DETECTION OF TIME-VARYING SUPPORT VIA RANK EVOLUTION APPROACH FOR EFFECTIVE JOINT SPARSE RECOVERY

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ABSTRACT

Efficient recovery of sparse signals from few linear projections is a primary goal in a number of applications, most notably in a recently-emerged area of compressed sensing. The multiple measurement vector (MMV) joint sparse recovery is an extension of the single vector sparse recovery problem to the case when a set of consequent measurements share the same support. In this contribution we consider a modification of the MMV problem where the signal support can change from one block of data to another and the moment of change is not known in advance. We propose an approach for the support change detection based on the sequential rank estimation of a windowed block of the measurement data. We show that under certain conditions it allows for an unambiguous determination of the moment of change, provided that the consequent data vectors are incoherent to each other.

***Index Terms*—** sparse recovery, multiple measurement vector, time-varying support, stationarity window

1. INTRODUCTION

Recovery of sparse signals, i.e., signals that can be approximated by only few non-zero coefficients in some representation, from a small number of linear measurements is of great interest in a wide variety of applications ranging from image to array processing. In this paper we consider the multiple measurement vector (MMV) sparse signal representation which is a straightforward extension of a basic sparse signal model, in a similar manner referred to as single measurement vector (SMV), to a finite number of jointly sparse vectors sharing the same support, i.e., positions of their non-zero elements [1, 2]. It has been shown that such a jointly-sparse structure allows for efficient simultaneous MMV recovery with improved performance compared to an equivalent SMV setting [3–5].

Exploitation of the sparse joint recovery is however significantly challenged if the signal support exhibit variations over time which is a common case in many practical scenarios. A currently prevalent way to account for the possibly changing support is to incorporate a dynamic update mechanism of one form or another directly into the recovery step. While the details of the available algorithms may vary as well as the exact considered signal model, the basic idea is to apply recursive reconstruction that appropriately penalizes the presence of innovations in the newly-arrived measurements compared to already available ones [6–8]. In this contribution we propose a different approach to tackling the problem of time-varying support that does not require numerous iterative reconstruction steps: we first detect the moment of the support change based on the analysis of the measurement data, and then apply existing MMV recovery algorithms to a sequence of identified this way independent MMVs.

It has been shown that in the static MMV setup the signal sparsity can be assessed via estimation of the effective rank of the block of measurement data provided that the individual signal vectors are incoherent to each other [9]. Capitalizing on the results obtained in [9], we propose to use the rank information in order to estimate the moment of the support change in the quasi-static scenario considered here. To do so we analyze the evolution of the consequent rank estimates of a sliding window of measurement data and derive conditions on the window size that ensure a unique determination of the exact moment of change in a noise-free setting. Additionally we numerically show how the presence of such an estimate improves the performance of the support recovery step. Note that although we consider the specific class of signals that fulfill the incoherence assumption, [10] indicates that this restriction can be overcome by appropriate measurement design.

2. NOTATIONS AND MMV DATA MODEL

2.1. Some notations

First we introduce several notations to be used throughout the paper. A vector $\boldsymbol{x}$ of length N is called k-sparse if exactly $k \ll N$ of its entries are non-zero. The support of a vector is a set of the positions of its non-zero elements, while a support $\mathcal{S}_\mathrm{X}$ of an $N \times T$ matrix $\boldsymbol{X}$ is a union over all the supports of its columns $\boldsymbol{x}_i$ so that

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$$\mathcal{S}_{\mathrm{X}} = \mathrm{supp}\,(\boldsymbol{X}) \triangleq \bigcup_{i=1}^{T} \mathrm{supp}\,(\boldsymbol{x}_i)\,. \tag{1}$$

Finally, a matrix $\boldsymbol{X}$ is called jointly k-sparse if it has exactly k non-zero rows and all its columns are k-sparse, so that

$$\forall i \in [1,T] \quad \mathcal{S}_{\mathrm{X}} = \mathrm{supp}(\boldsymbol{x}_i). \tag{2}$$

