MULTISTAGE GENERALIZED ADAPTIVE NOTCH FILTER WITH IMPROVED ACCURACY

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ABSTRACT

Generalized adaptive notch filters (GANFs) are estimators of coefficients of quasi-periodically time-varying systems. Current state of the art GANFs can deliver highly accurate estimates of system variations' frequency, but underperform in terms of accuracy of the coefficient estimates. The paper proposes a novel multistage GANF with accuracy improved in this aspect. The processing pipeline consists of three stages. The preliminary (pilot) frequency estimates are obtained first, then treated with a specially designed linear filter and used to guide the coefficient tracking GANF, which works out the estimates of system coefficients. The proposed solution has considerably better performance than a single stage GANF or a simple two-stage approach consisting of the pilot frequency estimator and the amplitude tracking GANF only.

***Index Terms*—** generalized adaptive notch filters, estimation algorithms, adaptive signal processing

1. INTRODUCTION

Consider the problem of estimating coefficients of a quasi-periodically varying, complex-valued system governed by

$$y(t) = \boldsymbol{\phi}^{\mathrm{T}}(t)\boldsymbol{\theta}(t) + v(t) \tag{1}$$

where t denotes discrete time, $y(t)$ is the system output, $v(t)$ is measurement noise, $\boldsymbol{\phi}(t) = [\phi_1(t)\ \phi_2(t)\ \dots\ \phi_n(t)]^{\mathrm{T}}$ is the regression vector, which is assumed to form a wide-sense stationary sequence, $\boldsymbol{\theta}(t) = [\theta_1(t)\ \theta_2(t)\ \dots\ \theta_n(t)]^{\mathrm{T}}$ consists of time varying system coefficients,

$$\boldsymbol{\theta}(t) = \boldsymbol{\beta}(t) f(t), \qquad f(t) = e^{j\sum_{\tau=1}^{t}\omega(\tau)}\ , \tag{2}$$

where $\boldsymbol{\beta}(t) = [\beta_1(t)\ \beta_2(t)\ \dots\ \beta_n(t)]^{\mathrm{T}}$ denotes the vector of complex "amplitudes", $f(t)$ is the phase term and $\omega(t)$ is the instantaneous angular frequency. Both $\boldsymbol{\beta}(t)$ and $\omega(t)$ are assumed to be slowly time-varying quantities.

Systems falling into the scope of (1)-(2) often appear in RF applications when the Doppler effect comes into play [1]. In most cases $y(t)$ and $v(t)$ are baseband complex envelopes of the received signal and the receiver noise, respectively, $\boldsymbol{\phi}(t)$ consists of delayed replicas of the transmitted signal, $\boldsymbol{\beta}(t)$ is the moving scatterer's "impulse response" and $\omega(t)$ is the scatterer's Doppler frequency.

Estimation of coefficients of (1)-(2) can be accomplished using many approaches, among which the generalized adaptive notch filter (GANF) takes an important place [2]. The word "generalized" stems from the fact that, when $n = 1$, $\boldsymbol{\phi}(t) = \phi(t) \equiv 1$, the problem (1) simplifies to tracking of a nonstationary complex sinusoid embedded in wideband noise

$$s(t) = \beta(t)e^{j\sum_{\tau=1}^{t}\omega(t)} \tag{3}$$
$$y(t) = s(t) + v(t), \tag{4}$$

which is often solved using adaptive notch filters, see e.g. [3].

GANFs are capable of providing excellent frequency tracking performance – under Gaussian random-walk type assumptions they can reach a fundamental Cramér-Rao type lower bound, which limits tracking accuracy of any scheme [2]. However, they leave a room for improvement in terms of estimation of the system coefficients. This can be attributed to the fact that optimization of coefficient and frequency tracking characteristics of the GANF are conflicting goals [2].

