# MATCHING PURSUIT VIA CONTINUOUS RESOLUTION CELL REJECTION IN PRESENCE OF UNRESOLVED RADAR TARGETS

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## ABSTRACT

In this article a new matching pursuit algorithm with continuous radar resolution cell rejection is proposed. It allows matching pursuit to work well even if more than one target is present in some resolution cell (unresolved targets) of the radar matched filter: it prevents its tendency to generate spurious sidelobes or miss a weaker target hidden in stronger target sidelobes. The FMCW radar case is particularly investigated which offers a very natural and computationally inexpensive solution to the problem that can also be applied in spectral analysis. The extension of the proposed approach to any radar waveform is also investigated.

***Index Terms*—** Matching Pursuit, Radar Signal Processing, Discrete Prolate Spheroidal Sequences,Spectral Estimation , Unresolved targets

## I. INTRODUCTION

Traditionally, the radar signal processing chain consists of a (single target) matched filter processing followed by a detection step [1] in order to both detect and estimate the target's range and radial velocity (Doppler shift). The matched filter is usually evaluated on a grid whose step is equal to the resolution capability of the radar.

This procedure, when done iteratively and followed by a rejection step (estimated target contribution removal) can be seen as a clean [2] or matching pursuit algorithm [3]. Matching pursuit is popular in compressive sensing, which has gained interest in the radar community [4]. The principle of matching pursuit is the following: estimate the strongest components of the signal (in the matched filter magnitude sense) and reject them iteratively from the original signal.

A fundamental hypothesis for this algorithm to be efficient is that there is no unresolved targets: that is to say no more than one dominant scatterer is present inside each resolution cell. Otherwise, the estimation can become so biased that the rejection step can only perform poorly and spurious detection may occur. It may also prevent detection of weaker targets hidden in the strongest sidelobes. High resolution in radar is only possible in not too complex scenario [5], so that separation of unresolved scatterers is usually not possible. To the best of our knowledge, the issue of unresolved targets in radar has mainly been investigated for (monopulse) Angle of Arrival estimation (see [6] and references inside). [7] proposes to reject several components inside the matched filter resolution cell, which can be costly and may not be efficient for high SNR targets.

In this article, in order to deal with unresolved targets in active radar, we propose a new matching pursuit algorithm with a (continuous) rejection on the estimated cell. The philosophy is the following: when a target has been detected in a resolution cell, then all possible contributions belonging to this cell should be removed from the measured signal. We call it "continuous" rejection since all contributions whose continuous parameters fall inside the considered cell must be rejected, contrary to the classical processing where only the cell center contribution is removed. We will particularly investigate the Frequency Modulated Continuous Waveform (FMCW) radar case, which is of great interest for low cost devices: it offers a simple and natural frame to solve our problem that becomes a multiple tone frequencies estimation. As we will see, in this case the projection is realized thanks to the Discrete Prolate Spheroidal Sequences (DPSS) [8] and that the proposed processing becomes particularly simple and computationally inexpensive. We will also show that, relying on a common approximation, the proposed scheme for FMCW can also be generalized to any waveform. Note that DPSS have recently been introduced for compressed sensing [9], showing their nice properties to build a good compressive dictionary. The proposed work extends [10] where the rejection is investigated on the Doppler axis only and for passive radar. Although [11] has suggested the use of DPSS for interference cancellation in compressed sensing for radio, the goal was not to perform continuous rejection. We will show that the choice of DPSS arises naturally and how it can be adapted to the classic active radar framework.

The outline of the paper lies as follows: in the second section we propose the radar model. In section III the continuous rejection projector is investigated, first for the FMCW radar case, and then it is shown how it can be adapted to any waveform. Finally, a simulation in section IV show that the proposed method outperforms the classic matched filter processing in presence of unresolved targets.