2.2. Multiple measurement vector data model

The multiple measurement vector (MMV) data model is formulated as

$$\boldsymbol{Y} = \boldsymbol{B} \cdot \boldsymbol{X}, \tag{3}$$

where $\boldsymbol{Y} = [\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_T] \in \mathbb{C}^{M \times T}$ is a matrix of measurements $\boldsymbol{y}_i \in \mathbb{C}^{M\times 1}$ of some k-sparse signals $\boldsymbol{x}_i \in \mathbb{C}^{N\times 1}$. The matrix $\boldsymbol{B} \in \mathbb{C}^{M\times N}$ in (3) is the sensing matrix with $N > M > k$. Finally a collection of vectors $\boldsymbol{x}_i$ form a jointly k-sparse matrix $\boldsymbol{X}$, so that $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_T]$ and the support of $\boldsymbol{X}$ satisfies (2).

The goal of the MMV problem associated with (3) is to recover the matrix $\boldsymbol{X}$ from the measurements $\boldsymbol{Y}$ for a known sensing matrix $\boldsymbol{B}$ [1,5]. It can be formulated as

$$\hat{\boldsymbol{X}} = \underset{\boldsymbol{X}}{\mathrm{argmin}}\, |\mathrm{supp}\,(\boldsymbol{X})\,| \text{ s.t. } \boldsymbol{B}\cdot\boldsymbol{X} = \boldsymbol{Y}. \tag{4}$$

A sufficient and necessary condition for signal recovery in the MMV setting is known to be

$$|\mathrm{supp}(\boldsymbol{X})| = k < \frac{\mathrm{spark}(\boldsymbol{B}) - 1 + \mathrm{rank}(\boldsymbol{Y})}{2}, \tag{5}$$

where spark$(\boldsymbol{B})$ is the smallest number of columns of $\boldsymbol{B}$ that are linearly dependent [1]. Note that although (5) does not explicitly impose any conditions on the rank of $\boldsymbol{X}$ henceforth we focus on the best-case MMV scenario when rank$(\boldsymbol{X}) = |\mathcal{S}_{\mathrm{X}}| = k$, i.e., individual vectors $\boldsymbol{x}_i$ are non-collinear and $T > |\mathcal{S}_{\mathrm{X}}|$ [5]. It can be shown that (5) then becomes

$$\mathrm{spark}(\boldsymbol{B}) > k + 1. \tag{6}$$

2.3. MMV with quasi-static varying support

The jointly sparse input signal $\boldsymbol{X}$ from (3) can be seen as a collection of independent data snapshots $\boldsymbol{x}_i$. Obviously, the key feature allowing the usage of (4) for the recovery of $\boldsymbol{X}$ is that the support of each individual vector $\boldsymbol{x}_i$ does not change from snapshot to snapshot. However, in many practical applications, the support of the input signal can exhibit variations from one block of data to another, preserving the jointly sparse structure of (2) for some number of snapshots. Thus, we call such MMV signals with varying but quasi-static support "quasi-stationary." It is worth noting that although for the sake of convenience we commonly refer to the individual snapshots $\boldsymbol{x}_i$ as if they were taken in time, they could be as well interpreted as taken in frequency, space or any other domain.

In this contribution we examine the problem of joint sparse recovery of quasi-stationary MMV signals. For simplicity we consider a scenario where the support changes only once while an extension to an arbitrary number of such changes is straightforward. Thus, the input matrix $\boldsymbol{X}$ consists of two sub-matrices $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ which are k_1 and k_2 jointly sparse so that

$$\boldsymbol{X} = [\boldsymbol{X}_1 \boldsymbol{X}_2], \tag{7a}$$

$$|\mathcal{S}_{\mathrm{X}_1}| = k_1 \le k_{\max} \text{ and } |\mathcal{S}_{\mathrm{X}_2}| = k_2 \le k_{\max} \tag{7b}$$