The paper proposes a novel, multistage GANF. Capitalizing on the excellent frequency tracking performance of a standard GANF, and using a special processing pipeline, we can improve quality of coefficient estimates without adversely affecting frequency tracking performance.

The organization of the text is as follows. Section 2 introduces the proposed multistage structure. Section 3 addresses the problem of optimizing performance of the proposed filter. Section 4 presents simulation results. Section 5 concludes.

2. PROPOSED SOLUTION

Fig. 1 shows the block diagram of the proposed solution. The algorithm consists of three steps, whose detailed description will be presented in the next three subsections.

2.1. Pilot filter

The purpose of the pilot filter is to work out initial, highly accurate frequency estimates which will be used later in the

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Fig. 1. Block diagram of the proposed multistage generalized adaptive notch filter.

processing. The pilot is a standard GANF, adopted from [2]

$$\begin{aligned}
\hat{f}(t) &= e^{j\hat{\omega}(t-1|t-1)}\hat{f}(t-1) \\
\epsilon(t) &= y(t) - \boldsymbol{\phi}^{\mathrm{T}}(t)\hat{f}(t)\hat{\boldsymbol{\beta}}(t-1) \\
\hat{\boldsymbol{\beta}}(t) &= \hat{\boldsymbol{\beta}}(t-1) + \mu\boldsymbol{\Phi}^{-1}\boldsymbol{\phi}^{*}(t)\hat{f}^{*}(t)\epsilon(t) \\
\delta(t) &= \mathrm{Im}\frac{\epsilon^{*}(t)\boldsymbol{\phi}^{\mathrm{T}}(t)\hat{f}(t)\hat{\boldsymbol{\beta}}(t-1)}{\hat{\boldsymbol{\beta}}^{\mathrm{H}}(t-1)\boldsymbol{\Phi}\hat{\boldsymbol{\beta}}(t-1)} \\
\hat{\omega}(t|t) &= \hat{\omega}(t|t-1) - \gamma\delta(t) \\
\hat{\boldsymbol{\theta}}(t) &= \hat{\boldsymbol{\beta}}(t)\hat{f}(t)
\end{aligned} \tag{5}$$

where $*$ denotes complex conjugation, H stands for Hermitian transposition, $\hat{f}(t)$ is a phase term, $\epsilon(t)$ denotes prediction error, $\boldsymbol{\Phi}$ is a positive definite correlation matrix of regression vectors (assumed to be known a'priori), $\boldsymbol{\Phi} = \mathrm{E}[\boldsymbol{\phi}^{*}(t)\boldsymbol{\phi}^{\mathrm{T}}(t)] > \mathbf{0}$, $\hat{\omega}(t|t)$ denotes the estimate of the instantaneous frequency, $\hat{\omega}(t+1|t)$ is its one-step ahead prediction, and $\hat{\boldsymbol{\theta}}(t)$ is the estimate of $\boldsymbol{\theta}(t)$. The parameters $\mu > 0$, $\gamma > 0$, $\gamma \ll \mu$ are small adaptation gains which govern rates of amplitude and frequency adaptation, respectively.

2.2. Frequency postprocessor

Although this may seem doubtful at the moment, frequency estimates yielded by the pilot should be postprocessed prior to estimation of system coefficients. The postprocessor takes the form of a linear filter

$$\bar{\omega}(t|t) = P(q^{-1})\hat{\omega}(t|t) \tag{6}$$

where the transfer function $P(q^{-1})$ will be found later.

2.3. Frequency guided filter

The coefficient tracking filter takes the form

$$\begin{aligned}
\bar{f}(t) &= e^{j\bar{\omega}(t|t)}\bar{f}(t-1) \\
\bar{\epsilon}(t) &= y(t) - \boldsymbol{\phi}^{\mathrm{T}}(t)\bar{f}(t)\bar{\boldsymbol{\beta}}(t-1) \\
\bar{\boldsymbol{\beta}}(t) &= \bar{\boldsymbol{\beta}}(t-1) + \mu_{\mathrm{a}}\boldsymbol{\Phi}^{-1}\boldsymbol{\phi}^{*}(t)\bar{f}^{*}(t)\bar{\epsilon}(t) \\
\bar{\boldsymbol{\theta}}(t) &= \bar{\boldsymbol{\beta}}(t)\bar{f}(t)
\end{aligned} \tag{7}$$

where $\mu_a > 0$ denotes its adaptation gain.

