DOUBLE RELAY COMMUNICATION PROTOCOL FOR BANDWIDTH MANAGEMENT IN CELLULAR SYSTEMS

Rodolfo Torrea-Duran[1,2], *Fernando Rosas*[1,3], *Zuhaib Khan Zii Khan*[1,3],
Sofie Pollin[1,3], *Paschalis Tsiaflakis*[4], *and Marc Moonen*[1,2]

[1]KU Leuven, Department of Electrical Engineering (ESAT), 3000 Leuven, Belgium
[2]STADIUS Center for Dynamical Systems, Signal Processing and Data Analytics
[3] TELEMIC, Telecommunications and Microwaves
[4]Bell Labs, Alcatel-Lucent, Copernicuslaan 50, B-2018, Antwerp, Belgium

ABSTRACT

The continuously growing demand for wireless connectivity has turned bandwidth into a scarce resource that should be carefully managed. A common solution is to assign disjoint portions of the bandwidth to different users, but the portion size decreases as the number of users grows. An alternative solution is to introduce spatial diversity through coordinated base stations, but such systems are very sensitive to timing and frequency synchronization offsets. To tackle these problems, we use principles of network coding for bandwidth management in a double relay cellular system of two base stations and two users. We propose a three-time-slot transmission strategy and a MMSE reception strategy. It avoids the need of tight frequency or timing synchronization through a simple communication protocol without using additional bandwidth or infrastructure. By finding a balance between spatial diversity and transmission time, our approach achieves the system capacity and fairness in all SNR conditions.

Index Terms— double relay, MMSE, time-multiplexing

1. INTRODUCTION

The continuously growing demand for wireless connectivity has turned bandwidth into a scarce resource that should be carefully managed. Traditionally, the management of the available bandwidth has been done by assigning disjoint portions of the bandwidth to different users. This is the case of the well-known frequency or time division multiple access approaches (FDMA or TDMA) [1]. The weakness of these approaches is that the amount of resources that each user receives decreases linearly with the number of users. An alternative solution is to use various coordinated base stations, which introduces additional spatial degrees of freedom that boost the capacity of the wireless link [2]. However, such systems are very sensitive to timing and frequency synchronization offsets which severely degrade their performance [3].

An attractive solution to this problem is provided by network coding [4–6]. Network coding introduces relay nodes to receive signals from multiple sources and to broadcast a combination of them to multiple destinations. In this way, different users can utilize all the available bandwidth simultaneously, increasing the network throughput compared to FDMA and TDMA approaches. Transmissions under network coding are coordinated through a simple communication protocol, avoiding the need for tight frequency or timing synchronization between the sources.

Although originally proposed for point-to-point communications [7], the broadcast nature of wireless networks provides a suitable scenario for network coding. However, applying it in cellular systems is not straightforward, as it would require the deployment of additional relay nodes [8, 9].

In this paper, we develop a communication protocol for bandwidth management in a cellular system based on principles of network coding. Our approach consists of a three-time-slot transmission strategy in which two users receive data from two base stations, achieving spatial diversity gain while avoiding the need of additional infrastructure by using the base stations as a double relay (the extension to an arbitrary number of base stations or users will be addressed in a future paper). In contrast to [10], our approach is oriented towards cellular systems and our analysis is not based on error statistics, but on the achievable capacity. Our approach is also novel in proposing a reception strategy based on minimum mean square error (MMSE) estimation. We prove that apart from achieving the system capacity in all signal-to-noise-ratio (SNR) conditions, our approach shows fairness among users. We focus on single-antenna base stations, leaving the analysis of multiple-antenna systems for future work.

The rest of the paper is organized as follows. Section 2 de-

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scribes the system model and problem formulation. Section 3 shows the evaluation of our approach and the comparison with other time-multiplexing approaches. Finally, section 4 draws some conclusions.

Notation: Matrices and vectors are denoted by upper and lower case boldface letters, respectively; $\mathbf{A}^T$, $\mathbf{A}^H$, and $|\mathbf{A}|$ denote the transpose, Hermitian, and determinant of $\mathbf{A}$, respectively; $\mathbf{I}$ is the 3x3 identity matrix; $\mathbb{E}\{\}$ is the expected value operator.

