ASYMPTOTIC NORMALITY OF CYCLIC AUTOCORRELATION ESTIMATE WITH ESTIMATED CYCLE FREQUENCY

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ABSTRACT

For an almost-cyclostationary signal, mean-square consistent and asymptotically complex normal estimators of the cyclic statistics exist, provided that the signal has finite or practically finite memory and the cycle frequency is perfectly known. In the paper, conditions are derived to obtain a mean-square consistent and asymptotically complex normal estimator of the cyclic autocorrelation function with estimated cycle frequency. For this purpose, a new lemma on conditioned cumulants of complex-valued random variables is derived. As an example of application, the problem of detecting a rapidly moving source emitting a cyclostationary signal is addressed and the case of a low Earth orbit satellite considered.

Index Terms— Cyclostationarity; Asymptotic Normality; Doppler effect

1. INTRODUCTION

Cyclostationarity properties of modulated signals have been successfully exploited in several problems in communications and radar/sonar [8]. In particular, in recent years, cyclostationarity-based detectors have been adopted in spectrum sensing for cognitive radio [1], [10], [11], [12], [13], [17], [19], [20].

If a cycle frequency of the signal-of-interest exists which is not shared with the interfering signals, then cyclostationarity-based detection and estimation algorithms provide satisfactory performance, provided that a sufficiently long observation interval is adopted for the estimation of the cyclic statistical functions. In fact, under mild assumptions on the finite or practically finite memory of the process and on the lack of cycle frequency cluster points, mean-square consistent and asymptotically complex normal estimators of the cyclic statistical functions exist [4], [5], [7, Chap. 15], [14, Sec. 2.4.2]. These estimators assume that the cycle frequency is exactly known. In the case of non perfect knowledge of the cycle frequency, the estimates are significantly asymptotically biased.

In the paper, the problem of estimating the cyclic autocorrelation function at a value of cycle frequency which is in turn an estimate is addressed. Conditions are provided such that the cyclic correlogram with estimated cycle frequency is a mean-square consistent and asymptotically complex normal estimate of the cyclic autocorrelation function. For this purpose, a new lemma on conditioned cumulants is proved.

As application of the obtained results, the statistical test for presence of cyclostationarity introduced in [3] and exploited in cognitive radio in [10], [13], [17], is considered to detect the presence of a cyclostationary moving source.

The techniques in [3], [10], [13], [17] exploit the knowledge of a single or multiple cycle frequencies and of a single or multiple lags at which the cyclic autocorrelation function of the signal to be detected is significantly non zero. When the cycle frequency is deterministic and perfectly known, the cyclic autocorrelation estimate is asymptotically complex normal and a significance test on a properly normalized version of the estimate can be performed [3].

In the case of a rapidly moving transmitter, the transmitted cycle frequency is modified at the receiver due the Doppler effect. In such a case, an estimated cycle frequency should be used in the detection statistic. By exploiting the results of the paper, one is guaranteed that the asymptotic normality of the cyclic autocorrelation estimate is preserved even if the cycle frequency is not perfectly known, provided that the estimate is sufficiently accurate. Thus, detectors designed assuming a known cycle frequency can be adopted by plugging the cycle frequency estimate in place of the known cycle frequency. In the paper, it is shown that a sufficient accuracy is achieved by the cycle frequency estimator proposed in [2]. As an example, the problem of detection the signal transmitted by a low earth orbit (LEO) satellite is considered.

2. ALMOST-CYCLOSTATIONARY PROCESSES

2.1. Statistical Characterization

A second-order complex-valued stochastic process $x(t)$ is said to be *almost-cyclostationary (ACS)* in the wide sense if its first- and second-order moments are almost-periodic functions of t. Thus, for the second-order moments the following

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(generalized) Fourier series expansion holds

$$\mathcal{R}_{\boldsymbol{x}}(t,\tau) \triangleq \mathrm{E}\big\{x(t+\tau)\, x^{(*)}(t)\big\} = \sum_{\alpha\in A} R_{\boldsymbol{x}}^{\alpha}(\tau)\, e^{j2\pi\alpha t}\,. \quad (1)$$

