

ENHANCED JOINT DATA DETECTION AND TURBO MAP CHANNEL ESTIMATION USING RANDOMLY ROTATED CONSTELLATIONS

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ABSTRACT

In this paper, we propose a joint data detection and a turbo maximum-*a-posteriori* (MAP) time-varying channel estimation in Slotted ALOHA MIMO systems using rotated constellations diversity. Our main idea is to use a randomly rotated and unrotated constellation, together with coding and interleaving, for each user in order to increase the diversity order and to improve the collision resolution at the receiver side. The burst-by-burst turbo-MAP channel estimator proposed is based on Space Alternating Expectation Maximization (SAGE) algorithm. Our proposed approach allows an efficient separation of colliding packets even if they are received with equal powers. Simulation results are given to support our claims.

Index terms : Time variable channel, Slotted ALOHA, Collision resolution, Maximum *a-posteriori*, SAGE.

1. INTRODUCTION

In this paper, we consider wireless Slotted ALOHA (SA) MIMO systems, without spread-spectrum transmission technique, over fast time-varying channel. For these systems, users do not coordinate their transmissions, which may collide at the receiver. A suitable strategy to separate the different user signals need to be applied in order to not deteriorate the MIMO system performance. In our study, without spread spectrum, colliding packets can only be distinguished through their different channel realizations, which are considered as signatures for the interfering users. In [1], a successive interference cancellation (SIC) based signal detector is proposed for space time block coding (STBC) to combat fast fading. Their contribution consists on treating undetected symbols as noise using a Gaussian approximation.

Our objective in this work is to propose an iterative receiver incorporating time-varying channel estimation, multi-user detection and data decoding for SA MIMO systems, where users transmit their data sequences according to Alamouti scheme. We note that a turbo-MAP

channel estimator has been proposed in [2] for the case of a single user Alamouti scheme over time-varying channel. The main contributions in our work compared to [2] are three : 1- In our channel modelisation, we take into account the shadowing and path-loss effects, 2- Unlike [2] where symbols modulation is performed with a fixed unrotated PSK constellation, we consider here a randomly rotated and unrotated constellation for each user in order to enhance the collided packets separation, 3- We introduce a practical implementation to solve the exact problem of MAP estimation of all channels seen from the interfering users following the SAGE algorithm[3]. Certainly, the derivation of the SAGE-based turbo-MAP channel estimation should take into account the rotating and unrotating of the constellation for each user.

The rest of the paper is organized as follows. In Section 2, we introduce the system model and the channel representation using a discrete version of the Karhunen-Loeve (KL) expansion theorem. In section 3, we derive the iterative turbo processing receiver for collision resolution. In section 4, we provide some simulation results to illustrate the performance of our receiver in terms of bit error rate.

2. SYSTEM MODEL

2.1. System description

We consider a cellular network where M users communicate with a base station equipped with N_r spatially decorrelated receive antennas. We assume that users use the Slotted ALOHA protocol to randomly access the channel. We assume that each user applies Alamouti space-time coding before transmission. The Alamouti scheme considers the symbols of the information sequence to be transmitted by pairs in the transmission procedure. The transmitted block from each user is organized into equal time slots of $2K$ modulated symbols, each with time positions $p_k = kT$, $k = 0, \dots, 2K - 1$. Here, T denotes the symbol period. Each block comprises $2K_p$ pilot symbols. In the following, we denote