3. OPTIMIZATION

3.1. Tracking analysis

To optimize the performance of the proposed GANF one needs to come up with analytical results concerning its tracking behavior first. This can be done using the approximating linear filter method – a linearization approach, introduced in [4], for the purpose of analyzing adaptive notch filters.

Suppose that

(A1) The vector $\boldsymbol{\beta}(t)$ is constant, $\boldsymbol{\beta}(t) \equiv \boldsymbol{\beta}_0$.

(A2) Instantaneous frequency $\omega(t)$ is governed by

$$\omega(t) = \frac{n(t)}{A(q^{-1})}$$

where $A(q^{-1})$ is a (marginally) stable polynomial of order n_A, $A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_{n_A} q^{-n_A}$ and $n(t)$ is a zero mean Gaussian white noise with variance σ_n^2, independent of $v(t)$ and $\boldsymbol{\phi}(t)$.

(A3) Measurement noise $v(t)$, independent of $n(t)$ and $\boldsymbol{\phi}(t)$, is a zero mean, wide sense stationary complex circular Gaussian white noise with variance σ_v^2, $v(t) \sim \mathcal{CN}(0, \sigma_v^2)$.

First, denote by $\Delta\hat{\omega}(t) = \hat{\omega}(t|t) - \omega(t)$ the pilot's frequency estimation error and let $e(t) = \boldsymbol{\beta}_0^{\mathrm{H}}\boldsymbol{\phi}^{*}(t)f^{*}(t)v(t)/b_0^2$, where $b_0^2 = \boldsymbol{\beta}_0^{\mathrm{H}}\boldsymbol{\Phi}\boldsymbol{\beta}_0$. One can show that [2, 5]

$$\Delta\hat{\omega}(t) = \frac{F(q^{-1}) - 1}{1 - q^{-1}}w(t) + F(q^{-1})C(q^{-1})e_I(t) \ , \tag{8}$$

where $w(t) = \omega(t) - \omega(t-1)$ is a sequence of one step changes of instantaneous frequency, q^{-1} is the backward shift operator, $q^{-1}w(t) = w(t-1)$, $C(q^{-1}) = 1 - q^{-1}$, $e_I(t) = \mathrm{Im}[e(t)]$,

$$F(q^{-1}) = \frac{\gamma}{1 - (\lambda + \delta)q^{-1} + \lambda q^{-2}} \tag{9}$$

and $\delta = 1 - \gamma$, $\lambda = 1 - \mu$. Note that, under (A1)-(A3), $e_I(t)$ is a (real valued) zero mean Gaussian white noise with variance $\sigma_{e_I}^2 = \sigma_v^2/(2b_0^2)$.

Second, using (8) it is straightforward to show that

$$\hat{\omega}(t|t) = F(q^{-1})u(t) \tag{10}$$

where

$$u(t) = \omega(t) + C(q^{-1})e(t)$$

can be interpreted as the underlying pseudomeasurement sequence for the frequency estimation problem.

Combining (6) with (10) one can obtain that

$$\bar{\omega}(t|t) = P(q^{-1})F(q^{-1})u(t) = L(q^{-1})u(t) \tag{11}$$

which emphasizes the pseudomeasurement interpretation of the pilot filter (10).