In (1), superscript (*) denotes an optional complex conjugation and subscript $\boldsymbol{x} \triangleq [x\, x^{(*)}]$. If the conjugation is present, then $\mathcal{R}_{xx^*}(t,\tau)$ is the *autocorrelation function*. If the conjugation is absent, then $\mathcal{R}_{xx}(t,\tau)$ is the *conjugate autocorrelation function*. Both functions are necessary for a complete second-order characterization in the wide-sense of complex-valued processes [16]. The set A is the countable set, depending on $(*)$, of the possibly incommensurate (conjugate) cycle frequencies α, and the Fourier coefficients

$$R_{\boldsymbol{x}}^{\alpha}(\tau) \triangleq \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \mathcal{R}_{\boldsymbol{x}}(t,\tau)\, e^{-j2\pi\alpha t}\mathrm{d}t \qquad (2)$$

are referred to as *(conjugate) cyclic autocorrelation functions*. If $A = \{k/T_0\}_{k\in\mathbb{Z}}$, for some $T_0 > 0$, the process $x(t)$ is said to be *cyclostationary* with period T_0 [8].

ACS processes are an appropriate model for almost-all modulated signals adopted in communications and radar/sonar [8].

2.2. Cyclic Autocorrelation Estimator

The natural estimator of the (conjugate) cyclic autocorrelation function at cycle frequency α is the *(conjugate) cyclic correlogram*

$$R_{\boldsymbol{x}}^{(T)}(\alpha,\tau) \triangleq \frac{1}{T}\int_{-T/2}^{T/2} x(t+\tau)\, x^{(*)}(t)\, e^{-j2\pi\alpha t}\mathrm{d}t\,. \quad (3)$$

Let as consider a zero-mean process $x(t)$ satisfying the following assumptions for every conjugation configuration in the definitions of kth-order statistics for $k \geqslant 2$.

AS1. The 2nd- and 4th-order cumulants are uniformly almost-periodic functions of t for all values of the lag parameters [14, As. 2.4.2] with sequences of the suprema of the Fourier coefficients that are summable [14, As. 2.4.3].

AS2. For every k, the kth-order moments are bounded [14, As. 2.4.4, 2.4.16].

AS3. For every k, the kth-order cumulants are summable functions with respect to the lag parameters [14, As. 2.4.8, 2.4.15].

AS4. There is no cluster of cycle frequencies [14, As. 2.4.10].

The asymptotic complex normality of the cyclic correlogram is proved in [14, Sec. 2.4.2] for the generalized almost-cyclostationary processes. In the special case of ACS process such a result specializes into the following one. See also [4], [5], [7, Chap. 15].

Theorem 2.1 *Under Assumptions AS1–AS4, the (conjugate) cyclic correlogram $R_{\boldsymbol{x}}^{(T)}(\alpha,\tau)$ is a mean-square consistent estimator of the (conjugate) cyclic autocorrelation function $R_{\boldsymbol{x}}^{\alpha}(\tau)$. Moreover, the random variables*

$$V_i^{(T)} \triangleq \sqrt{T}\big[R_{\boldsymbol{x}}^{(T)}(\alpha_i,\tau_i) - R_{\boldsymbol{x}}^{\alpha_i}(\tau_i)\big] \quad i = 1,\dots,M \quad (4)$$

are asymptotically jointly complex normal. □

2.3. Cycle Frequency Estimators

In this section, (conjugate) cycle frequency estimators that are considered in the following are briefly described and their properties summarized.

Let $D(\alpha_0,\delta\alpha) \triangleq [\alpha_0 - \delta\alpha, \alpha_0 + \delta\alpha]$ be an interval containing only one (conjugate) cycle frequency, say α_0. That is, if $\alpha \in D(\alpha_0,\delta\alpha)$, then $R_{\boldsymbol{x}}^{\alpha}(\tau) \not\equiv 0$ only for $\alpha = \alpha_0$.

In [2] the discrete-time counterpart of the estimator

$$\alpha^{(T)} = \arg\max_{\alpha\in D(\alpha_0,\delta\alpha)} \int_{-\tau_{\mathrm{M}}}^{\tau_{\mathrm{M}}} \big|R_{\boldsymbol{x}}^{(T)}(\alpha,\tau)\big|^2\mathrm{d}\tau \qquad (5)$$

is proposed, where τ_{M} is such that $R_{\boldsymbol{x}}^{\alpha_0}(\tau)$ is significantly nonzero in $[-\tau_{\mathrm{M}},\tau_{\mathrm{M}}]$. It is shown that under mild regularity assumptions expressed in terms of summability of cumulants, for $T\to\infty$ it results $T^{3/2}\big[\alpha^{(T)}-\alpha_0\big] \to \mathcal{N}(0,\overline{\sigma}^2)$ with $\overline{\sigma}^2 = \lim_{T\to\infty} T^3\,\mathrm{E}\{|\alpha^{(T)}-\alpha_0|^2\}$. Moreover, $T\big[\alpha^{(T)}-\alpha_0\big] \to 0$ almost surely.