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## II. SIGNAL MODEL

We consider a common narrowband radar transmitting a signal $s(t)$ (waveform). The received signal $x(t)$ is then

$$x(t) = \sum_{q=1}^{Q} \rho_q s(t-\tau_q) e^{j2\pi\nu_q t} + n(t), \tag{1}$$

where $\tau_q$ denotes the unknown round trip time delay for the radar signal scattered by the $q-th$ target, $\nu_q = 2v_q/\lambda$ is the unknown Doppler shift, where $\lambda$ is the carrier wavelength and $v_q$ the target radial velocity. $\rho_q$ denotes the unknown target amplitude and $n(t)$ is an additive noise, considered as centered white complex Gaussian of known variance $\sigma^2$. A sequence of $P$ replicas (each of period $T_p$) is sent and the corresponding received signal is then sampled at period $T_s = T_p/N$. We consider the overall observation vector

$$\mathbf{y} = \begin{bmatrix} \mathbf{x}_0^T & ... & \mathbf{x}_{P-1}^T \end{bmatrix}^T, \tag{2}$$

where, denoting $t_n = nT_s$

$$\mathbf{x}_p = \begin{bmatrix} x(t_1 + pT_p) & ... & x(t_N + pT_p) \end{bmatrix}^T. \tag{3}$$

Since it is usually assumed that the Doppler phase $2\pi\nu_q t$ does not vary significantly during a period $T_p$ [1] and that $\rho_q$ is constant [1] during the acquisition

$$\mathbf{y} = \sum_{q=1}^{Q} \rho_q \mathbf{u}(\tau_q, \nu_q) + \mathbf{n}, \tag{4}$$

$$\mathbf{u}(\tau_q, \nu_q) = \mathbf{a}_P\left(2\pi\nu_q T_p\right) \otimes \mathbf{s}(\tau_q), \tag{5}$$

$$\mathbf{a}_P(\omega) = \begin{bmatrix} 1 & e^{j\omega} & ... & e^{j(P-1)\omega} \end{bmatrix}^T, \tag{6}$$

$$\mathbf{s}(\tau) = \begin{bmatrix} s(t_1 - \tau) & ... & s(t_N - \tau) \end{bmatrix}^T, \tag{7}$$

where $\otimes$ is the Kronecker product and $\mathbf{n}$ the stacked noise vector. Note that the model (4) is separable in $\tau$ and $\nu$.

In case of FMCW radar, the waveform is $s(t) = e^{j\alpha t^2}$. The transmitted signal is then multiplied by the sent waveform replica, forming the beat signal observation vector

$$\mathbf{y} = \begin{bmatrix} \mathbf{x}_0^T \odot \mathbf{s}^H(0) & ... & \mathbf{x}_{P-1}^T \odot \mathbf{s}^H(0) \end{bmatrix}^T, \tag{8}$$

where $\odot$ is the Hadamard product (element wise). Then,

$$\mathbf{y} = \sum_{q=1}^{Q} \rho_q \mathbf{a}_P\left(\omega(\nu_q)\right) \otimes \mathbf{a}_N(\omega(\tau_q)) + \mathbf{n}, \tag{9}$$

$$\omega(\nu) = 2\pi\nu T_p, \quad \omega(\tau) = -2\alpha\tau T_s. \tag{10}$$

**Algorithm 1** Matching pursuit in radar

$\mathbf{y}_{(0)} = \mathbf{y}$, $\tau_{(0)} = \{\emptyset\}$,$\nu_{(0)} = \{\emptyset\}$

At the $i-th$ iteration

1) Matched filter : estimate the parameters (on a grid)

$$\left\{\hat{\tau}_{(i)}, \hat{\nu}_{(i)}\right\} = \arg\max_{\tau,\nu} \frac{|\mathbf{y}_{(i)}^H \mathbf{u}(\tau,\nu)|^2}{||\mathbf{u}(\tau,\nu)||^2}$$

2) Reject the estimated component from the signal:

$$\mathbf{y}_{(i+1)} = \mathbf{y}_{(i)} - \mathbf{\Pi}(\mathbf{u}(\hat{\tau}_{(i)}, \hat{\nu}_{(i)}))\mathbf{y}_{(i)},$$

where $\mathbf{\Pi}(\mathbf{u})$ is defined in (11).

3) Repeat until the maximum iteration number is reached or until with $(\tau, \nu)$ belonging to a grid

$$\max_{\tau,\nu} \frac{|\mathbf{y}_{(i)}^H \mathbf{u}(\tau,\nu)|^2}{||\mathbf{u}(\tau,\nu)||^2} \le \gamma,$$

where $\gamma$ is a threshold set according to a desired probability of false alarm ($P_{fa}$).

## III. MATCHING PURSUIT WITH CONTINUOUS REJECTION

Matching pursuit can be viewed in radar context as a clean algorithm [2] as summarized in Algorithm 1.