$$\mathcal{S}_{\mathrm{X}_1} \neq \mathcal{S}_{\mathrm{X}_2}, \tag{7c}$$

where $\boldsymbol{X}_1 \in \mathbb{C}^{N\times t_1}$ and $\boldsymbol{X}_2 \in \mathbb{C}^{N\times t_2}$ with $t_1 > k_{\max}$ and $t_2 > k_{\max}$, $k_{\max}$ is the maximum possible sparsity order and $t_1 + t_2 = T$. If the instance t_1 when the support changes from $\mathcal{S}_{\mathrm{X}_1}$ to $\mathcal{S}_{\mathrm{X}_2}$, referred to as the stationarity window of $\boldsymbol{X}_1$, is perfectly known, the recovery of $\boldsymbol{X}$ can be formulated according to (4) as a set of two MMV problems:

$$\hat{\boldsymbol{X}} : \begin{cases} \hat{\boldsymbol{X}}_1 = \underset{\boldsymbol{X}_1}{\mathrm{argmin}}\, |\mathrm{supp}\,(\boldsymbol{X}_1)\,| \text{ s.t. } \boldsymbol{B}\cdot\boldsymbol{X}_1 = \boldsymbol{Y}_1 \\ \hat{\boldsymbol{X}}_2 = \underset{\boldsymbol{X}_2}{\mathrm{argmin}}\, |\mathrm{supp}\,(\boldsymbol{X}_2)\,| \text{ s.t. } \boldsymbol{B}\cdot\boldsymbol{X}_2 = \boldsymbol{Y}_2 \end{cases}, \tag{8}$$

where $\hat{\boldsymbol{X}} = [\hat{\boldsymbol{X}}_1 \hat{\boldsymbol{X}}_2]$, $\boldsymbol{Y}_1 = [\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_{t_1}]$, and $\boldsymbol{Y}_2 = [\boldsymbol{y}_{t_1+1}, \boldsymbol{y}_{t_1+2}, \ldots, \boldsymbol{y}_T]$.

However, when the exact moment of the support change is unknown or the change itself is not anticipated, direct application of (4) to the recovery of the signal of (7a)-(7c) will lead to a performance degradation. The reason behind this is that $\boldsymbol{X}$ from (7a)-(7c) is not exactly jointly sparse anymore. Therefore, in order to efficiently apply joint recovery of the form of (8) to the signals with supports that can vary over time, a method to estimate stationarity windows of individual signals is highly required.

3. DETECTION OF THE QUASI-STATIC VARYING SUPPORT VIA RANK EVOLUTION ANALYSIS

The main idea behind the proposed approach is to exploit the linear independence of the individual vectors $\boldsymbol{x}_i$ and formulate the task of the support change detection in terms of the rank estimation of a set of blocks of measurement data. To do so we analyze the relation between the resulting ranks of the consequent windows of measurement data and the supports of the corresponding blocks of input data depending on the size and the position of the blocks.

We begin by defining a window $\boldsymbol{Y}_p^{\mathrm{w}}$ as a block of measurement data of size $M \times m$ so that

$$\boldsymbol{Y}_p^{\mathrm{w}} = \boldsymbol{B} \cdot \boldsymbol{X}_p^{\mathrm{w}}, \tag{9}$$

where $\boldsymbol{X}_p^{\mathrm{w}} = [\boldsymbol{x}_p, \boldsymbol{x}_{p+1}, \ldots, \boldsymbol{x}_{p+m-1}]$, $p \in [1, T-m+1]$, and the window size $m > 1$ is a parameter we choose. From