2. SYSTEM MODEL

Consider the case in which two base stations communicate with two users sharing the same bandwidth. Assume that users and base stations are half-duplex, i.e. they cannot transmit and receive simultaneously and no wired backhaul link exists between the base stations.[1] Symbol s_1 is intended to be transmitted from base station 1 (BS1) to user 1 (U1), while symbol s_2 is intended to be transferred from base station 2 (BS2) to user 2 (U2). Both symbols are assumed to be uncorrelated. Our goal is to find an efficient way to perform these transmissions.

We assume that each base station can overhear the transmission of the other base station in a reliable way. For simplification purposes, the mathematical derivations neglect the possible decoding errors between the base stations, and leave this analysis to a future extension of this work. These errors are less relevant than the errors that may occur at the users' side since base stations are usually equipped with more powerful receivers (i.e. with greater sensitivity and smaller noise figure) and often count with line-of-sight.

In the following, section 2.1 presents the baseline TDMA approaches with and without spatial diversity, while sections 2.2 and 2.3 present the proposed transmission and reception strategies, respectively.

2.1. Baseline TDMA Approaches

In a basic TDMA approach, the communication is done in turns, i.e. first BS1 transmits s_1 to U1 while BS2 is inactive, and then BS2 transmits s_2 to U2 while BS1 is inactive, hence using 2 time-slots per symbol pair. We refer to this as TDMA-2. The weakness of this approach is the absence of spatial diversity: if the channel between a user and its base station is in a deep fading then it will be impossible to transfer information between them.

Let us define P and Q as the transmit powers of BS1 and BS2 and introduce the variables $\gamma = \frac{\sigma_x^2}{\sigma_n^2}P$ and $\eta = \frac{\sigma_x^2}{\sigma_n^2}Q$, where $\sigma_x^2 = \mathbb{E}\{|s_1|^2\} = \mathbb{E}\{|s_2|^2\}$ and σ_n^2 is the received noise power. Then, the capacity per time-slot of TDMA-2 for

[1]Therefore, it is not possible to use a transmission scheme which requires coordinated base stations, e.g. Alamouti.

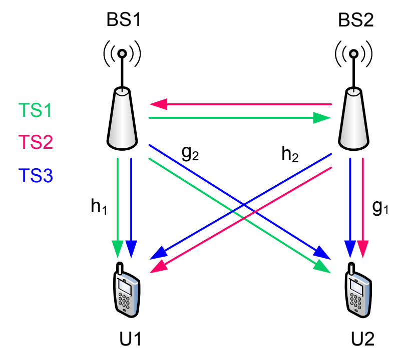

Fig. 1. System model.

U1 and U2 can be directly computed as

$$\begin{aligned} C_{\text{U1}}^{\text{TDMA}-2} &= \frac{1}{2}\mathbb{E}\{\log_2\left(1+\gamma|h_1|^2\right)\} \\ C_{\text{U2}}^{\text{TDMA}-2} &= \frac{1}{2}\mathbb{E}\{\log_2\left(1+\eta|g_1|^2\right)\}, \end{aligned} \tag{1}$$

where the expected value is calculated over the distribution of the channel coefficients. We define h_1 and h_2 as the direct and interfering channel gains of U1 from BS1 and BS2, and g_1 and g_2 as the direct and the interfering channel gains of U2 from BS2 and BS1.

One way of increasing the spatial diversity would be to use the overhearing capabilities of the system to share the transmitted symbols between the base stations. In this way, each symbol can reach its destination following more than one signal path. For instance, BS1 transmits s_1 to both U1 and BS2 in time-slot 1, then BS2 transmits s_2 to both U2 and BS1 in time-slot 2. The same happens in time-slots 3 and 4, but this time BS1 transmits s_2 and BS2 transmits s_1. We refer to this as TDMA-4. In comparison to TDMA-2, TDMA-4 achieves diversity gain at the cost of increasing the transmission time. The capacity per time-slot of TDMA-4 for U1 and U2 can be directly computed as

$$\begin{aligned} C_{\text{U1}}^{\text{TDMA}-4} &= \frac{1}{4}\mathbb{E}\{\log_2\left(1+\gamma|h_1|^2+\eta|h_2|^2\right)\} \\ C_{\text{U2}}^{\text{TDMA}-4} &= \frac{1}{4}\mathbb{E}\{\log_2\left(1+\eta|g_1|^2+\gamma|g_2|^2\right)\}. \end{aligned} \tag{2}$$