d_m the distance between user m and the base station. For each user, the average energy received per symbol is equal to E_s . The interfering users will be indexed $0, \dots, M-1$. The channel is assumed to be fast time-varying. Hence, at time p_k , it may be modelled by a complex multiplicative distortion $\tilde{c}_{l,j}^{m,2k}$ where $l = 0$ or 1 represents the transmit antenna index and $j = 0, \dots, N_r - 1$ represents the receive antenna index. We note that the fading is assumed to be independent between the channels corresponding to different transmit and receive antennas and correlated in time within each antenna. Each multiplicative distortion vector $\tilde{\mathbf{c}}_{l,j}^m = (\tilde{c}_{l,j}^{m,0}, \tilde{c}_{l,j}^{m,1}, \dots, \tilde{c}_{l,j}^{m,2K-1})^T$, $l = 0, 1$, $j = 0, \dots, N_r - 1$, $m = 0, \dots, M-1$ is characterized by its average power as well as its underlying Doppler Power Spectrum (DPS). Depending on the environment, the shape of the DPS is either classical or flat. The classical DPS is typically met in outdoor environments. The corresponding autocorrelation function, for one path with average power $\phi(0)$ is given by $\phi(\tau) = \phi(0) J_0(\pi B_D \tau)$ where B_D is the Doppler spread of the channel and $J_0(\cdot)$ is the 0th-order Bessel function of the first kind. We note that the time slot index is omitted in this work to simplify the notations in the system model. The interfering signal coming from user m is attenuated by relative shadowing and path loss coefficients to the signal of user 0 and denoted respectively by w_m^s and w_m^p . The coefficient w_m^p follows respectively a propagation model of $w_m^p = (d_0/d_m)^{\frac{\alpha}{2}}$, $m = 0, \dots, M-1$, where α is the path loss exponent that depends on the environment. The coefficient w_m^s is modeled as log-normal distribution with variance $\sigma_{s,m}^2$. Next, we assume that path loss and shadowing are considered to be invariant during a time slot and can change from one slot to another. In this work, we also assume that these coefficients are known by the receiver. The orthogonal Alamouti scheme assumes the channel to be constant during each pair of consecutive even and odd indexed symbols. Consequently, the components of the received vector \mathbf{y}_j , $j = 0, \dots, N_r - 1$ for two consecutive time symbols $2kT$ and $(2k+1)T$, due to transmitted sequences from the M interfering users, can be written as

$$y_j^{2k} = \sum_{m=0}^{M-1} \left(c_{0,j}^{m,2k} s_{m,2k} + c_{1,j}^{m,2k} s_{m,2k+1} \right) + b_j^{2k} \quad (1)$$

and

$$y_j^{2k+1} = \sum_{m=0}^{M-1} \left(-c_{0,j}^{m,2k+1} s_{m,2k+1} + c_{1,j}^{m,2k+1} s_{m,2k} \right) + b_j^{2k+1}, \quad (2)$$

where $c_{l,j}^{m,2k} = w_m^s w_m^p \sqrt{\frac{E_s}{2}} \tilde{c}_{l,j}^{m,2k}$, $l = 0, 1$, $j = 0, \dots, N_r - 1$, $k = 0, 1, \dots, K-1$ and $(\cdot)^*$ denotes complex conjugate.

Here, b_j^{2k} and b_j^{2k+1} denote respectively the additive noises, at times $2kT$ and $(2k+1)T$, assumed to be Gaussian distributed with zero mean and variance σ_b^2 .

2.2. Rotated and unrotated constellations

In the above expression, $s_{m,2k+l}$ denotes the rotated or unrotated version of the transmitted symbol $x_{m,2k+l}$. Basically, this technique uses a predefined set Ω_m encompassing rotated and unrotated constellations. The set Ω_m^1 contains the initial unrotated constellation. The set Ω_m^2 contains the randomly rotated constellation with a certain angle θ which is applied in the complex plane to the original signal constellation. As depicted in figure 1, we propose to use for each user a trellis coded modulation (TCM) encoding, symbol interleaving and random rotation mapper before applying the Alamouti ST encoder. We consider that each user m has a bit random generator which generates a bit sequence denoted $\mathbf{q}_m = (q_{m,0}, q_{m,1}, \dots, q_{m,2K-1})^T$. When $q_{m,k} = 0$, $k = 0, \dots, 2K-1$, the constellation of the k -th transmitted symbol keeps unchanged (i.e belongs to Ω_m^1) and when $q_{m,k} = 1$, the constellation of the k -th transmitted symbol is rotated (i.e belongs to Ω_m^2).