This technique is efficient when no more than one scatterer is present in each resolution cell of the matched filter (resolved targets). Otherwise the classic matched filter may fail to separate several scatterers inside the resolution cell - quite likely in common radar signal processing [5]- and then the estimation step will be so biased that the rejection will perform poorly. Then the matching pursuit tends to produce spurious detections or can even misdetect weaker targets hidden in stronger target sidelobes.

This is why, to allow the matching pursuit to perform in presence of unresolved targets we propose to replace step (2) in Algorithm 1 by a (continuous) rejection of all possible contributions inside the detected resolution cell.

### III-A. Building the continuous projector

We would like to build the projector $\mathbf{\Pi}(\mathbf{U}(\tau_0, \nu_0))$ on all the components of $\mathbf{u}(\tau, \nu)$ for all $(\tau, \nu)$ in the resolution cell $\left[\tau_0 - \frac{\Delta_\tau}{2}, \tau_0 + \frac{\Delta_\tau}{2}\right] \times \left[\nu_0 - \frac{\Delta_\nu}{2}, \nu_0 + \frac{\Delta_\nu}{2}\right]$, $\Delta_\tau$ and $\Delta_\nu$ being respectively the delay and Doppler resolution. We recall that the orthogonal projector on any matrix $\mathbf{A}$ is:

$$\mathbf{\Pi}(\mathbf{A}) = \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^\dagger \mathbf{A}^H, \tag{11}$$

where $(.)^\dagger$ denotes the Moore-Penrose inverse, which is equal to the classic inverse when the matrix is invertible.

In order to build the continuous projector we will first consider a projector on $2K+1$ contributions inside the resolution cell and let $K$ tend to infinity. Let us consider the following $NP \times (2K+1)^2$ matrix

$$\mathbf{U}_K(\tau_0, \nu_0) = \mathbf{A}_{P,K}\left(\omega(\nu_0), \Delta_{\omega(\nu_0)}\right) \otimes \mathbf{S}_{N,K}(\tau_0, \Delta_\tau), \quad (12)$$

where the $k-$th column of those matrices is

$$[\mathbf{S}_{N,K}(\tau_0, \Delta_\tau)]_k = \mathbf{s}(\tau_k), \; 1 \leq k \leq 2K+1 \quad (13)$$

with $\tau_k = \tau_0 - (K-k+1)\frac{\Delta_\tau}{2K}$ and

$$[\mathbf{A}_{P,K}(\omega_0, \Delta_\omega)]_k = \mathbf{a}_P(\omega_k), \; 1 \leq k \leq 2K+1 \quad (14)$$

with $\omega_k = \omega_0 - (K-k+1)\frac{\Delta_\omega}{2K}$ , respectively.

The sought projector $\mathbf{\Pi}(\mathbf{U}(\tau_0, \nu_0))$ can be seen as the limit of the $NP \times NP$ orthogonal projector $\mathbf{\Pi}(\mathbf{U}_K(\tau_0, \nu_0))$:

$$\mathbf{\Pi}(\mathbf{U}(\tau_0, \nu_0)) = \lim_{K \to +\infty} \mathbf{\Pi}(\mathbf{U}_K(\tau_0, \nu_0)), \quad (15)$$

which exists when $\underset{K}{sup} \, ||\mathbf{U}_K^\dagger(\tau_0, \nu_0)|| < +\infty$ [12].

We propose to evaluate it thanks to the following remarks:

*Remark* 1. For any two matrices $\mathbf{A}$ and $\mathbf{B}$, we have

$$\mathbf{\Pi}(\mathbf{A} \otimes \mathbf{B}) = \mathbf{\Pi}(\mathbf{A}) \otimes \mathbf{\Pi}(\mathbf{B}). \quad (16)$$

*Remark* 2. For any $N \times K$ matrix $\mathbf{A}$ we have $\mathbf{A} = \mathbf{U}_A \mathbf{D}_A \mathbf{V}_A^H$, where $\mathbf{D}_A$ is a $N \times K$ diagonal matrix containing the $N_0$ singular values of $\mathbf{A}$, and $\mathbf{U}_A$ and $\mathbf{V}_A$ are $N \times N$ and $K \times K$ orthonormal matrices, respectively. So,

$$\begin{aligned} \mathbf{\Pi}(\mathbf{A}) &= \mathbf{U}_A \mathbf{I}_{1:N_0} \mathbf{U}_A^H, & (17) \\ &= \mathbf{\Pi}(\mathbf{A}\mathbf{A}^H), & (18) \end{aligned}$$

where $\mathbf{I}_{1:N_0}$ is a diagonal matrix with its first $N_0$ components being equal to 1, the others are zeros. But $\mathbf{U}_A$ also contains the eigenvectors of the $N \times N$ matrix $\beta \mathbf{A}\mathbf{A}^H$ for any $\beta$.