Window size m		Range of index p	Value of $\lvert\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}\rvert$		
			1. $\mathcal{S}_{\mathrm{X}_1} \subset \mathcal{S}_{\mathrm{X}_2}$	2. $\mathcal{S}_{\mathrm{X}_2} \subset \mathcal{S}_{\mathrm{X}_1}$	3. otherwise
1	$m \leq \min(t_1, t_2)$	$1 \leq p \leq p_m$	k_1	k_1	k_1
		$p_m + 1 \leq p \leq t_1$	k_2	k_1	$k_1 + d_\circ$
		$t_1 + 1 \leq p \leq T - m + 1$	k_2	k_2	k_2
2	$m > t_1$	$1 \leq p \leq T - m + 1$	k_2	k_1	$k_1 + d_\circ$
3	$t_2 < m \leq t_1$	$1 \leq p \leq p_m$	k_1	k_1	k_1
		$p_m + 1 \leq p \leq T - m + 1$	k_2	k_1	$k_1 + d_\circ$

Table 1: Cardinality of the support set $\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}$ depending on the size of the window m, the sliding step p and the three possible types of intersection of the supports $\mathcal{S}_{\mathrm{X}_1}$ and $\mathcal{S}_{\mathrm{X}_2}$.

the above definition it immediately follows that

$$\operatorname{rank}(\boldsymbol{Y}_p^{\mathrm{w}}) \leq \operatorname{rank}\left(\boldsymbol{X}_p^{\mathrm{w}}\right) \leq |\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}|. \tag{10}$$

Thus, the ultimate goal of this work is to determine: 1) the conditions on the sensing matrix $\boldsymbol{B}$ and the window size m under which a change in the support of $\boldsymbol{X}_p^{\mathrm{w}}$ caused by the transition from $\boldsymbol{X}_1$ to $\boldsymbol{X}_2$ uniquely corresponds to a change in the rank of $\boldsymbol{Y}_p^{\mathrm{w}}$; 2) exact relations between the moments of these changes.

To accomplish this we examine two parts of the inequality (10) independently. The left-hand part of (10) turns into an equality, i.e., $\operatorname{rank}\left(\boldsymbol{Y}_p^{\mathrm{w}}\right) = \operatorname{rank}\left(\boldsymbol{X}_p^{\mathrm{w}}\right)$, when the spark of $\boldsymbol{B}$ is $> \max_p \left(\operatorname{rank}(\boldsymbol{X}_p^{\mathrm{w}})\right)$. It can be shown that a sufficient condition for this to be fulfilled is

$$\operatorname{spark}(\boldsymbol{B}) > \min(k_1 + k_2, m), \tag{11}$$

which, compared to (6), introduces an additional term of $\min(k_1 + k_2, m) - k_{\max} - 1$ in case of $m \geq k_{\max}$. As the right-hand part of (10) is slightly more complicated, we analyze it in two steps: first, we study the value of $|\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}|$, after which we establish its relationship to the rank of $\boldsymbol{X}_p^{\mathrm{w}}$.

3.1. Evolution of the window support

For the value of $|\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}|$ we can distinguish a number of exclusive cases depending on the window size m, the relation between the supports $\mathcal{S}_{\mathrm{X}_1}$ and $\mathcal{S}_{\mathrm{X}_2}$, and the range of the index p. These are summarized in Table 1 where the following short-hand notations are used

- last sliding index $p_m = t_1 - m + 1$ corresponding the the case when $\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}} = \mathcal{S}_{\mathrm{X}_1}$,
- cardinality $d_\cap = |\mathcal{S}_{\mathrm{X}_1} \cap \mathcal{S}_{\mathrm{X}_2}|$ of the intersection of the supports $\mathcal{S}_{\mathrm{X}_1}$ and $\mathcal{S}_{\mathrm{X}_2}$,
- cardinality $d_\circ = k_2 - d_\cap$ of the relative complement of the support of $\boldsymbol{X}_1$ in the support of $\boldsymbol{X}_2$.