2.2. Three-Time-Slot Transmission Strategy

By exploiting network coding principles, it is possible to achieve diversity gain using only three time-slots. In the first time-slot (TS1), BS1 transmits s_1 to U1, U2, and BS2. In the second time-slot (TS2), BS2 transmits s_2 to U1, U2, and BS1. In the third time-slot (TS3), each base station acts as relay to transmit the received symbol (s_2 for BS1 and s_1 for BS2) to U1 and U2. This is shown in Fig. 1. Assuming a channel coherence time larger than 3 time-slots and equal power allocation along time-slots, the received signals for U1 in the three time-slots can be expressed as:

$$\begin{aligned} y_{\mathrm{U1}}^{(1)} &= \sqrt{P}h_1 s_1 + n_{\mathrm{U1}}^{(1)} \\ y_{\mathrm{U1}}^{(2)} &= \sqrt{Q}h_2 s_2 + n_{\mathrm{U1}}^{(2)} \\ y_{\mathrm{U1}}^{(3)} &= \sqrt{P}h_1 s_2 + \sqrt{Q}h_2 s_1 + n_{\mathrm{U1}}^{(3)}, \end{aligned} \tag{3}$$

where $y_{\mathrm{U1}}^{(t)}$ is the received signal for U1 in time-slot t, P and Q are the transmit powers of BS1 and BS2, and $n_{\mathrm{U1}}^{(t)}$ is the AWGN noise for U1 in time-slot t. A similar set of equations describes the received signals for U2. The advantage of this approach is not only the reduced number of time-slots for transmission, but also the spatial diversity achieved from the transmission of two base stations, while no additional infrastructure nor bandwidth is needed.

To obtain a performance upper bound, let us now derive the single-user capacity of the proposed transmission strategy. The received signal $\mathbf{y} = \begin{bmatrix} y_{\mathrm{U1}}^{(1)} & y_{\mathrm{U1}}^{(2)} & y_{\mathrm{U1}}^{(3)} \end{bmatrix}^T$, can be expressed in matrix form as

$$\begin{bmatrix} y_{\mathrm{U1}}^{(1)} \\ y_{\mathrm{U1}}^{(2)} \\ y_{\mathrm{U1}}^{(3)} \end{bmatrix} = \begin{bmatrix} \sqrt{P}h_1 & 0 \\ 0 & \sqrt{Q}h_2 \\ \sqrt{Q}h_2 & \sqrt{P}h_1 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_{\mathrm{U1}}^{(1)} \\ n_{\mathrm{U1}}^{(2)} \\ n_{\mathrm{U1}}^{(3)} \end{bmatrix}, \tag{4}$$

or in vector form as

$$\mathbf{y} = \mathbf{a}s_1 + \mathbf{b}s_2 + \mathbf{n} = \mathbf{a}s_1 + \mathbf{w}, \tag{5}$$

where we define the vectors $\mathbf{a} = \begin{bmatrix} \sqrt{P}h_1 & 0 & \sqrt{Q}h_2 \end{bmatrix}^T$, $\mathbf{b} = \begin{bmatrix} 0 & \sqrt{Q}h_2 & \sqrt{P}h_1 \end{bmatrix}^T$, $\mathbf{n} = \begin{bmatrix} n_{\mathrm{U1}}^{(1)} & n_{\mathrm{U1}}^{(2)} & n_{\mathrm{U1}}^{(3)} \end{bmatrix}^T$, and $\mathbf{w} = \mathbf{b}s_2 + \mathbf{n}$, which is defined as the interference-plus-noise vector of U1. Assuming a Gaussian input distribution, the optimal capacity per time-slot of U1 can be calculated as

$$C_{\mathrm{U1}}^{\mathrm{Opt}} = \frac{1}{3}\left[H(\mathbf{y}) - H(\mathbf{w})\right] = \frac{1}{3}\log\frac{|\mathbf{R}_y|}{|\mathbf{R}_w|}, \tag{6}$$

where $H(\mathbf{y})$ and $H(\mathbf{w})$ are the entropies of $\mathbf{y}$ and $\mathbf{w}$, and $\mathbf{R}_y$ and $\mathbf{R}_w$ are the covariance matrices of $\mathbf{y}$ and $\mathbf{w}$, respectively. The last equality in (6) is obtained by using the well-known expression for the entropy of a multivariate complex Gaussian distribution [11]. For given channel coefficients, $\mathbf{R}_w$ can be computed as