Thanks to Remark 1, $\mathbf{\Pi}(\mathbf{U}_K(\tau_0, \nu_0))$ in (12) is directly deduced from $\mathbf{\Pi}(\mathbf{S}_{N,K})$ and $\mathbf{\Pi}(\mathbf{A}_{P,K})$: this structure enables to lower its computational cost. Remark 2 shows that $\mathbf{\Pi}(\mathbf{S}_{N,K})$ and $\mathbf{\Pi}(\mathbf{A}_{P,K})$ , whose analytical expression is not trivial, can be evaluated thanks to the eigenvectors of the matrices $\frac{\Delta_\tau}{2K+1}\mathbf{S}_{N,K}\mathbf{S}_{N,K}^H$ and $\frac{\Delta_\omega}{2K+1}\mathbf{A}_{P,K}\mathbf{A}_{P,K}^H$.

As a consequence, when $K$ grows to infinity, we propose to deduce $\mathbf{\Pi}(\mathbf{U}(\tau_0, \nu_0))$ in (15) from the eigenvectors of

$$\begin{aligned} \mathbf{S}_N(\tau_0, \Delta_\tau) &= \lim_{K \to +\infty} \frac{\Delta_\tau}{2K+1} \mathbf{S}_{N,K} \mathbf{S}_{N,K}^H \\ &= \lim_{K \to +\infty} \sum_{k=1}^{2K+1} \frac{\Delta_\tau}{2K+1} \mathbf{s}(\tau_k) \mathbf{s}^H(\tau_k) \\ &= \int_{\tau_0 - \frac{\Delta_\tau}{2}}^{\tau_0 + \frac{\Delta_\tau}{2}} \mathbf{s}(\tau) \mathbf{s}^H(\tau) \, d\tau, \quad (19) \end{aligned}$$

and

$$\begin{aligned} \mathbf{A}_P(\omega_0, \Delta_\omega) &= \lim_{K \to +\infty} \frac{\Delta_\omega}{2K+1} \mathbf{A}_{P,K} \mathbf{A}_{P,K}^H \\ &= \int_{\omega_0 - \frac{\Delta_\omega}{2}}^{\omega_0 + \frac{\Delta_\omega}{2}} \mathbf{a}_P(\omega) \mathbf{a}_P^H(\omega) \, d\omega. \quad (20) \end{aligned}$$

We have $(\mathbf{A}_P(\omega_0, \Delta_\omega))_{kl} = e^{j(k-l)\omega_0} \Delta_\omega \text{sinc}\left((k-l)\Delta_\omega\right).$

### III-B. FMCW radar case

For FMCW radar, the delay estimation is turned into the estimation of the beat signal frequencies. According to (9) $\mathbf{S}_{N,K}(\tau_0, \Delta_\tau)$ is replaced by $\mathbf{A}_{N,K}\left(\omega(\tau_0), \Delta_{\omega(\tau_0)}\right)$. In this case the projector (15) is deduced from the eigenvectors of two matrices with the following structure

$$[\mathbf{M}_N(\omega_i)]_{kl} = e^{j(k-l)\omega_i} \frac{1}{N} \text{sinc}\left(\frac{k-l}{N}\right), \quad (21)$$

where $\omega_i = i/N$, with $i \in \{0, 1, ..., N-1\}$ since in radar, the angular frequencies $\omega(\tau)$ and $\omega(\nu)$, related to delay $\tau$ and Doppler $\nu$, are sampled on a grid (normalized by $2\pi T_s$ and $2\pi T_p$, respectively) defined by their corresponding resolution step ($\Delta_{\omega(\tau)} = 1/N$ and $\Delta_{\omega(\nu)} = 1/P$). Obviously,

$$\mathbf{M}_N(\omega) = \left[\mathbf{a}_N(\omega) \mathbf{a}_N^H(\omega)\right] \odot \mathbf{B}_{N, \frac{1}{2N}}, \quad (22)$$

$$(\mathbf{B}_{N,W})_{k,l} = 2W \text{sinc}(2W(k-l)), \; (1 \leq k, l \leq N). \quad (23)$$

We denote by $\{\mathbf{u}_k(N, W), 1 \leq k \leq N\}$ the eigenvectors of $\mathbf{B}_{N,W}$ with the corresponding eigenvalues $\{\lambda_k(N, W), 1 \leq k \leq N\}$, sorted by decreasing magnitude.