Third, let $\Delta\bar{\boldsymbol{\theta}}(t) = \bar{\boldsymbol{\theta}}(t) - \boldsymbol{\theta}(t)$ and $\Delta\bar{\boldsymbol{\beta}}(t) = \Delta\bar{\boldsymbol{\theta}}(t)f^{*}(t)$. Steady-state tracking accuracy of the guided filter may be quantified using the following weighted mean square error

$$J = \lim_{t\to\infty} \mathrm{E}[\Delta\bar{\boldsymbol{\beta}}^{\mathrm{H}}(t)\boldsymbol{\Phi}\Delta\bar{\boldsymbol{\beta}}(t)] \ . \tag{12}$$

It can be shown that, under weak conditions, J is related to the transfer function $L(q^{-1})$ as follows (see appendix A in [6] for details)

$$J = \int_0^{2\pi} \frac{b_0^2\lambda_a^2}{|\Lambda_a(e^{-j\omega})|^2} \times \Bigg[\left|L(e^{-j\omega}) - 1\right|^2 S_\omega(e^{-j\omega}) + \left|L(e^{-j\omega})C(e^{-j\omega})\right|^2 \frac{\sigma_{e_I}^2}{2\pi} + 2\frac{\mu_a^2}{\lambda_a^2}\frac{\sigma_{e_I}^2}{2\pi} + \frac{\mu_a}{\lambda_a}\left[L(e^{-j\omega})C(e^{-j\omega}) + L^*(e^{-j\omega})C^*(e^{-j\omega})\right]\frac{\sigma_{e_I}^2}{2\pi}\Bigg] d\omega\ , \tag{13}$$

where $L(e^{-j\omega}) = L(q^{-1})|_{q=e^{j\omega}}$, denotes the frequency response of the filter $L(q^{-1})$, $C(e^{-j\omega}) = C(q^{-1})|_{q=e^{j\omega}}$, $\lambda_a = 1-\mu_a$, $\Lambda_a(e^{-j\omega}) = 1-\lambda_a e^{-j\omega}$ and $S_\omega(e^{-j\omega})$ denotes power spectral density of $\omega(t)$.

3.2. Optimization

Consider first the problem of finding the transfer function $L_\omega(q^{-1})$ which minimizes frequency tracking errors

$$\mathrm{E}[|\Delta\bar{\omega}(t)|^2] \to \min\ ,$$

where $\Delta\bar{\omega}(t) = \bar{\omega}(t|t) - \omega(t)$. These results will allow us to gain more insight into the solution which minimizes (13) at a later stage.

Under (A1)-(A3) the optimal, in the mean-squared sense, causal estimator of $\omega(t)$ takes the form of the Wiener filter

$$L(q^{-1}) = \frac{X_+(q^{-1})}{W(q^{-1})}\ , \tag{14}$$

where $W(q^{-1})$ denotes a stable, non-minimum phase causal transfer function obtained by spectral decomposition of power spectral density of $u(t)$, further denoted as $S_u(q^{-1})$

$$W(q^{-1})W(q) = S_u(q^{-1}) = \frac{\sigma_n^2}{2\pi A(q^{-1})A(q)} + \frac{\sigma_{e_I}^2}{2\pi}C(q^{-1})C(q) \tag{15}$$

and $X_+(q^{-1})$ is the causal part [correspondingly, $X_-(q)$ denotes the anticausal part] of crosspectral density between $\omega(t)$ and $u(t)$

$$X(q^{-1}) = X_+(q^{-1}) + X_-(q) = \frac{\sigma_n^2}{2\pi A(q^{-1})A(q)W(q)}\ . \tag{16}$$

Note that $W(q^{-1})$ may be represented in the following form

$$W(q^{-1}) = \frac{V(q^{-1})}{A(q^{-1})}\ , \tag{17}$$

where

$$V(q^{-1}) = v_0 + v_1 q^{-1} + \cdots + v_{n_A+1}q^{-n_A-1}$$

is a polynomial of order $n_A + 1$ such that

$$V(q^{-1})V(q) = \frac{\sigma_n^2}{2\pi} + \frac{\sigma_{e_I}^2}{2\pi}A(q^{-1})A(q)C(q^{-1})C(q)\ . \tag{18}$$