Other (conjugate) cycle frequency estimators that have been considered for cognitive radio applications are the cycle frequency domain profile (CDP) [12], and the estimator proposed in [11]. The latter is an approximate and computationally efficient version of the estimator in [2].

3. CYCLIC CORRELOGRAM WITH ESTIMATED CYCLE FREQUENCIES

The proof of the asymptotic complex normality of the (conjugate) cyclic correlogram with estimated cycle frequency is based on the following lemma.

Lemma 3.2 *Let $X_1,\dots,X_k$, and Y be complex-valued random variables. The kth-order cumulant of $X_1,\dots,X_k$ can be expressed as*

$$\begin{aligned}
&\mathrm{cum}\,\{X_1,\dots,X_k\}\\
&\quad= \mathrm{E}_Y\Big\{\mathrm{cum}\,\{X_1,\dots,X_k \mid Y\}\Big\}\\
&\qquad -\sum_{\substack{\boldsymbol{P}\\ p>1}}(-1)^{p-1}(p-1)!\,\Bigg[\mathrm{E}_Y\Big\{\prod_{i=1}^{p}\mathrm{E}_{X_{\mu_i}|Y}\Big\{\prod_{\ell\in\mu_i} X_\ell|Y\Big\}\Big\}\\
&\qquad -\prod_{i=1}^{p}\mathrm{E}_Y\Big\{\mathrm{E}_{X_{\mu_i}|Y}\Big\{\prod_{\ell\in\mu_i} X_\ell|Y\Big\}\Big\}\Bigg]
\end{aligned} \qquad (6)$$

where $\boldsymbol{P}$ *is the set of all distinct partitions of* $\{1, \ldots, k\}$*, each constituted by the subsets* $\mu_1, \ldots, \mu_p$*, and* X_{μ_i} *is the* $|\mu_i|$*-dimensional vector whose entries have indices in* μ_i*,* $|\mu_i|$ *being the size of the set* μ_i*.*

Proof: *For cumulants of complex random variables defined according to [14, Sec. 1.4.2], [18, App. A], the Leonov-Shiryaev relationship [9] links cumulants and moments:*

$$\operatorname{cum}\{X_1, \ldots, X_k\} = \sum_{\boldsymbol{P}} (-1)^{p-1}(p-1)! \prod_{i=1}^{p} \mathrm{E}\Big\{ \prod_{\ell \in \mu_i} X_\ell \Big\} \tag{7}$$

By expressing the moments in (7) as moments conditioned to Y *and then averaged over* Y*, and accounting for the fact that* $\mu_i \bigcap \mu_j = \emptyset$ *for* $i \neq j$*, after some manipulations (6) follows. The result can be easily extended to the case of* Y *random vector.* □

Theorem 3.3 *Let* $\alpha_i^{(T)}$ *be estimators of the (conjugate) cycle frequencies* $\alpha_{0,i}$ *defined accordingly with (5), based on non-overlapping (conjugate) cycle frequency sets* $D_i(\alpha_{0,i}, \delta\alpha_i)$*, and converging almost-surely to* $\alpha_{0,i}$*,* $i = 1, \ldots, M$ *as* $T \to \infty$*. Under assumptions AS1–AS4, the random variables*

$$V_i^{(T)} \triangleq \sqrt{T}\Big[R_{\boldsymbol{x}}^{(T)}\big(\alpha_i^{(T)}, \tau_i\big) - R_{\boldsymbol{x}}^{\alpha_{0,i}}(\tau_i)\Big] \quad i = 1, \ldots, M \tag{8}$$

are asymptotically ($T \to \infty$) *zero-mean jointly complex normal with asymptotic covariance matrix with entries* Σ_{ij} *and asymptotic conjugate covariance matrix with entries* $\Sigma_{ij}^{(c)}$ *having the same expressions [14, eqs. (2.146) and (3.137)] for the case of known (conjugate) cycle frequencies.*