*Remark* 3. Let $\mathbf{A}$ and $\mathbf{B}$ be two $N \times N$ Hermitian matrices with $\mathbf{A} = \left[\mathbf{a}_N(\omega) \mathbf{a}_N^H(\omega)\right] \odot \mathbf{B}$. Then, due to the particular structure of $\mathbf{a}_N(\omega)$, the eigenvector decomposition of $\mathbf{A}$ is simply deduced from the decomposition of $\mathbf{B}$: $\mathbf{A} = \sum_{n=1}^N \lambda_n \mathbf{a}_n \mathbf{a}_n^H$ where $\mathbf{a}_n = \mathbf{a}_N(\omega) \odot \mathbf{b}_n$ with $\mathbf{B} = \sum_{n=1}^N \lambda_n \mathbf{b}_n \mathbf{b}_n^H$.

According to Remark 3 and (22), the eigenvectors of $\mathbf{M}_N(\omega)$ can be deduced from those of $\mathbf{B}_{N, \frac{1}{2N}}$. Finally

$$\mathbf{\Pi}(\mathbf{U}(\tau_0, \nu_0)) = \mathbf{\Pi}(\mathbf{M}_P(\omega(\nu_0))) \otimes \mathbf{\Pi}(\mathbf{M}_N(\omega(\tau_0))), \quad (24)$$

where, for any $\omega$,

$$\mathbf{\Pi}(\mathbf{M}_N(\omega)) = \mathbf{V}_{1:N_0}(\omega) \mathbf{V}_{1:N_0}^H(\omega), \quad (25)$$

$$\mathbf{V}_{1:N_0}(\omega) = \left(\mathbf{1}_{N_0}^T \otimes \mathbf{a}_N(\omega)\right) \odot \mathbf{U}_{1:N_0}\left(N, \frac{2}{N}\right), \quad (26)$$

$$\mathbf{U}_{1:N_0}(N, W) = \left[\; \mathbf{u}_1(N, W) \quad ... \quad \mathbf{u}_{N_0}(N, W) \;\right]. \quad (27)$$

where $\mathbf{1}_N$ is a $N \times 1$ vector composed of ones.

Then, all essential information is embedded in the eigenvectors of $\mathbf{B}_{N,W}$ for $W = \Delta/2$ which are nothing but the well known DPSS [8].

Note that the projector (25) is also the minimizer of the projection residue over the cell

$$\mathcal{E}(\mathbf{\Pi}_{N_0}) = \int_{\omega - \frac{\Delta}{2}}^{\omega + \frac{\Delta}{2}} \left\|\mathbf{a}_N(\omega') - \mathbf{\Pi}_{N_0} \mathbf{a}_N(\omega')\right\|^2 d\omega', \quad (28)$$

for any orthonormal projector $\mathbf{\Pi}_{N_0}$ of rank $N_0$ [9], and

$$\mathcal{E}\left(\mathbf{V}_{1:N_0}(\omega) \mathbf{V}_{1:N_0}^H(\omega)\right) = \sum_{k=N_0+1}^{N} \lambda_k\left(N, \frac{\Delta}{2}\right), \quad (29)$$