As a result the following lemma can be formulated

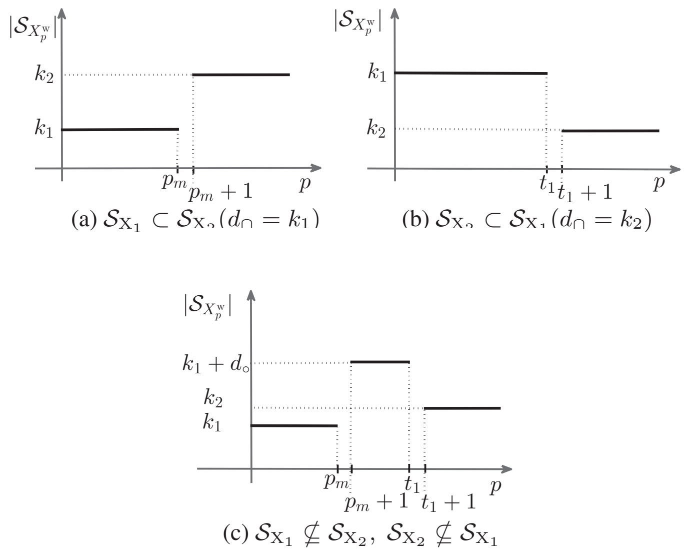


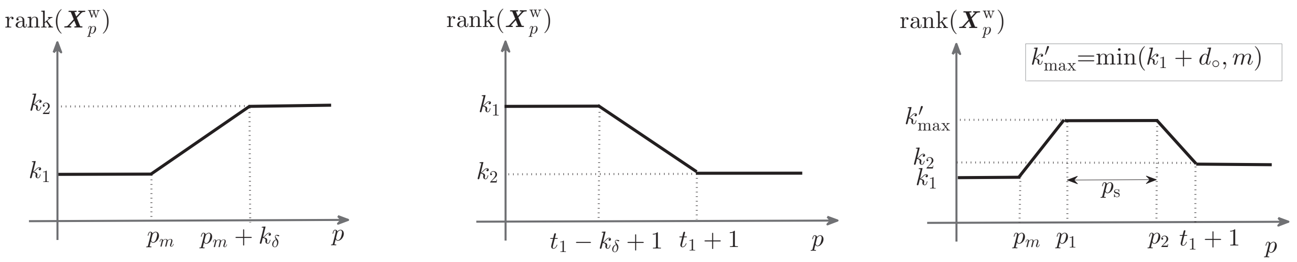
Fig. 1: Cardinality of the support set $\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}$ versus sliding index p for $m \in [2, \min(t_1, t_2)]$.

Lemma 1. *For any supports $\mathcal{S}_{\mathrm{X}_1}$ and $\mathcal{S}_{\mathrm{X}_2}$ there is at least one index $g = t_1 + \delta$ where δ depends only on m such that $|\mathcal{S}_{\mathrm{X}_g^{\mathrm{w}}}| \neq |\mathcal{S}_{\mathrm{X}_{g+1}^{\mathrm{w}}}|$ if and only if $m \in [2, \min(t_1, t_2)]$*

Lemma 1 establishes a condition on the window size m that guarantees a presence of the change in the cardinality of the window support. Moreover, an index corresponding to this change, i.e., the value of g for which $|\mathcal{S}_{\mathrm{X}_g^{\mathrm{w}}}| \neq |\mathcal{S}_{\mathrm{X}_{g+1}^{\mathrm{w}}}|$, is defined exclusively by t_1, m and the type of intersection between $\mathcal{S}_{\mathrm{X}_1}$ and $\mathcal{S}_{\mathrm{X}_2}$. Table 1 demonstrates how the cardinality $|\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}|$ evolves with the index p. This evolution is characterized by a distinct behavior for the three types of intersection between $\mathcal{S}_{\mathrm{X}_1}$ and $\mathcal{S}_{\mathrm{X}_2}$ as illustrated in Figure 1.