$$\mathbf{R}_w = \mathbb{E}\{(\mathbf{b}s_2 + \mathbf{n})(\mathbf{b}s_2 + \mathbf{n})^H\} = \mathbf{b}\mathbf{b}^H\sigma_x^2 + \mathbf{I}\sigma_n^2 \tag{7}$$

and $\mathbf{R}_y$ as

$$\mathbf{R}_y = \mathbb{E}\{(\mathbf{a}s_1 + \mathbf{w})(\mathbf{a}s_1 + \mathbf{w})^H\} = \mathbf{a}\mathbf{a}^H\sigma_x^2 + \mathbf{b}\mathbf{b}^H\sigma_x^2 + \mathbf{I}\sigma_n^2. \tag{8}$$

Hence, equation (6) can be re-written as

$$\begin{aligned} C_{\mathrm{U1}}^{\mathrm{Opt}} &= \frac{1}{3}\log|\mathbf{R}_w^{-1}(\mathbf{R}_w + \mathbf{a}\mathbf{a}^H\sigma_x^2)| \\ &= \frac{1}{3}\log\left(1 + \mathbf{a}^H\mathbf{R}_w^{-1}\mathbf{a}\sigma_x^2\right). \end{aligned} \tag{9}$$

Then, using equations (7), (8), (9), and the results presented in the appendix, $C_{\mathrm{U1}}^{\mathrm{Opt}}$ can be expressed as

$$C_{\mathrm{U1}}^{\mathrm{Opt}} = \frac{1}{3}\mathbb{E}\left\{\log_2\left(1 + \gamma|h_1|^2 + \eta|h_2|^2\frac{1+\eta|h_2|^2}{1+\gamma|h_1|^2+\eta|h_2|^2}\right)\right\}, \tag{10}$$

where γ and η are defined in section 2.1. Similarly, $C_{\mathrm{U2}}^{\mathrm{Opt}}$ can be expressed as

$$C_{\mathrm{U2}}^{\mathrm{Opt}} = \frac{1}{3}\mathbb{E}\left\{\log_2\left(1 + \eta|g_1|^2 + \gamma|g_2|^2\frac{1+\gamma|g_2|^2}{1+\eta|g_1|^2+\gamma|g_2|^2}\right)\right\}. \tag{11}$$

Interestingly, when $\gamma|h_1|^2 \ll \eta|h_2|^2$ (low SNR) equation (10) can be approximated with the expression $C_{\mathrm{U1}}^{\mathrm{Opt}} \approx \frac{1}{3}\mathbb{E}\{\log_2\left(1+\eta|h_2|^2\right)\}$. Also, when $\gamma|h_1|^2 \gg \eta|h_2|^2$ (high SNR) it can be approximated with the expression $C_{\mathrm{U1}}^{\mathrm{Opt}} \approx \frac{1}{3}\mathbb{E}\{\log_2\left(1+\gamma|h_1|^2\right)\}$. The same happens for U2. In other words, the system achieves diversity for both users in low and high SNR conditions. It is worth to notice that the symmetry of equations (10) and (11) shows the fairness of the proposed strategy. This suggests that our approach is able to find an efficient balance between spatial diversity and transmission time.

2.3. MMSE Reception Strategy

In this section the MMSE reception strategy for the proposed transmission strategy will be studied. The MMSE receiver is well known for being an efficient low-complexity linear receiver. Furthermore, it has been shown that it provides sufficient statistics to detect the input signal when it follows a Gaussian distribution [1].