Proof: *The proof is only sketched here for the lack of space. Starting from the asymptotic complex normality of the random variables* $V_i^{(T)}$*s conditioned to the* $\alpha_i^{(T)}$*s, as* $T \to \infty$ *we have*

1) $\mathrm{E}\{V_i^{(T)}\} \to 0$*;*

2a) $\operatorname{cov}\{V_i^{(T)}, V_j^{(T)}\} \to \Sigma_{ij}$ *finite;*

2b) $\operatorname{cov}\{V_i^{(T)}, V_j^{(T)*}\} \to \Sigma_{ij}^{(c)}$ *finite;*

3) $\operatorname{cum}\{V_1^{(T)}, \ldots, V_k^{(T)}\} \to 0$ *for* $k \geqslant 3$.

1) is proved starting from the expression of the bias of the (conjugate) cyclic correlogram [14, Theorem 2.4.6] and exploiting the properties of the estimate $\alpha_i^{(T)}$*.*

2a) and 2b) are proved starting from the expressions of the covariance and conjugate covariance of the (conjugate) cyclic correlograms [14, Theorems 2.4.7 and 3.7.1] and using Lemma 3.2 with $k = 2$*.*

3) is proved using [14, Lemma 2.4.17], Lemma 3.2 with $k \geqslant 3$*, and exploiting the asymptotic independence of* $\alpha_i^{(T)}$ *and* $\alpha_j^{(T)}$ *for* $i \neq j$*.*

Thus, the logarithm of the joint characteristic function of $V_i^{(T)}$*,* $i = 1, \ldots, k$*, is a quadratic form as* $T \to \infty$*. That is, the* $V_i^{(T)}$ *are asymptotically complex normal.* □

4. NUMERICAL RESULTS

The empirical marginal cumulative distribution functions (CDFs) of the real and imaginary parts of $V_i^{(T)}$, normalized to the respective variances σ_{R} and σ_{I}, are evaluated by 1000 Monte Carlo trials for a data-record length $T = 2^{13}T_s$, where T_s is the sampling period. The signal $x(t)$ is a pulse-amplitude modulated signal with binary stationary white modulating sequence, raised cosine pulse with excess bandwidth $\eta = 0.35$, and bit period $T_p = 8T_s$.

In Fig. 1, results for known cycle frequency $\alpha = \alpha_{\text{true}}$ ($V_i^{(T)}$ as in (4)) and estimated cycle frequency $\alpha = \alpha^{(T)}$ ($V_i^{(T)}$ as in (8)), for $\tau = 0$, are compared with the CDF of a standard normal random variable. Accordingly with the results of Theorem 3.3, the curves of the CDFs for estimated cycle frequency practically overlap those for known cycle frequency and both very accurately fit the CDF of a standard normal random variable.

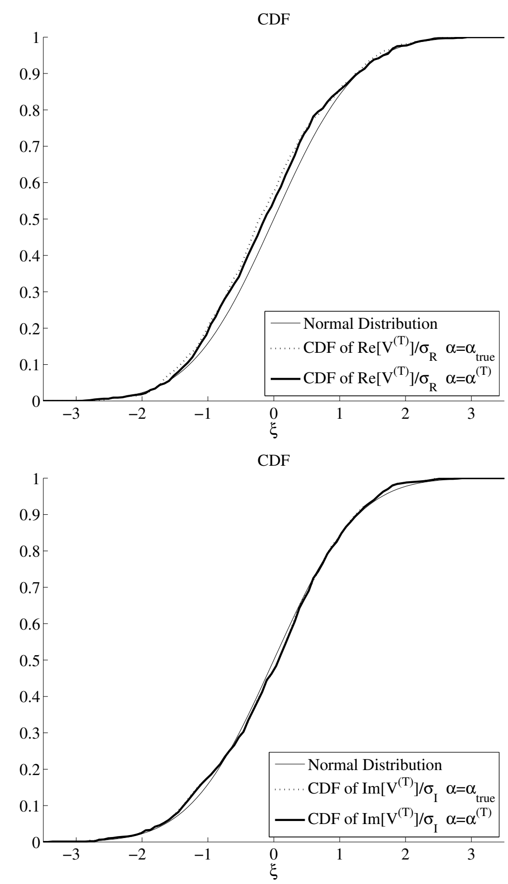

Fig. 1. Marginal CDF of the real and imaginary parts of $V_i^{(T)}$, normalized to the respective variances σ_{R} and σ_{I}, for known cycle frequency $\alpha = \alpha_{\text{true}}$ and estimated cycle frequency $\alpha = \alpha^{(T)}$, with $\tau = 0$, compared with the CDF of a standard normal random variable.