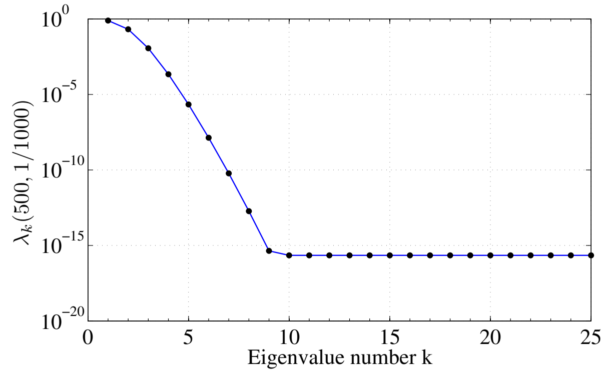


**Fig. 1**. Eigenvalues (sorted in decreasing order) of $\mathbf{B}_{500,\frac{1}{1000}}$.

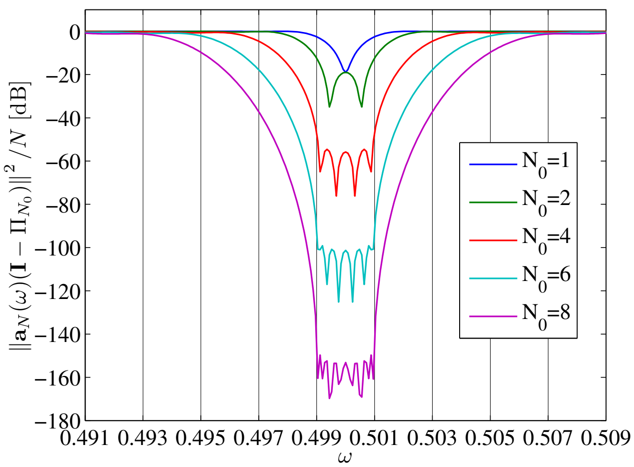


**Fig. 2**. Residue after projection on $\mathbf{\Pi}_{N_0}$, built with the $N_0$ strongest eigenvectors of $\mathbf{M}_{500}(1/2)$ according to (25).

which gives us a means to quantify the quality of the rejection inside the cell.

One interesting property of the DPSS is that the magnitude of the eigenvalues decreases very quickly, as shown in Figure 1. For that reason, only a few (e.g. $N_0 \leq 8$) DPSS are needed to compute the projectors $\mathbf{\Pi}(\mathbf{M}_N(\omega))$ and $\mathbf{\Pi}(\mathbf{M}_P(\omega))$, with a low computational time. Moreover, these DPSS (corresponding to $\mathbf{B}_{N,\frac{1}{2N}}$ and $\mathbf{B}_{P,\frac{1}{2P}}$, respectively) can be computed offline for once. The online additional computational cost of the proposed method is then very small and resorts to simple matrix products (no svd is required): once a detection occurs in a cell the corresponding 2D continuous projector $\mathbf{\Pi}(\mathbf{U}(\tau_0, \nu_0))$ in (24) is simply deduced from two 1D projectors, built with their corresponding $N_0$ "baseband" DPSS and translated to the cell position thanks to (26).

The selected number of eigenvectors $N_0$ in the single dimensional projector (25) has an impact on the rejection performance. In Figure 2, we plotted the residue after projection for several values of $N_0$. As we can see, the more eigenvalues we take, the better the rejection in the cell, but also the weaker the magnitude in the nearest cells, which may cause bias or even misdetection in those cells. This suggests that $N_0$ should be chosen depending on the desired rejection level: that is to say the estimated SNR in the cell.

### III-C. Generalization to any waveform

Writing the discrete Fourier transform (DFT) of the sequence $\{s(t_n), 1 \leq n \leq N\}$ by $\{s(f_k), 1 \leq k \leq N\}$ with $f_k = \frac{k-1}{NT_s}$, we can use the following common approximation

$$s(t_n - \tau) \approx \sum_{k=1}^{N} s(f_k) e^{-j2\pi f_k \tau} e^{j2\pi f_k t_n}. \tag{30}$$

Then if we denote $\mathbf{s}_f = \begin{bmatrix} s(f_1) & \dots & s(f_N) \end{bmatrix}^T$, we have

$$\mathbf{s}(\tau) \approx \mathbf{F}_N^H \left[ \mathbf{s}_f \odot \mathbf{a}_N \left( -\frac{2\pi}{NT_s}\tau \right) \right], \tag{31}$$

where $\mathbf{F}_N$ is the $N \times N$ Fourier matrix. We have then, inserting (31) in (19) and using (20), with $\Delta_\tau = 1/N$

$$\mathbf{S}_N(\tau_0, \Delta_\tau) \approx \mathbf{F}_N^H \check{\mathbf{S}}_N(\tau_0) \mathbf{F}_N, \tag{32}$$

$$\check{\mathbf{S}}_N(\tau) = \mathbf{s}_f \mathbf{s}_f^H \odot \mathbf{M}_N \left( -\frac{2\pi}{NT_s}\tau \right). \tag{33}$$

$\mathbf{\Pi}(\mathbf{S}_N(\tau_0, \Delta_\tau))$ can then be deduced as in (25) from the $N_0$ principal eigenvalues of $\mathbf{F}_N^H \check{\mathbf{S}}_N(\tau) \mathbf{F}_N$, linked with the DPSS thanks to (33) and (22).