3.2. Evolution of the window rank

The next step is to determine how the behavior of $\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}$ translates into the rank of $\boldsymbol{X}_p^{\mathrm{w}}$. Since $\boldsymbol{X}$ is full rank, any of it sub-matrices including $\boldsymbol{X}_p^{\mathrm{w}}$ will also be full rank and thus



(a) $\mathcal{S}_{\mathrm{X}_1} \subset \mathcal{S}_{\mathrm{X}_2} (d_\cap = k_1)$ (b) $\mathcal{S}_{\mathrm{X}_2} \subset \mathcal{S}_{\mathrm{X}_1} (d_\cap = k_2)$ (c) $\mathcal{S}_{\mathrm{X}_1} \not\subseteq \mathcal{S}_{\mathrm{X}_2},\ \mathcal{S}_{\mathrm{X}_2} \not\subseteq \mathcal{S}_{\mathrm{X}_1}$

Fig. 2: Rank of the sliding window $\boldsymbol{X}_p^{\mathrm{w}}$ versus sliding index p for $m \in [\max(k_1, k_2) + 1, \min(t_1, t_2)]$ where $k_\delta = |k_1 - k_2|$, $p_1 = p_m + \min(d_{\mathrm{mk}}, d_\circ)$, $p_2 = p_1 + p_s$ and $p_s = \min\left(H^{\mathrm{s}}_{\mathrm{m}}(d_\circ - d_{\mathrm{mk}}), d_{\mathrm{mk}}\right) + \min\left(d_\cap - H^{\mathrm{s}}_{\mathrm{m}}(d_\circ - d_{\mathrm{mk}}), d_{\mathrm{mk}} - H^{\mathrm{s}}_{\mathrm{m}}(d_\circ - d_{\mathrm{mk}})\right)$.

$$\mathrm{rank}(\boldsymbol{X}_p^{\mathrm{w}}) = \begin{cases} \min(|\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}|, m) & , p \in [1, p_m] \\ k' & , p \in [p_m + 1, t_1], \\ \min(|\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}|, m) & , p \in [t_1 + 1, T - m + 1] \end{cases} \tag{12}$$

where $k' \leq |\mathcal{S}_{\mathrm{X}_p^{\mathrm{w}}}| \in \mathbb{N}$. In order to give an expression for the value of k' we first introduce some additional notations:

- the number of columns p' of $\boldsymbol{X}_2$ in $\boldsymbol{X}_p^{\mathrm{w}}$,

$$p' = \begin{cases} m & , \text{if } p > t_1 - 1 \\ p - t_1 + m - 1 & , \text{if } p \in [p_m + 1, t_1 - 1] \\ 0 & , p < p_m + 1 \end{cases}, \tag{13}$$

- modified Heaviside function $H_{\mathrm{m}}(t)$ and and scaled modified Heaviside function $H^{\mathrm{s}}_{\mathrm{m}}(t)$ defined as

$$H_{\mathrm{m}}(t) = \frac{H^{\mathrm{s}}_{\mathrm{m}}(t)}{t} = \begin{cases} 1 & , \text{if } \quad t > 0 \\ 0 & , \text{if } \quad t \leq 0 \end{cases}, \tag{14}$$

- the redundancy $d_{\mathrm{mk}} = H^{\mathrm{s}}_{\mathrm{m}}(m - k_1)$ of the window size m with regard to k_1.

Taking into account these, it can be shown that the value of k' is given by (15).

Although (15) is an exact formula, it is rather cumbersome to analyze in the full form. However, it can be significantly simplified if considered independently for the three types of the support intersection identified in Section 3.1 and the two cases for the value of d_{mk}, namely $d_{\mathrm{mk}} > 0$ and $d_{\mathrm{mk}} \leq 0$. This way one can comprise a table showing the evolution of the rank of $\boldsymbol{X}_p^{\mathrm{w}}$ in a form similar to Table 1. While we omit presenting the full table here due to the space limitation, we summarize the main result in the following lemma instead

Lemma 2. *For any supports $\mathcal{S}_{\mathrm{X}_1}$ and $\mathcal{S}_{\mathrm{X}_2}$ there is at least one index $n = t_1 + \delta'$ where $\delta' \in \mathbb{Z}$ such that rank($\boldsymbol{X}_n^{\mathrm{w}}$) $\neq$ rank($\boldsymbol{X}_{n+1}^{\mathrm{w}}$) if and only if* $m \in [\max(k_1, k_2) + 1, \min(t_1, t_2)]$.