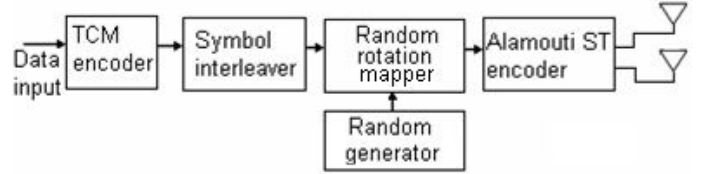


Fig. 1. Structure of the proposed coded and randomly rotated data transmission for each user.

2.3. Convenient channel representation

In our study, for MAP channel estimation, we need a convenient decorrelated representation of all distorting channels during each received block. This representation of each channel is directly derived from [2]. Consequently, we can use, up to a multiplicative factor $\sqrt{\frac{E_s}{2}}$, the decimated distortion vectors $\tilde{\mathbf{h}}_{l,j}^m = \sqrt{\frac{E_s}{2}} (\tilde{c}_{l,j}^{m,0}, \tilde{c}_{l,j}^{m,2}, \dots, \tilde{c}_{l,j}^{m,2K-2})^T$, $l = 0, 1$, $j = 0, \dots, N_r - 1$, $m = 0, \dots, M-1$, with half size of the multiplicative distortion vector $\tilde{\mathbf{c}}_{l,j}^m$. The m -th normalized discrete channel vector $\tilde{\mathbf{h}}_{l,j}^m$ can be expressed as follows

$$\tilde{\mathbf{h}}_{l,j}^m = \sum_{k=0}^{K-1} \tilde{g}_{l,j}^{m,k} \mathbf{u}_k, \quad (3)$$

where $\{\mathbf{u}_k\}_{k=0}^{K-1}$ are the normalized eigenvectors of the covariance matrix $\mathbf{F} = E [\tilde{\mathbf{h}}_{l,j}^m \tilde{\mathbf{h}}_{l,j}^{mH}]$ of $\tilde{\mathbf{h}}_{l,j}^m$, $\{\tilde{g}_{l,j}^{m,k}\}_{k=0}^{K-1}$,

$l = 0, 1, j = 0, \dots, N_r - 1, m = 0, \dots, M - 1$ are independent complex zero-mean Gaussian coefficients. Let $(\cdot)^H$ and $E[\cdot]$ denote respectively the Hermitian transposition and the expectation operator. We note that \mathbf{F} is independent of l, j and m since the $2MN_r$ channels obey the same statistics. The variances of $\left\{ \tilde{g}_{l,j}^{m,k} \right\}_{k=0}^{K-1}$, arranged in decreasing order, are equal to the eigenvalues $\left\{ \tilde{\lambda}_k \right\}_{k=0}^{K-1}$ of the Hermitian matrix \mathbf{F} . The system $\left\{ \mathbf{u}_k \right\}_{k=0}^{K-1}$ constitutes an orthonormal base of the complex space of K dimensions. Based on this, we rewrite the channel vector seen at the receiver as follows

$$\mathbf{h}_{l,j}^m = w_m^s w_m^p \sum_{k=0}^{K-1} \tilde{g}_{l,j}^{m,k} \mathbf{u}_k = \sum_{k=0}^{K-1} g_{l,j}^{m,k} \mathbf{u}_k \quad (4)$$

where $g_{l,j}^{m,k} = w_m^s w_m^p \tilde{g}_{l,j}^{m,k}$. In the following, we assume that the vectors $\mathbf{g}_{l,j}^m = \left(g_{l,j}^{m,0}, g_{l,j}^{m,1}, \dots, g_{l,j}^{m,K-1} \right)^T$, are referred to as the convenient representations of each of the $2MN_r$ discrete channels for each user m during each received block.