Following [1], the MMSE receiver for U1 can be derived in two steps. The first step consists in whitening the colored noise term $\mathbf{w}$ from equation (5). This is achieved by filtering the received signal in (5) with the matrix $\mathbf{R}_w^{-1/2}$, obtaining

$$\mathbf{R}_w^{-1/2}\mathbf{y} = \mathbf{R}_w^{-1/2}\mathbf{a}s_1 + \mathbf{R}_w^{-1/2}\mathbf{w}. \tag{12}$$

The second step consists in performing maximum ratio combining (MRC) over the remaining signal, which is an optimal way of processing the signal when the additive noise is white [1]. This is achieved by taking the inner product of the signal of (12) and the vector $\mathbf{R}_w^{-1/2}\mathbf{a}$, giving the following:

$$\begin{aligned} \hat{z} = (\mathbf{R}_w^{-1/2}\mathbf{a})^H\mathbf{R}_w^{-1/2}\mathbf{y} &= \mathbf{a}^H\mathbf{R}_w^{-1}\mathbf{a}s_1 + \mathbf{a}^H\mathbf{R}_w^{-1}\mathbf{w} \\ &= \hat{z}_{\mathrm{sig}} + \hat{z}_{\mathrm{noise}}. \end{aligned} \tag{13}$$

From equation (13), the signal power can be calculated as

$$\begin{aligned} \mathbb{E}\{|\hat{z}_{\mathrm{sig}}|^2\} &= \mathbb{E}\{(\mathbf{a}^H\mathbf{R}_w^{-1}\mathbf{a}s_1)(\mathbf{a}^H\mathbf{R}_w^{-1}\mathbf{a}s_1)^H\} \\ &= (\mathbf{a}^H\mathbf{R}_w^{-1}\mathbf{a})^2\sigma_x^2 \end{aligned} \tag{14}$$

and the noise power as

$$\begin{aligned} &\mathbb{E}\{|\hat{z}_{\mathrm{noise}}|^2\} = \mathbb{E}\{(\mathbf{a}^H\mathbf{R}_w^{-1}\mathbf{w})(\mathbf{a}^H\mathbf{R}_w^{-1}\mathbf{w})^H\} \\ &= (\mathbf{R}_w^{-1/2}\mathbf{a})^H\mathbb{E}\{\mathbf{R}_w^{-1/2}\mathbf{w}(\mathbf{R}_w^{-1/2}\mathbf{w})^H\}\mathbf{R}_w^{-1/2}\mathbf{a} = \mathbf{a}^H\mathbf{R}_w^{-1}\mathbf{a}. \end{aligned} \tag{15}$$

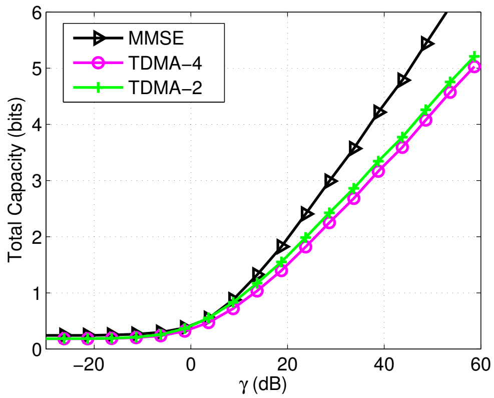

Fig. 2. Total capacity per time-slot with $\eta = -3dB$.

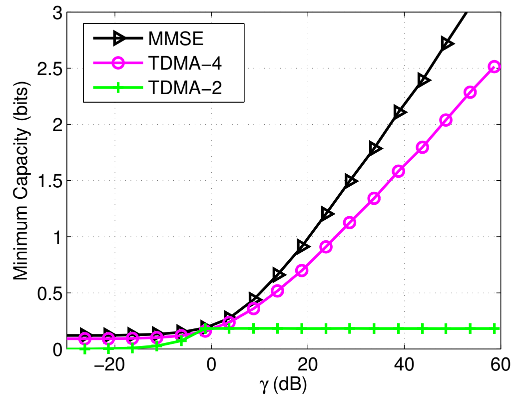

Fig. 3. Minimum capacity per time-slot with $\eta = -3dB$.

Using equations (14) and (15), the SNR of U1 using the MMSE receiver can be found to be

$$\mathrm{SNR}_{\mathrm{U1}}^{\mathrm{MMSE}} = \frac{\mathbb{E}\{|\hat{z}_{\mathrm{sig}}|^2\}}{\mathbb{E}\{|\hat{z}_{\mathrm{noise}}|^2\}} = \mathbf{a}^H \mathbf{R}_w^{-1} \mathbf{a} \sigma_x^2. \tag{16}$$

A similar analysis can be done for U2. Since the SNR of equation (16) is equal to the argument of the right-hand side term of (9), it is clear that the MMSE receiver is able to achieve the system capacity for both users.