It follows that $X(q^{-1})$, $X_+(q^{-1})$, and $X_-(q)$ take the following forms

$$X(q^{-1}) = X_+(q^{-1}) + X_-(q) = \frac{\sigma_n^2}{2\pi A(q^{-1})V(q)}$$
$$X_+(q^{-1}) = \frac{Y_+(q^{-1})}{A(q^{-1})} \qquad X_-(q) = q\frac{Y_-(q)}{V(q)} \tag{19}$$

where $Y_+(q^{-1})$ and $Y_-(q)$ are polynomials of q^{-1} and q, respectively.

Combining equation (14) with (19) leads to

$$L_\omega(q^{-1}) = \frac{Y_+(q^{-1})}{V(q^{-1})}\ . \tag{20}$$

The following two properties, which both stem from eq. (18), will prove useful

$$\frac{\sigma_n^2}{2\pi} = V(q^{-1})V(q)\Big|_{q=1} = V(1)V(1)$$
$$\frac{\sigma_{e_I}^2}{2\pi} = v_{n_A+1}\ . \tag{21}$$

We are now in a position to move on to minimization of (13). First, factor $L(q^{-1})$ as

$$L(q^{-1}) = \frac{\Lambda_a(q^{-1})H(q^{-1})}{W(q^{-1})}\ , \tag{22}$$

where $H(q^{-1}) = \sum_{k=0}^{\infty} h(k)q^{-k}$ is a causal transfer function whose exact form is yet to be found. Note that the adopted factorization does not impose any constraints other than $L(q^{-1})$ being causal – a proper choice of $H(q^{-1})$ can always "undo" both $\Lambda_a(q^{-1})$ and $W(q^{-1})$. The purpose of introducing $\Lambda_a(q^{-1})$ and $W(q^{-1})$ into (22) is that enables one to cancel some terms in the formulas which will appear during analysis, thus making it simpler.

Substituting (22) and (A2) into (13) yields

$$J = \int_0^{2\pi} \frac{b_0^2\lambda_a^2}{|\Lambda_a(e^{-j\omega})|^2} \times \Bigg[\left|\frac{\Lambda_a(e^{-j\omega})H(e^{-j\omega})}{W(e^{-j\omega})} - 1\right|^2 \frac{\sigma_n^2}{2\pi|A(e^{-j\omega})|^2} + \left|\frac{\Lambda_a(e^{-j\omega})H(e^{-j\omega})C(e^{-j\omega})}{W(e^{-j\omega})}\right|^2 \frac{\sigma_{e_I}^2}{2\pi} + 2\frac{\mu_a^2}{\lambda_a^2}\frac{\sigma_{e_I}^2}{2\pi} + \frac{\mu_a}{\lambda_a}\frac{\Lambda_a(e^{-j\omega})H(e^{-j\omega})C(e^{-j\omega})}{W(e^{-j\omega})}\frac{\sigma_{e_I}^2}{2\pi} + \frac{\mu_a}{\lambda_a}\frac{\Lambda_a^*(e^{-j\omega})H^*(e^{-j\omega})C^*(e^{-j\omega})}{W^*(e^{-j\omega})}\frac{\sigma_{e_I}^2}{2\pi}\Bigg] d\omega\ . \tag{23}$$