3. ITERATIVE TURBO PROCESSING RECEIVER FOR COLLISION RESOLUTION

3.1. MAP channel estimation using the SAGE algorithm and Bahl approach

In this subsection, for notation simplicity, we will replace $\left\{ \mathbf{g}_{l,j}^m \right\}_{l,m,j}$ by \mathbf{g} and $\left\{ \mathbf{y}_j \right\}_j$ by \mathbf{y} , for $l = 0, 1, j = 0, \dots, N_r - 1$ and $m = 0, \dots, M - 1$. The MAP estimate $\hat{\mathbf{g}}$ of the equivalent representation of the discrete channel \mathbf{g} is given by

$$\hat{\mathbf{g}} = \arg \max_{\mathbf{g}} p(\mathbf{g}|\mathbf{y}). \quad (5)$$

In such a situation, the iterative SAGE algorithm provides an iterative scheme to reach the solution. This algorithm inductively reestimates, the discrete channel distortion one by one, the $2MN_r$ vectors $\left\{ \mathbf{g}_{l,j}^m \right\}_{l,m,j}$ so that a monotonous increase in the *a-posteriori* conditional PDF in (5) is guaranteed. This monotonous increase is realized via the maximization of the auxiliary function defined by

$$Q(\mathbf{g}, \mathbf{g}') = \sum_{\mathbf{s}} p(\mathbf{y}, \mathbf{s}, \mathbf{g}) \log p(\mathbf{y}, \mathbf{s}, \mathbf{g}'), \quad (6)$$

with respect to \mathbf{g}' , where $\mathbf{s} = \{s_m\}_m$, $s_m = [s_{m,0} \ s_{m,1} \ \dots \ s_{m,2K-1}]$ and $m = 0, \dots, M - 1$. The latter sum is operated over all possible transmitted data vectors. Given the received vectors \mathbf{y} , the SAGE

algorithm starts with an initial guess $\hat{\mathbf{g}}^{(0)}$ of \mathbf{g} . The evolution from the estimate $\hat{\mathbf{g}}^{(d)}$ to the new estimate $\hat{\mathbf{g}}_m^{(d+1)}$ is performed in several stages via a maximization of an auxiliary function in (6). In each stage, the SAGE algorithm iteratively alternates between two steps, an expectation step (E-step) followed by a maximization step (M-step). After some calculations, omitted here for lack of space, we calculate the derivatives of $Q(\mathbf{g}^{(d)}, \mathbf{g}')$ with respect to \mathbf{g}'_m and we equate them to zero, we obtain

$$\begin{aligned} \hat{g}_{l,j}^{m,k(d+1)} = & \xi_k \sum_{k'=0}^{K-1} \left[\hat{y}_{m,j}^{2k'(d)} \left(\hat{s}_{m,2k'+l}^{(d)} \right)^* + \right. \\ & \left. (-1)^l \hat{y}_{m,j}^{2k'+1(d)} \hat{s}_{m,2k'-l+1}^{(d)} \right] u_{kk'}^*, \end{aligned} \quad (7)$$

where $\xi_k = 1 / (1 + \sigma_b^2 / \lambda_k)$,

$$\hat{y}_{m,j}^{2k(d)} = y_j^{2k} - \sum_{\substack{m'=0 \\ m' \neq m}}^{M-1} \left(\hat{c}_{0,j}^{m',2k(d)} \hat{s}_{m',2k}^{(d)} + \hat{c}_{1,j}^{m',2k(d)} \hat{s}_{m',2k+1}^{(d)} \right), \quad (8)$$

$$\begin{aligned} \hat{y}_{m,j}^{2k+1(d)} = & y_j^{2k+1} - \\ & \sum_{\substack{m'=0 \\ m' \neq m}}^{M-1} \left(-\hat{c}_{0,j}^{m',2k+1(d)} \hat{s}_{m',2k+1}^{(d)*} + \hat{c}_{1,j}^{m',2k+1(d)} \hat{s}_{m',2k}^{(d)*} \right) \end{aligned} \quad (9)$$