3. PERFORMANCE EVALUATION

In this section we compare the approaches studied in Section 2 in terms of total and minimum capacity per time-slot of the system. The total capacity is the sum of the capacities of U1 and U2, which measures the performance of the whole system. The minimum capacity is the smallest capacity of both users and measures the fairness of the approach. We analyze the performance under various values of γ for two fixed values of η: low interference (-3dB) and high interference (17dB). Our evaluations use an average of Rayleigh fading channels with $\mathbb{E}\{|h_1|^2\} = \mathbb{E}\{|h_2|^2\} = \mathbb{E}\{|g_1|^2\} = \mathbb{E}\{|g_2|^2\} = 1$, $\sigma_x^2 = 1$ and $\sigma_n^2 = 1$.

Results show that the proposed approach (referred to as MMSE) has the best performance in terms of total capacity for most values of γ and η compared to the baseline approaches, confirming the results of our analysis presented in section 2.3 (see Fig. 2 and 4). This holds also for the minimum capacity, which indicates that MMSE also achieves fairness between users (see Fig. 3 and 5).

TMDA-4 uses an additional time-slot compared to the MMSE approach, hence it presents a decrease of a factor close to $\frac{3}{4}$ for both the total and the minimum capacity. As TDMA-4 also exploits spatial diversity, it shows fairness among users.

Interestingly, TDMA-2 can achieve a larger total capacity than all the other studied approaches in the region where $\gamma \approx \eta$, especially when both γ and η are high (see Fig. 4). This is due to the fact that it is better to use less time-slots for transmission than exploiting diversity because in this region the total capacity increases linearly with TDMA-2 and not logarithmically. Although TDMA-2 shows a high performance in terms of total capacity, it is not a fair approach as its minimum capacity is usually far below the other approaches. In fact, it shows a flooring when $\gamma > \eta$ because the increase in γ benefits one user until its capacity reaches the capacity of the other user.

As a final remark, although the MMSE approach is the best in most SNR and interference conditions, an adaptive approach that switches between using three time-slots with diversity and using two time-slots without diversity (only when $\gamma \approx \eta$) may provide some additional performance gain especially for the total capacity. The analysis of such approach is a topic of ongoing work.

4. CONCLUSIONS

In this paper we have proposed a double relay communication protocol for bandwidth management in a cellular system with two base stations and two users based on principles of network coding. The proposed approach consists of a three-time-slot transmission strategy and a MMSE reception strategy. It avoids the need of tight frequency or timing synchronization between base stations through a simple communication protocol, and it does not require additional infrastructure as it uses the base stations as relays. The proposed approach is able to find an efficient balance between spatial diversity and transmission time. We have shown that it achieves the system capacity in all SNR conditions. Furthermore, it reaches fairness among users when compared to other time-multiplexing approaches.

A. APPENDIX

In this section, an alternative expression for the SNR of U1 given by

$$\mathrm{SNR}_{\mathrm{U1}} = \mathbf{a}^H \mathbf{R}_w^{-1} \mathbf{a} \sigma_x^2 \tag{17}$$

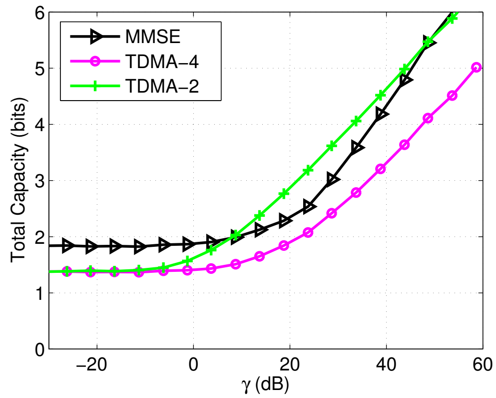

Fig. 4. Total capacity per time-slot with $\eta = 17dB$.