Thus, using the common DFT approximation (30), the proposed approach for FMCW can be generalized to any waveform provided the DFT samples of the signal waveform are known, which is common.

## IV. NUMERICAL RESULTS

In this section we will focus on FMCW radar. This kind of radar offers several advantages: the range matched filter is turned into an FFT of the beat signal (analogical multiplication between received and transmitted signal), which is simple and very fast. Moreover, the beat signal can usually be sampled at much lower rate than the Nyquist rate of the waveform $s(t)$, which is suited to low cost devices.

In Figure 3 we consider a scenario described in Table I: we consider three targets where the first one is unresolved (4 off-grid scatterers in the same resolution cell) with a very strong SNR and the others are resolved targets with a much weaker SNR. According to the scenario described in caption of Figure 3 we have a corresponding range resolution $\Delta_r = 8.5$ m and a radial velocity resolution $\Delta_v = 3$ m/s. We defined the SNR of a target (or scatterer) by $SNR_q = |\rho_q|^2 ||\mathbf{s}(0)||^2/\sigma^2$ . Four algorithms are compared:

- "MP": is a classic matching pursuit performed either "on-grid" or with an additional "off-grid" local opti-

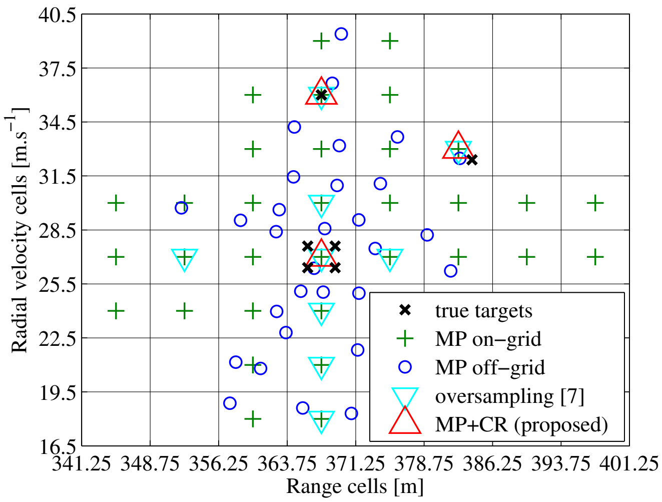


**Fig. 3**. Matching pursuit output for some unresolved targets according to Table I. $T_s = 2\,\mu s$, $T_p = 1$ ms, $\lambda = 0.3$ m, $\alpha = \pi \times 200$ MHz/s, $N = P = 500$, $P_{fa} = 10^{-6}$, $N_0 = 4$.

| Cell [range[m], velocity[m/s]] | [367.5,27] | [367.5,36] | [382.5,27] |
|---|---|---|---|
| Number of scatterers | 4 | 1 | 1 |
| Scatterers SNR (each) [dB] | 55 | 15 | 20 |
| Estimated target SNR [dB] | 61.3 | 13.6 | 18.6 |

**Table I**. Target parameters and estimated target SNR for the proposed method.

mization of the matched filter in step (1) in Algorithm 1, to deal with off-grid targets.

- Algorithm [7].
- "MP+CR": is the proposed matching pursuit with the continuous rejection (projector (24)) in step (2).

As we can see in Figure 3 the proposed method is able to distinguish all three targets, even the two weakest located close to the strong unresolved target. The results in Table I show that the proposed method offers good estimation of the target SNR. As expected, the classic matching pursuit fails and produces a lot of spurious peaks: since the first target is unresolved, the estimates are strongly biased and the algorithm needs a lot of iterations to remove the true contribution. Due to the first high SNR target, algorithm [7] produces false detection too.

Note that the proposed algorithm can be adapted to off-grid targets similarly to what is done for "MP off-grid".

## V. CONCLUSION

In this paper, a new matching pursuit algorithm for unresolved targets in active radar thanks to a (continuous) projection step on the entire resolution cell is proposed. It has been shown to perform well in presence of unresolved targets, contrary to the classic matching pursuit or clean algorithm. We particularly investigate the FMCW radar case, well suited for low-cost radar devices, where the problem can be turned into a spectral estimation problem: we showed that the continuous projector naturally leads to consider the Discrete Prolate Spheroidal Sequences and offers a simple and computationally inexpensive solution. The extension to any waveform is also investigated.

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