and

$$\hat{s}_{m,2k+l}^{(d)} = \sum_{\Omega_m^q} s_{m,2k+l} p(\mathbf{s}|\mathbf{y}, \hat{\mathbf{g}}^{(d)}), \quad (10)$$

for $l = 0, 1, j = 0, \dots, N_r - 1, q = q_{m,2k+l}, k = 0, 1, \dots, K - 1$ and $m = 0, \dots, M - 1$. The coded structure of each normalized transmitted vector \mathbf{s} leads to a dependence of the conditional probabilities $p(\mathbf{s}|\mathbf{y}, \hat{\mathbf{g}}^{(d)})$ on all the components of \mathbf{y} . To compute these conditional probabilities and their associated soft decision outputs, we refer to the turbo processing proposed in [2]. In [2], the authors deduced that searching for symbols $s_{m,k}, k = 0, 1, \dots, 2K - 1$, with a symbol MAP detector based on $p(s_{m,k}|\mathbf{y}, \hat{\mathbf{g}}^{(d)})$, is equivalent to searching for symbols $\bar{s}_{m,\mu}, \mu = 0, 1, \dots, 2K - 1$, based on the conditional probabilities $p(\bar{s}_{m,\mu}|\bar{\Lambda}_{m,j}^{(d)}, \hat{\mathbf{g}}^{(d)})$, as follows

$$p(s_{m,k}|\mathbf{y}, \hat{\mathbf{g}}^{(d)}) = \rho p(\bar{s}_{m,\mu}|\bar{\Lambda}_{m,j}^{(d)}, \hat{\mathbf{g}}^{(d)}), \quad (11)$$

where $\bar{\Lambda}_{m,j}^{(d)} = \left\{ \bar{\Lambda}_{m,j}^{k(d)} \right\}_{k=0}^{2K-1}$ is the symbol to bit demapper and de-interleaved version of the equivalent observation $\Lambda_{m,j}^{(d)} = \left\{ \Lambda_{m,j}^{k(d)} \right\}_{k=0}^{2K-1}$, referred as an ST decoder, $\bar{s}_{m,\mu}$ is the encoder output at time position μ and

ρ is a proportionally factor which only depends on channel realizations and common to all possible transmitted sequences.

From (6), we compute respectively $\Lambda_{m,j}^{2k(d)}$ and $\Lambda_{m,j}^{2k+1(d)}$ as

$$\Lambda_{m,j}^{2k(d)} = \frac{\hat{y}_{m,j}^{2k(d)} \left(\hat{h}_{0,j}^{m,k(d)} \right)^* + \left(\hat{y}_{m,j}^{2k+1(d)} \right)^* \hat{h}_{1,j}^{m,k(d)}}{\sigma_b^2}, \quad (12)$$

$$\Lambda_{m,j}^{2k+1(d)} = \frac{\hat{y}_{m,j}^{2k(d)} \left(\hat{h}_{1,j}^{m,k(d)} \right)^* - \left(\hat{y}_{m,j}^{2k+1(d)} \right)^* \hat{h}_{0,j}^{m,k(d)}}{\sigma_b^2}, \quad (13)$$

for $j = 0, \dots, N_r - 1$, $k = 0, 1, \dots, K - 1$ and $m = 0, 1, \dots, M - 1$.