5. APPLICATION TO THE DETECTION OF A MOVING SOURCE

In this section, as an example of application, the problem of detecting a moving cyclostationary source is addressed.

Let us consider a LEO satellite at altitude $h = 200$ km and with orbital speed $\mathrm{v}_o = 28061.5$ km h^{-1}. It transmits a direct-sequence spread-spectrum (DSSS) signal with pseudo-noise (PN) modulating sequence, chip period $T_p = 0.06$ μs, bit period $T_b = N_c T_p$, with $N_c \geqslant 2$, and carrier frequency $f_c = 2$ GHz. Within observation intervals of interest in the applications, the relative radial speed v_ϱ between the satellite and a receiver on the Earth can be considered constant. Thus, due to the Doppler effect, the transmitted signal experiences a linearly time-varying delay. It reflects into a frequency shift of the carrier and a time scaling of the complex envelope. That is, denoted by $x(t)$ the complex envelope of the transmitted signal, the complex envelope of the received signal is given by

$$r(t) = a\, x(s(t-\tau_0))\, e^{j2\pi\nu t} + n(t)\,. \tag{9}$$

In (9), a is a complex gain that accounts for attenuation and possible mismatch between the phases of the transmitter and receiver oscillators, τ_0 is the propagation delay, s is a time-scale or time-stretch factor given by $s = 1 - v_\varrho/c$ with c medium propagation speed, and ν is a frequency shift due to the Doppler effect on the carrier and possible frequency mismatch between transmitter and receiver oscillators. The term $n(t)$ is a disturbance signal that accounts for both additive Gaussian noise and nonstationary interference.

The statistical test for presence of cyclostationarity presented in [3] compares with a threshold the test statistic

$$\mathcal{T} = T\, \boldsymbol{z}^{\mathrm{T}}(\alpha,\tau)\, \boldsymbol{C}^{-1}(\alpha,\tau)\, \boldsymbol{z}(\alpha,\tau) \tag{10}$$

where $\boldsymbol{z}(\alpha,\tau)$ is the column vector whose entries are the real and imaginary parts of $R^{(T)}_{rr^*}(\alpha,\tau)$ and $\boldsymbol{C}(\alpha,\tau)$ is a consistent estimate of the covariance matrix of $\boldsymbol{z}(\alpha,\tau)$. If $\mathcal{T}$ exceeds the threshold, then $r(t)$ is declared to exhibit cyclostationarity at (α,τ) (hypothesis H_1 verified); if $\mathcal{T}$ is below the threshold, then $r(t)$ is declared to not exhibit cyclostationarity at (α,τ) (hypothesis H_0 verified). Starting from a desired false-alarm probability P_{fa}, the threshold can be analytically evaluated since the test statistic $\mathcal{T}$ has a χ^2_2 distribution under H_0 due to the asymptotic complex normality of $V^{(T)} = \sqrt{T}[R^{(T)}_{rr^*}(\alpha,\tau) - R^{\alpha}_{rr^*}(\tau)]$.

In the case of no motion between transmitter and receiver, the test can be performed by estimating the cyclic autocorrelation function at the pair $(\alpha_0,\tau) = (1/T_p, T_p/2)$ since for these values of cycle frequency and lag, the DSSS signal $x(t)$ with PN modulating sequence and rectangular pulse exhibits a strong degree of cyclostationarity [8].

In the case of relative motion between transmitter and receiver, if α_0 is a cycle frequency of $x(t)$ then, due to the Doppler effect, the received signal exhibits cyclostationarity at cycle frequency $\alpha = s\alpha_0$ [14, Sec. 7.7]. The value of s is close to 1 in practical applications and the cycle frequency α at the receiver can be approximated to α_0 only if

$$|\alpha - \alpha_0| = |1-s|\,|\alpha_0| \ll 1/T \tag{11}$$

where $1/T$ is the cycle frequency resolution for a data-record length T.