Similar to Lemma 1, Lemma 2 provides a condition on the window size m that ensures a presence of a change in the rank of a sliding window $\boldsymbol{X}_p^{\mathrm{w}}$ with its moment being determined by the value of t_1. Furthermore, the behavior exhibited by rank($\boldsymbol{X}_p^{\mathrm{w}}$) essentially repeats that of $|\mathcal{S}_{\mathrm{X}_{\mathrm{p}}^{\mathrm{w}}}|$ except for the presence of monotonic slopes within the sliding index range of $[t_1 - m + 2, t_1 + 1]$ as demonstrated in Fig. 2. The number, length and type of the slopes are uniquely determined by the type and size of intersection of the supports $\mathcal{S}_{\mathrm{X}_1}$ and $\mathcal{S}_{\mathrm{X}_2}$. However, in contrast to the case of the support evolution, not all the moments of change in the rank of $\boldsymbol{X}_p^{\mathrm{w}}$ are solely determined by t_1 and m anymore. It turns out that some of them additionally depend on the cardinality of the support intersection and/or their difference as shown in Fig. 2.

3.3. Estimation of the stationarity window

Taking into account (11) and the fact that the largest value that spark($\boldsymbol{B}$) can take is $M+1$ we can formulate a final condition on the window size m

$$\max(k_1, k_2) < m < \min(M + 1, \min(t_1, t_2) + 1). \tag{16}$$

When (11) and (16) hold, the value of t_1 can be found by identifying the index p that corresponds to the first and/or the last change in the values of the rank of $\boldsymbol{Y}_p^{\mathrm{w}}$. This is because for the cases of the support intersection corresponding to (a) and (c) from figure 2, the index of the first change in the rank of $\boldsymbol{X}_p^{\mathrm{w}}$ is $p_m = t_1 - m + 1$ where the window block size m is known, whereas for (b) and (c) the index of the last change is given by $t_1 + 1$. Finally, rank($\boldsymbol{Y}_p^{\mathrm{w}}$) = rank($\boldsymbol{X}_p^{\mathrm{w}}$) $\forall p \in [1, T - m + 1]$ due to (11).

Thus, given the vector $\boldsymbol{r} = [r_1, r_2, \cdots, r_p, \cdots, r_{T-m+1}]$ of rank estimates of $\boldsymbol{Y}_p^{\mathrm{w}}$, one can formulate a following rule

$$t_1 = \begin{cases} p_{\mathrm{fc}} + m - 1 & , \text{if } \forall p \in [p_{\mathrm{fc}}, p_{\mathrm{lc}}]\ r_p < r_{p+1} \\ p_{\mathrm{lc}} - 1 & , \text{otherwise} \end{cases}, \tag{17}$$

$$k' = \min(k_1, m - p') + \min\left(d_\circ, \min(p', H^{\mathrm{s}}_{\mathrm{m}}(d_{\mathrm{mk}})\right) + \left(1 - H_{\mathrm{m}}(d_{\mathrm{mk}})\right) \cdot \min(d_\circ, p') + H_{\mathrm{m}}(p' - d_{\mathrm{mk}}) \cdot \\ \Big(H_{\mathrm{m}}(p' - d_{\mathrm{mk}}) \cdot \min\left(H^{\mathrm{s}}_{\mathrm{m}}\left(d_\circ - d_{\mathrm{mk}}\right), p' - d_{\mathrm{mk}}\right) + \min\left(d_\cap, H^{\mathrm{s}}_{\mathrm{m}}\left(p' - H^{\mathrm{s}}_{\mathrm{m}}(d_{\mathrm{mk}}) - H^{\mathrm{s}}_{\mathrm{m}}(d_\circ - H^{\mathrm{s}}_{\mathrm{m}}(d_{\mathrm{mk}}))\right)\right)\Big). \tag{15}$$

