \text{tr}(\lambda_n^o \boldsymbol{\theta}_{b_k, n} \mathbf{T}_{b_k} \boldsymbol{\theta}_{b_k, n} \mathbf{T}_{b_k}) \right] \quad (21)$$

*Proof:* Due to the lack of space the proof of Theorem 2 is given in a supporting document [16].

Theorem 2 gives an approximation for  $\mathbf{G}$  matrix which is used to get the approximated downlink power allocation  $\boldsymbol{\delta}^o$  defined by (8). Theorem 1 and 2 jointly define an algorithm that gives the approximations for the optimal UL/DL power allocations and the corresponding detection/beamforming vectors can be achieved by plugging the powers  $\lambda_k^o$  and  $\boldsymbol{\delta}^o$  in (6) and (7), respectively.

The error in approximations of the proposed algorithm causes variations in the resulted SINRs. Thus, the SINR constraints cannot be guaranteed and the resulted SINR might be higher or lower than the target SINRs. In the next section, we introduce another approximation approach that satisfies the SINR constraints with slightly higher transmission power.

## B. Approximation of intercell interference terms

The method proposed in this section relies on approximately optimal ICI values, where, the approximations just depend on statistics of channels. Thus, the obtained approximate ICIs remain valid for a given set of users until a change occurs in the statistics of the channel, i.e., when a user changes its location.

Recalling relations between UL and DL beamformers, the intercell interference term from the  $b$ th base station to user  $k$  is,

$$\epsilon_{b, k}^2 = \sum_{l \in \mathcal{U}_b} |\mathbf{h}_{b, k}^H \mathbf{w}_l|^2 = \sum_{l \in \mathcal{U}_b} \sqrt{\delta_{b, l}} |\mathbf{h}_{b, k}^H \hat{\mathbf{w}}_l|^2 \quad (22)$$

where  $\delta_{b,l}$  values can be found from (8) and the approximations for the cross-terms  $|\hat{\mathbf{w}}_k^H \mathbf{h}_{b,k,l}|^2$  are defined by the non-diagonal elements of  $\mathbf{G}$  matrix in (15). Therefore,

$$\epsilon_{b,k}^2 \simeq - \sum_{l \in \mathcal{U}_b} \sqrt{\delta_{b,l}} [\mathbf{G}]_{l,k} \quad (23)$$

This approximation allows derivation of approximately optimal ICI terms based on statistics of the user channels. Each BS needs knowledge about user specific average statistics, i.e., correlation properties from other BSs based on which each BS can locally and independently calculate the approximately optimal ICI values.

### C. Distributed approximately optimal algorithm

Using any fixed ICI value in (2) produces a special case that results in a suboptimal performance in general. In [7]–[9], an agreement on optimal fixed ICI values is achieved via the exchange of scalar ICI parameters, i.e., local copies of ICI terms or corresponding dual variables. Another straightforward decentralized approach is to enforce all inter-cell interference to zero [7]. In all cases, however, the intra-cell interference among local users can be optimally handled. Solving (2) with the approximated ICI values  $\epsilon_{b,k}$  developed in the previous subsection leads to an algorithm that benefits from both the locally optimal beamforming design and near optimal ICI knowledge. This property brings significant gains compared to other suboptimal methods like inter-cell interference nulling. The proposed algorithm is summarized in Algorithm 1.

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#### Algorithm 1 Approximation of the ICI values.

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- 1: Initialize the ICI values based on the exchanged correlation properties.
  - 2: **loop**
  - 3:   **if** Any change in the user statistics **then**
  - 4:     Exchange the updated correlation properties among coupled BSs.
  - 5:     Update the approximated  $\lambda_k$  values,  $m_{\Sigma_{b_k}}(-1)$  and its derivative from equations (12), (13) and (17).
  - 6:     Get approximated  $\delta$  values from (8).
  - 7:     Update the approximated ICIs based on (23).
  - 8:   **end if**
  - 9:   Use the approximated ICIs as a fixed  $\epsilon_{b,k}^2$  in (2) and solve the subproblems locally for getting the optimal downlink beamformers.
  - 10: **end loop**
- 

The local problems can be solved independently (till a change in statistics of the channels happen) by reformulating (2) as BS specific SOCP or solved iteratively as in [9]. The proposed algorithm guarantees the target SINRs because the feasible solution of the optimization problem defined by (2) always satisfies the constraints and the possible error in approximations is translated into a somewhat higher transmit power at BSs compared to the optimal centralized solution.

## V. NUMERICAL ANALYSIS

Two algorithms developed in the previous section for multicell system with large dimensions provide good approximations even when the dimensions of the problem (i.e. the number of users and antennas) are practically limited. In order to show the performance of the approximate algorithms, some numerical examples are presented in this section. Due to lack of space, we just present the results for the algorithm based on ICI approximation. This algorithm satisfies the target SINRs for all users; however, the error in approximations results a higher transmit power at BSs.