For each iteration d , the channel estimates for the m -th user is given by the following iterations

$$\hat{\mathbf{h}}_{l,j}^{m(d)} = \sum_{k=0}^{K-1} \hat{g}_{l,j}^{m,k(d)} \mathbf{u}_k. \quad (14)$$

3.2. Joint data detection and channel estimation with turbo processing

We apply the proposed data-aided decision-directed MAP algorithm for the initial channel estimation. Let S_P be the set of pilot symbols indices within a block. The pilot symbols are also considered in pairs and space-time encoded in the same manner as data symbols. We assume that the position of pilot symbols is optimum and common to all users and that their common number $N_p \geq M$. When used pilot sequences are orthogonal in space and in time, channel estimates of all users will be mutually interference free. Using the initial channel estimates from (14), the receiver can compute the total received power for each user as follows

$$\hat{P}_m^{(0)} = \sum_{l=0, j=0}^{1, N_r-1} \left\| \hat{\mathbf{h}}_{l,j}^{m(0)} \right\|^2. \quad (15)$$

At the output of the receiver, users are ranked according to their powers, so that the strongest user is ranked first, and the weakest is ranked last. Without loss of generality, we assume that the received user powers are organized in decreasing order, starting from user 0 to user $M - 1$.

As shown in figure 2, the obtained sequence $\Lambda_{m,j}^{(d)} = \left\{ \Lambda_{m,j}^{k(d)} \right\}_{k=0}^{2K-1}$ is de-interleaved, rotate and unrotate demapped to obtain $\bar{\Lambda}_{m,j}^{(d)} = \left\{ \bar{\Lambda}_{m,j}^{k(d)} \right\}_{k=0}^{2K-1}$. Then, $\bar{\Lambda}_{m,j}^{(d)}$ is applied at the input of the BCJR decoder[4].

At each iteration d , the SAGE algorithm only re-estimates the transmitted data of user m , $m = d \bmod (M - 1)$ while the symbols of the others users $m' \neq m$ are not updated

$$\hat{s}_{m', 2k+l}^{(d+1)} = \hat{s}_{m', 2k+l}^{(d)}. \quad (16)$$

The proposed algorithm iterates overall a few iterations until a convergence is achieved.

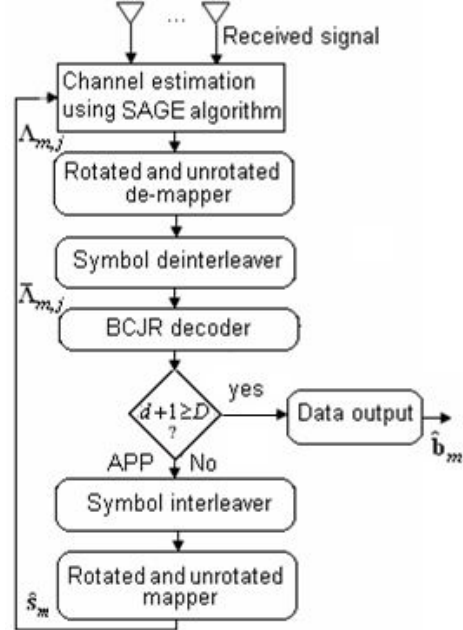


Fig. 2. Structure of the turbo-MAP using the SAGE algorithm.

3.3. Decision on information symbols

The iterative algorithm we have proposed leads to a joint improvement of channel estimation and multi-user symbol detection through iterations. After a fixed number of iterations D , we obtain the estimate $\hat{\mathbf{g}}_{l,j}^m$, $l = 0, 1$, $j = 0, \dots, N_r - 1$, $m = 0, \dots, M - 1$ of the discrete multi-path channel. This number D is chosen so that the reached estimate $\hat{\mathbf{g}}_{l,j}^{m(D)}$ guarantees an unnoticeable degradation in performance with respect to the optimum estimate $\hat{\mathbf{g}}_{l,j}^m$. The optimum MAP detection of trellis coded information carrying binary symbols is based on the maximisation of the *a-posteriori* probabilities (APP) $p(b_{m,k} = b | \bar{\Lambda}_{m,j}^{(D)}, \hat{\mathbf{g}}^{(D)})$, where $b = 0, 1$. This APP $p(b_{m,k} = b | \bar{\Lambda}_{m,j}^{(D)}, \hat{\mathbf{g}}^{(D)})$ is provided by the BCJR MAP algorithm.