Taking a complex-real derivative [7] of J with respect to h_k^*, equating to 0, and moving all terms without $H(e^{-j\omega})$ to the right hand side leads to the following necessary condition

$$\int_0^{2\pi} \frac{H(e^{-j\omega})}{|W(e^{-j\omega})|^2}\left[\frac{\sigma_n^2}{2\pi|A(e^{-j\omega})|^2} + |C(e^{-j\omega})|^2\frac{\sigma_{e_I}^2}{2\pi}\right]e^{j\omega k}d\omega$$
$$= \int_0^{2\pi}\frac{\sigma_n^2}{2\pi\Lambda_a(e^{-j\omega})W(e^{j\omega})|A(e^{-j\omega})|^2}e^{j\omega k}d\omega -$$
$$\int_0^{2\pi}\frac{\mu_a}{\lambda_a}\frac{\sigma_{e_I}^2 C(e^{j\omega})}{2\pi\Lambda_a(e^{-j\omega})W(e^{j\omega})}e^{j\omega k}d\omega \qquad k = 0, 1, \ldots$$

where we also employed the fact that $W^*(e^{-j\omega}) = W(e^{j\omega})$ and, similarly, $C^*(e^{-j\omega}) = C(e^{j\omega})$.

Observe that the term in braces on the left hand side equals $S_u(e^{-j\omega})$ [c.f. (15)]. This immediately simplifies the left hand side to

$$\int_0^{2\pi} H(e^{-j\omega})e^{j\omega k}d\omega = 2\pi h(k), \quad k = 0, 1, \ldots$$

Similarly, using eq. (16) transforms the right hand side to

$$\int_0^{2\pi}\frac{X_+(e^{-j\omega})}{\Lambda_a(e^{-j\omega})}e^{j\omega k}d\omega + \int_0^{2\pi}\frac{X_-(e^{j\omega})}{\Lambda_a(e^{-j\omega})}e^{j\omega k}d\omega +$$
$$-\int_0^{2\pi}\frac{\mu_a}{\lambda_a}\frac{\sigma_{e_I}^2 C(e^{j\omega})}{2\pi\Lambda_a(e^{-j\omega})W(e^{j\omega})}e^{j\omega k}d\omega \qquad k = 0, 1, \ldots \tag{24}$$

Therefore, we can say that the transfer function $H(q^{-1})$ is a sum of three terms

$$H(q^{-1}) = H_1(q^{-1}) + H_2(q^{-1}) + H_3(q^{-1}) \ , \tag{25}$$

corresponding to the integrals in (24), whose exact forms are yet to be found.

Caution is required at this stage, because $H_1(q^{-1})$, $H_2(q^{-1})$, $H_3(q^{-1})$ are all causal transfer functions. This stems from the fact that we restrict k in (22) to nonnegative values only. However, the integrals include noncausal terms, such as $X_-(e^{j\omega})$ or $C(e^{j\omega})$ – the resulting noncausal parts of inverse Fourier transforms should be discarded. The "right" solution to this problem is (see [6] for details)

$$\begin{aligned} H_1(q^{-1}) &= \frac{X_+(q^{-1})}{\Lambda_a(q^{-1})} \\ H_2(q^{-1}) &= \frac{X_-(\lambda_a)}{\Lambda_a(q^{-1})} \\ H_3(q^{-1}) &= -\frac{\mu_a}{\lambda_a}\frac{\sigma_{e_I}^2 C(\lambda_a)}{2\pi W(\lambda_a)\Lambda_a(q^{-1})} \ , \end{aligned} \tag{26}$$

where $X_-(\lambda_a)$ denotes the value of $X_-(q)$ evaluated at $q = \lambda_a$, $X_-(\lambda_a) = X_-(q)|_{q=\lambda_a}$. Similarly, $C(\lambda_a) = C(q)|_{q=\lambda_a}$ and $W(\lambda_a) = W(q)|_{q=\lambda_a}$.