A network with 7 cells is considered and users are equally distributed between cells. Exponential pathloss model is used for assigning the pathloss to each user,  $a_{b,k} = (d_0/d_{b,k})^2$  where  $d_{b,k}$  is distance between base station  $b$  and user  $k$ . The pathloss exponent is 2.3 and the reference distance ( $d_0$ ) is 1m. The pathloss from a base station to the boundary of the reference distance of the neighboring base station is fixed to 60dB. The correlation among channel entries is introduced using a simple exponential model

$$[\boldsymbol{\theta}_{b,k}]_{i,j} = \rho^{|i-j|} \quad (24)$$

where,  $\rho$  represents the correlation coefficient which is 0.8 for the following simulations. The users are dropped randomly for each trial and in total 1000 user drops are used for calculating the average transmit power. The number of antennas at each BS varies from 14 to 84 and the total number of users is equal to half the number of antennas at each BS. Thus, the spatial loading is fixed as the number of antennas is increased.

Fig. 1 and 2 illustrate the transmit powers versus the number of antennas for 0 dB and 10dB SINR target respectively. It is clear that the gap between the approximated and optimal algorithm (denoted as SOCP) diminishes as the number of antennas and users increase. Small gap in small dimensions indicates that the approximate algorithm can be applied to the practical scenarios with a limited number of antennas and users.

From the results it is clear that SOCP algorithm and the approximated ICI algorithm outperform the ZF method. Note that the number of antennas at each BS per number of served users is increasing while the gap between ZF and optimal and approximated method is fixed which is due to the fixed ratio of the number of antennas to the total number of users. The gap in performance is mainly due to the fact that the ZF algorithm wastes a degree of freedom for nulling the interference towards the distant users while the SOCP algorithm finds the optimal balance between interference suppression and maximizing the desired signal level. MF beamforming must be dealt with more care since the SINR target is below the target SINR and it can be guaranteed only asymptotically, i.e., when the ratio of the number of antennas to the number of users approaches infinity.

## VI. CONCLUSIONS

In this work, we used the theory of large-dimensional random matrices for the development of two algorithms for

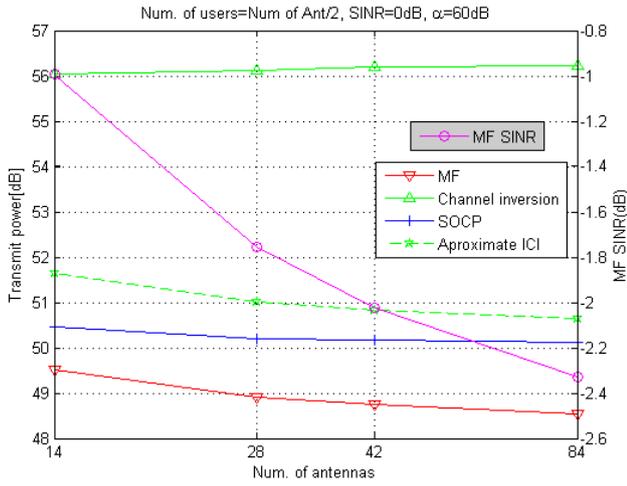


Fig. 1: Comparison of required transmit power for 0 dB SINR target.

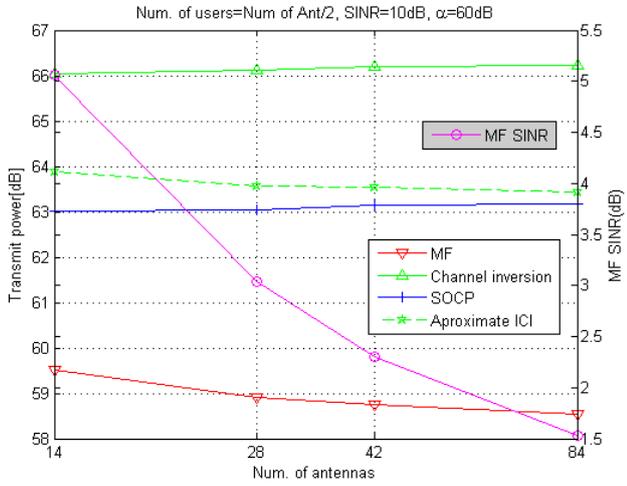


Fig. 2: Comparison of required transmit power for 10 dB SINR target

minimum power beamforming based on uplink-downlink duality and optimization decomposition. These algorithms give the simplified approximately optimal UL/DL power allocations and the corresponding detection/beamforming vectors based on other BSs channel statistics and local CSIs. The channel model utilized here is fairly general and applicable to various propagation environments. The algorithms can be further simplified depending on the characteristics of the propagation medium. The same correlation properties with different channel gains for all users or the case with diagonal correlation matrices are such examples. The simulation results indicate that these approximations are accurate even for small system dimensions. Thus, the framework presented in this paper can be used for both the theoretical analysis as well as the practical algorithm development.

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