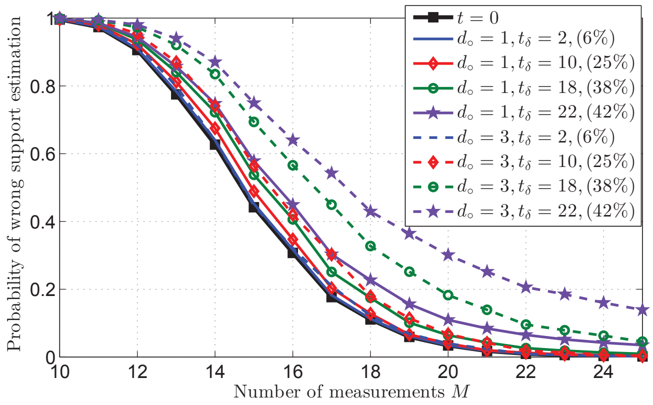


Fig. 3: Empirical probability of wrong support recovery P_{er} versus the number of measurements per snapshot M

where $r_p = \text{rank}(\boldsymbol{Y}_p^{\text{w}})$ and p_{fc} and p_{lc} are the indexes of the first and last change of values in $\boldsymbol{r}$, respectively.

4. SOME NUMERICAL RESULTS

In order to empirically evaluate how the absence of stationarity window estimate influences the performance of the joint sparse recovery in the case of MMV with time-varying quasi-static support we performed a series of simulations. For the generation of the sensing matrix $\boldsymbol{B}$ we considered a class of random Gaussian matrices whose elements were drawn independently from a complex normal distribution of unit variance, after which they were normalized so that every column had unit norm. The input signal $\boldsymbol{X}$ was generated according to (7a)-(7c) where $N = 128$, $t_1 = t_2 = 30$ and $k_1 = 6$. The values of it non-zero entries were drawn from a unit variance complex normal distribution. The supports of the sub-matrices $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$ were generated randomly according to the scenarios (a) and (c) from Fig. 2 where the value of k_2 was defined as $k_2 = d_\circ + k_1$ and $k_2 = k_1$, respectively. Also, we used the simultaneous orthogonal matching pursuit (SOMP) algorithm for signal recovery [2].

Figure 3 presents the results for the empirical probability of the wrong support recovery of $\boldsymbol{X}_1$ versus the number of measurements per snapshot M. The black square-marked solid line represents the case when t_1 was estimated via (17) and the support of $\boldsymbol{X}_1$ was then recovered from the first t_1 snapshots. The colored lines in turn show the results for the case when no estimate was available and $\boldsymbol{X}_1$ was recovered from $t_1 + t_\delta$ snapshots where t_δ is the number of snapshots added from $\boldsymbol{X}_2$. The presented results clearly show that in case of quasi-stationary input signals, failure to provide an information on the moment of the support change results in a performance degradation that increases both with the increase of the delay t_δ in detecting the change and the difference between the supports. On the contrary, the application of the proposed rank evolution approach allows to efficiently avoid this effect provided that the signal of interest exhibit linear independence between snapshots.

5. CONCLUSIONS

In this paper we have discussed joint recovery of full-rank quasi-stationary MMV. We have proposed an approach for support change detection that exploits rank information obtained from a windowed portion of measurement data and provided an extensive analysis of the window rank evolution with regard to the window size and possible types of the support change. The proposed approach allows to adapt the number of snapshots used for recovery in order to efficiently utilize the joint-sparse structure of MMV, provided that it is full rank. Although in the current work we have focused on the noise-free case only, the proposed approach can be extended to the noisy observations by application of effective rank estimation techniques.

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