4. SIMULATION RESULTS

We consider for this simulation that the destination can apply our approach to separate two collided packets ($M = 2$). If the number of interfering users in the same slot is more than two, it does not apply this approach and the packets are lost and retransmitted later on. In the simulations, each user transmits a packet that contains $2K = 128$ modulated symbols with $2K_p = 12$ pilot symbols taken from a rotated and unrotated QPSK constellation. The normalized Doppler spread is fixed at $B_D T = 1/128$. For each user, the input bit sequence is encoded with a TCM based on a rate-1/2 convolutional code. Simulation results are analysed over two different scenarios. For the first scenario, we consider the critical case of equal user powers. For the second scenario, the ratio of interfering users powers belongs to $[\varepsilon, \frac{1}{\varepsilon}]$, where $\varepsilon = 0.8$. Figure 3 (respectively, Figure 4) shows the behavior of the BER obtained using the iterative collision resolution procedure system as a function of the average E_s/σ_b^2 , when the first scenario (respectively, the second scenario) is considered with $\theta = \pi/4$ and $N_r = 1$ or 2. The Attenuation parameters in (1-2) are set to be $\alpha = 4$ and $\sigma_{s,1} = \sigma_{s,0} = 8\text{dB}$. As first benchmarks, we consider a single user transmitting according to Figure 1 and a turbo-MAP channel estimator (EC) or Perfect Channel State Information (PCSI) at the destination. Comparison is also done with a system considering the same proposed approach for transmission and reception but with unrotated QPSK constellation for all users. The results indicate that the performance of the proposed procedure for collision resolution approaches that of single user case even for comparable user powers. From Figures 3 and 4, we also notice a significant enhancement in performance when constellation is randomly rotated and unrotated for each user. This idea guarantees a better separation of colliding packets, even if they are received with equal powers.

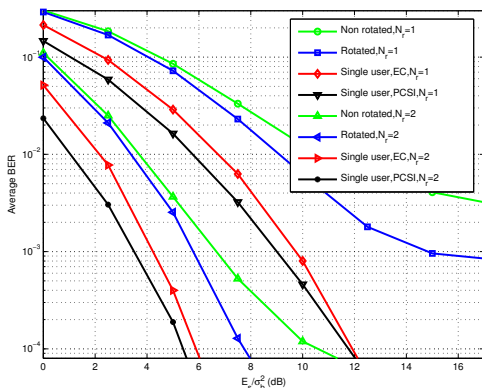


Fig. 3. Average bit error rate as a function of mean E_s/σ_b^2 considering the first scenario.

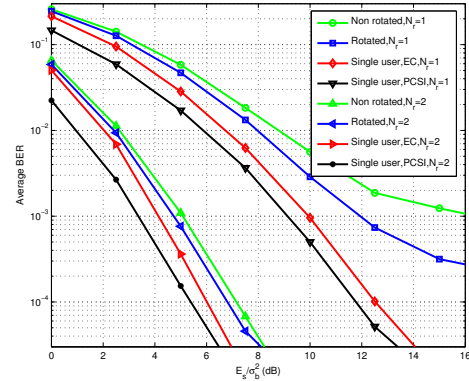


Fig. 4. Average bit error rate as a function of mean E_s/σ_b^2 considering the second scenario.

5. CONCLUSION

In this paper, we have proposed an enhanced multi-user detection and MAP channel estimation, in Slotted ALOHA MIMO systems, using randomly rotated constellations. Our main idea consists on using rotated and unrotated constellation, together with channel coding and interleaving, for each user in order to increase the diversity order and to enhance colliding packets differentiation. The proposed receiver performs an iterative channel estimation using the SAGE algorithm, incorporating the coded structure of each block in a turbo-processing fashion and taking into account the constellation rotating and unrotating. Based on simulation results, we have noticed that our proposed approach allows a reliable transmitted data recovery and efficient estimation of all channels connecting the interfering users and the base station, even if the collided packets are received with comparable powers.

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