Tracing back to $L(q^{-1})$ leads to [c.f. (11)]

$$\begin{aligned} L(q^{-1}) &= L_1(q^{-1}) + L_2(q^{-1}) + L_3(q^{-1}) \\ L_1(q^{-1}) &= \frac{X_+(q^{-1})}{W(q^{-1})} = L_\omega(q^{-1}) \\ L_2(q^{-1}) &= \frac{X_-(\lambda_a)}{W(q^{-1})} \\ L_3(q^{-1}) &= -\frac{\mu_a}{\lambda_a}\frac{\sigma_{e_I}^2 C(\lambda_a)}{2\pi W(\lambda_a)W(q^{-1})} \ . \end{aligned} \tag{27}$$

Using equations (16), (17), (20) and (21) allows one to rewrite $L_2(q^{-1})$ and $L_3(q^{-1})$ as (see [6] for details of the derivation)

$$\begin{aligned} L_2(q^{-1}) &= \left[\frac{V(1)V(1)}{V(\lambda_a)V(\lambda_a^{-1})} - L_\omega(\lambda_a^{-1})\right]\frac{V(\lambda_a^{-1})}{A(\lambda_a^{-1})}\frac{A(q^{-1})}{V(q^{-1})} \\ L_3(q^{-1}) &= -\frac{\mu_a}{\lambda_a}v_{n_A+1}C(\lambda_a)\frac{A(\lambda_a)}{V(\lambda_a)}\frac{A(q^{-1})}{V(q^{-1})} \ , \end{aligned} \tag{28}$$

where $V(\lambda_a^{-1}) = V(q^{-1})|_{q=\lambda_a}$, $A(\lambda_a^{-1}) = A(q^{-1})|_{q=\lambda_a}$, $L_\omega(\lambda_a^{-1}) = L_\omega(q^{-1})|_{q=\lambda_a}$.

3.3. Finalized algorithm

Suppose that (A1)-(A3) hold, $A(q^{-1}) = 1 - q^{-1}$ and that the pilot filter is well tuned, i.e. $F(q^{-1})$ in (10) equals $L_\omega(q^{-1})$ [2]. In such case the nominator and denominator of $F(q^{-1})$, further denoted as $N(q^{-1})$ and $D(q^{-1})$, must be equal, up to a constant multiplicative term α, to $Y_+(q^{-1})$ and $V(q^{-1})$,

$$F(q^{-1}) = \frac{N(q^{-1})}{D(q^{-1})} = \frac{\alpha Y_+(q^{-1})}{\alpha V(q^{-1})} \ . \tag{29}$$

Moreover, since the estimates yielded by the pilot filter take the form $\hat{\omega}(t|t) = L_\omega(q^{-1})u(t)$, the postprocessor can be reduced to [c.f. (6), (11), (28)]

$$\bar{\omega}(t|t) = \left[1 + (l_2 + l_3)\frac{A(q^{-1})}{N(q^{-1})}\right]\hat{\omega}(t|t) \tag{30}$$

$$\begin{aligned} l_2 &= \left[\frac{D(1)D(1)}{D(\lambda_a)D(\lambda_a^{-1})} - F(\lambda_a^{-1})\right]\frac{D(\lambda_a^{-1})}{A(\lambda_a^{-1})} \\ l_3 &= -\frac{\mu_a}{\lambda_a}\lambda C(\lambda_a)\frac{A(\lambda_a)}{D(\lambda_a)} \ . \end{aligned} \tag{31}$$

4. SIMULATION RESULTS

The simulated system, excited using a white 4-QAM sequence with $\mathbf{\Phi} = 2\boldsymbol{I}$, took the form

$$\begin{aligned} y(t) &= [\beta_1(t)\ \beta_2(t)]e^{j\sum_{\tau=1}^{t}\omega_i(\tau)} + v(t) \\ \beta_1(t) &= \left[1 + 0.5\sin\left(\frac{2\pi t}{2000}\right)\right](2 - j) \\ \beta_2(t) &= \left[1 + 0.5\cos\left(\frac{2\pi t}{2000}\right)\right](1 + 2j) \\ \omega(t) &= 0.1 + 0.5\sin\left(\frac{2\pi t}{2000}\right) \ . \end{aligned}$$