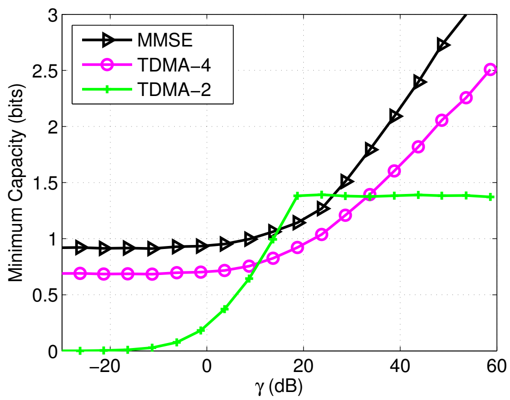

Fig. 5. Minimum capacity per time-slot with $\eta = 17dB$.

is derived. Using the Sherman-Morrison-Woodbury formula

$$(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} = \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{B}(\mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{C}\mathbf{A}^{-1}, \tag{18}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are arbitrary matrices, and substituting the following matrices $\mathbf{A} = \mathbf{I}\sigma_n^2$, $\mathbf{B} = \mathbf{b}$, $\mathbf{C} = \mathbf{b}^H\sigma_x^2$, $\mathbf{D} = -1$, then $\mathbf{R}_w^{-1}$ from equation (7) becomes

$$\mathbf{R}_w^{-1} = \frac{1}{\sigma_n^2}\mathbf{I} - \frac{\sigma_x^2}{\sigma_n^4}\left(1 + \frac{\sigma_x^2}{\sigma_n^2}\mathbf{b}^H\mathbf{b}\right)^{-1}\mathbf{b}\mathbf{b}^H. \tag{19}$$

Now, substituting $\mathbf{R}_w^{-1}$ in equation (17) we obtain

$$\begin{aligned}\mathrm{SNR}_{\mathrm{U1}} &= \mathbf{a}^H\left(\frac{1}{\sigma_n^2}\mathbf{I} - \frac{\sigma_x^2}{\sigma_n^4}\left(1 + \frac{\sigma_x^2}{\sigma_n^2}\mathbf{b}^H\mathbf{b}\right)^{-1}\mathbf{b}\mathbf{b}^H\right)\mathbf{a}\sigma_x^2 \\ &= \frac{\sigma_x^2}{\sigma_n^2}\mathbf{a}^H\mathbf{a} - \frac{\sigma_x^2}{\sigma_n^4}\left(1 + \frac{\sigma_x^2}{\sigma_n^2}\mathbf{b}^H\mathbf{b}\right)^{-1}|\mathbf{b}^H\mathbf{a}|^2\sigma_x^2.\end{aligned} \tag{20}$$

Using the definition of vectors $\mathbf{a}$ and $\mathbf{b}$ from section 2.2 we find that

$$\begin{aligned}\mathbf{a}^H\mathbf{a} &= P|h_1|^2 + Q|h_2|^2 \\ |\mathbf{b}^H\mathbf{a}|^2 &= PQ|h_1|^2|h_2|^2 \\ \mathbf{b}^H\mathbf{b} &= P|h_1|^2 + Q|h_2|^2.\end{aligned} \tag{21}$$

Substituting equations (21) in (20) we find that

$$\begin{aligned}&\mathrm{SNR}_{\mathrm{U1}} = \frac{\sigma_x^2}{\sigma_n^2}\left(P|h_1|^2 + Q|h_2|^2\right) - \ldots \\ &\left(1 + \frac{\sigma_x^2}{\sigma_n^2}(P|h_1|^2 + Q|h_2|^2)\right)^{-1} PQ|h_1|^2|h_2|^2\frac{\sigma_x^4}{\sigma_n^4} \\ &= \frac{\sigma_x^2}{\sigma_n^2}P|h_1|^2 + \frac{\sigma_x^2}{\sigma_n^2}Q|h_2|^2\left(\frac{1 + \frac{\sigma_x^2}{\sigma_n^2}Q|h_2|^2}{1 + \frac{\sigma_x^2}{\sigma_n^2}\left(P|h_1|^2 + Q|h_2|^2\right)}\right).\end{aligned} \tag{22}$$

Following an analogous derivation, $\mathrm{SNR}_{\mathrm{U2}}$ is given by

$$\begin{aligned}\mathrm{SNR}_{\mathrm{U2}} &= \frac{\sigma_x^2}{\sigma_n^2}Q|g_1|^2 + \ldots \\ &\frac{\sigma_x^2}{\sigma_n^2}P|g_2|^2\left(\frac{1 + \frac{\sigma_x^2}{\sigma_n^2}P|g_2|^2}{1 + \frac{\sigma_x^2}{\sigma_n^2}\left(Q|g_1|^2 + P|g_2|^2\right)}\right).\end{aligned} \tag{23}$$

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