In the experiment, additive white Gaussian noise (AWGN) $n(t)$ is present with slowly varying power spectral density level. The average SNR ranges from –24 dB to –12 dB. In order to obtain a reliable cyclic autocorrelation estimate and, hence, a satisfactory detection performance, a data record length $T = N_b T s$ is assumed, with $N_b = 2^{17}$ number of processed bits and $T_s = T_p/4$ sampling period. The value of $1-s$ is of the order of 10^{-5} in the considered satellite application [15]. Consequently, (11) is not satisfied and the cycle frequency at receiver needs to be estimated.

The cycle frequency estimator proposed in [2] (Section 2.3) with $\delta\alpha = (1/T_p)|v_\varrho|_{\max}/c$, $\tau_{\mathrm{M}} = T_p$, is adopted and the estimate $\alpha^{(T)}$ is plugged into the test statistics (10). The covariance matrix $\boldsymbol{C}$ is estimated by a subsampling technique accordingly with [14, Secs. 2.6.4.1, 6.3.5]. The consistency of the estimate can be proved by following the guidelines in [6] for the distribution estimate.

In Fig. 2, the probability of detection P_{d}, evaluated over 1000 Monte Carlo trials, for $P_{\mathrm{fa}} = 0.01$, is presented for the test statistics (10) as a function of SNR. The following cases are considered: (o) known true cycle frequency ($\alpha = \alpha_{\mathrm{true}} = s\alpha_0$); ($*$) estimated cycle frequency ($\alpha = \alpha^{(T)}$); ($\times$) neglected Doppler effect ($\alpha = \alpha_0$). In addition, the performance of the energy detector ($\Diamond$) is also considered. The detector with estimated cycle frequencies has performance practically equivalent to that of the detector with known cycle frequency and performs well for SNR $\geqslant$ –16 dB. This is in accordance with the result of Theorem 3.3 that Σ_{ij} and $\Sigma^{(c)}_{ij}$ have the same expressions of the case of deterministic known cycle frequency. In contrast, the detector that neglects the Doppler effect and assumes as cycle frequency that of the transmitted signal, has very poor performance. The poor performance of the energy detector is due to the variability of the power spectral density level.

Simulation results not reported here for the lack of space show that satisfactory performance can be obtained with the CDP method [12] and a slightly worse performance with the estimator proposed in [11].

6. CONCLUSION

The asymptotic joint complex normality of the (conjugate) cyclic correlograms with estimated cycle frequencies is proved. For this purpose, cycle frequency estimates almost-surely convergent to the true values with sufficiently fast rate must be used. The covariance and conjugate covariance of

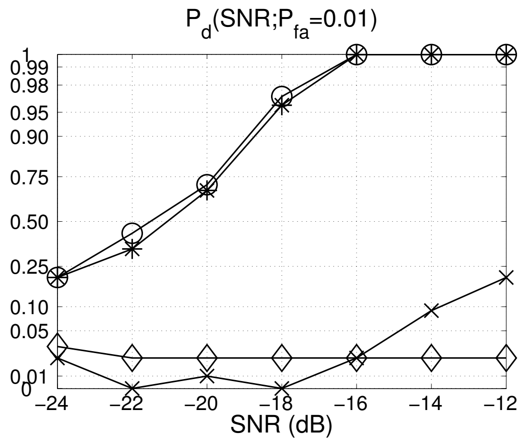

Fig. 2. P_{d} as a function of SNR for: (o) detector with true cycle frequency ($\alpha = \alpha_{\mathtt{true}} = s\alpha_0$); ($*$) detector with estimated cycle frequency ($\alpha = \alpha^{(T)}$); ($\times$) detector that neglects the Doppler effect ($\alpha = \alpha_0$); ($\Diamond$) energy detector.

the (conjugate) cyclic correlograms have the same expressions as in the case of perfectly known cycle frequencies. As an example of application, the detection of the signal emitted by a LEO satellite is considered. A widely adopted cyclostationarity-based detector that assumes the asymptotic normality of the estimate of the cyclic autocorrelation function is considered. It is shown that the performance of the detector designed for the case of known cycle frequency does not degrade if an accurate estimate of the cycle frequency is plugged into the test statistics in place of the known cycle frequency.

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