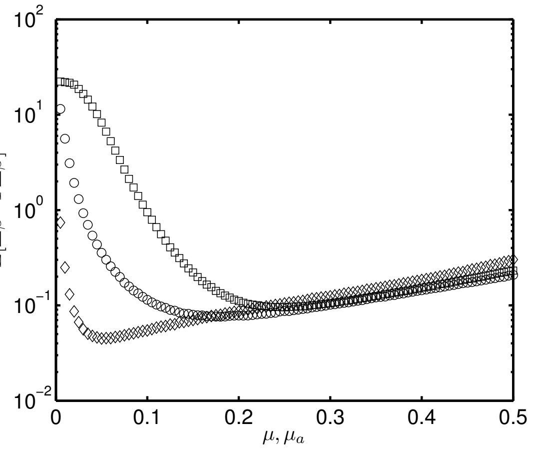

Fig. 2. Comparison of steady state mean-squared coefficient tracking errors yielded by three GANF variants for different values of adaptation gains under high SNR: Single-stage GANF (squares),Multistage GANF without frequency postprocessor (circles), Multistage GANF with frequency postprocessor (diamonds).

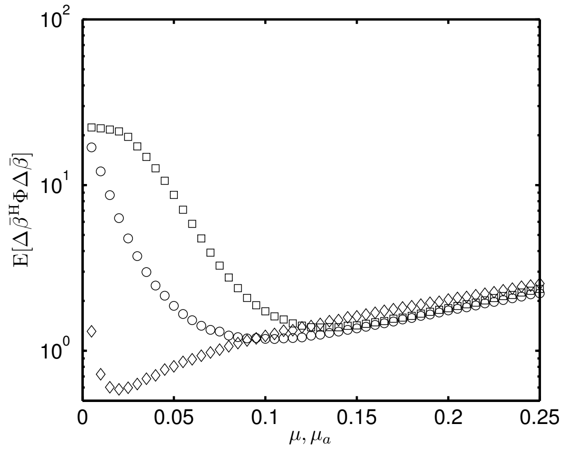

Fig. 3. Comparison of steady state mean-squared coefficient tracking errors yielded by three GANF variants for different values of adaptation gains under low SNR: Single-stage GANF (squares), Multistage GANF without frequency postprocessor (circles), Multistage GANF with frequency postprocessor (diamonds).

Three approaches were compared: the single-stage GANF (5), the cascade of the pilot GANF followed by the frequency guided filter (without the frequency postprocessor), and the proposed solution.

In the first case the adaptation gain μ took several values the interval $[0,\ 0.25]$; each time γ was set to $\mu^2/2$. In the case of the two multistage solutions, the optimal values of adaptation gains of the pilot filter were found using exhaustive numerical search in the interval $\mu \in [0,\ 0.5]$ with $\gamma = \mu^2/2$ and then distorted by a factor of two. This was made on purpose, to make the simulation more realistic. The parameter μ_a of the frequency guided filter took several values from the interval $[0,\ 0.25]$.

The simulation experiment was performed both for high and low values of signal to noise ratio. In the first case, the measurement noise variance was equal to $\sigma_v^2 = 0.2$ while in the low SNR case σ_v^2 was set to 6.32

The results of the simulation, depicted in Fig. 2 and Fig. 3, show that the proposed approach delivers the best performance, despite the fact that the pilot filter was deliberately mistuned. Furthermore, note that the performance gains are almost two-fold and that the overall 'operating window' of the proposed algorithm is much wider. These factors demonstrate the robustness of the proposed multistage GANF.

5. CONCLUSIONS

The problem of estimating coefficients of a quasi-periodic system was considered. The proposed approach is based on a previously introduced GANF and the concept of multistage processing. It consists of the pilot filter, which works out frequency estimates, a frequency postprocessor, which takes the form of a nontrivial linear filter, and a frequency guided filter, which tracks coefficients of the system of interest.

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