ANALOG JOINT SOURCE CHANNEL CODING FOR MIMO BROADCAST CHANNELS

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ABSTRACT

We study the application of analog Joint Source Channel Coding (JSCC) techniques for the transmission of discrete-time, continuous-amplitude source information symbols over the Multiple-Input Multiple-Output (MIMO) Broadcast Channel (BC). Two channel access methods are proposed with different requirements regarding channel knowledge at transmission: Code Division Multiple Access (CDMA) and linear Minimum Mean Square Error (MMSE). The obtained results show that the CDMA scheme performs rather well when the channel responses are unknown at transmission, whereas the linear MMSE access approaches the Optimal Performance Theoretically Attainable (OPTA) when the channel information is also available at transmission.

Index Terms— JSCC, BC, CDMA, MMSE Transceiver.

1. INTRODUCTION

This work addresses the transmission of discrete-time, continuous-amplitude data over a Multiple-Input Multiple-Output (MIMO) Broadcast Channel (BC). In this model, a centralized transmitter (base station) with multiple antennas sends independent information to several decentralized receivers (users) also with multiple antennas.

When analog information is transmitted over any communication channel, lossy source-channel systems are optimal if the source distortion and the channel cost lie on the optimal cost-distortion tradeoff curve. In general, the optimal cost-distortion tradeoff for single-user point-to-point communications can be achieved by source-channel coding separation [1]. Nevertheless, several works in the literature show that separation of source and channel coding is not necessarily optimum for several multiuser scenarios [2–4], and it does not hold for the particular case of BCs [5, 6].

Analog Joint Source Channel Coding (JSCC) is an alternative strategy where complexity and delay are drastically reduced by jointly optimizing the source and the channel encoder. The utilization of analog JSCC for multiuser communications has already been investigated in [7–9]. Nevertheless, these works focus on multiuser Gaussian channels. The design of practical analog mappings that approach the optimal bounds over fading channels is considerably more difficult. To circumvent this limitation, [10] proposes a distributed JSCC architecture for fading Multiple Access Channels (MAC). This approach is based on the use of single-user mappings together with an adequate channel access scheme. This strategy not only simplifies the design of analog JSCC schemes for multiuser communications, but also has been shown to approach the optimal distortion-cost tradeoff.

In this work, we exploit this idea for the broadcasting of independent data from the base station to multiple users over MIMO block-fading channels using analog JSCC. We consider two access methods to ensure a reliable transmission of the analog JSCC symbols: Code Division Multiple Access (CDMA) and linear Minimum Mean Square Error (MMSE).

2. SYSTEM MODEL

Let us consider the transmission of $N$ independent streams of discrete-time continuous-amplitude information symbols from a base station to $N$ distributed users over a shared block-fading BC. The base station and the users are equipped with $n_T$ transmit antennas and $n_R_i$ receive antennas, $i = 1, \ldots, N$, respectively. The received signal at time $k$ and at the $i$-th user can be hence expressed as

$$y_{ik} = \sqrt{\frac{P_i}{n_T}}H_i z_k + n_{ik},$$

where $P_i$ is the available transmit power, $z_k \in \mathbb{C}^{n_T \times 1}$ is the vector of transmitted symbols at time $k$, $H_i \in \mathbb{C}^{n_R_i \times n_T}$ represents the block-fading Multiple-Input Multiple-Output (MIMO) channel between the base station and the $i$-th user, and $n_{ik} \sim \mathcal{CN}(0, N_0 I_{n_R_i})$ is the Additive White Gaussian Noise (AWGN) at user $i$ and time $k$. The BC is assumed to remain static during the transmission of a packet of symbols, but independently varies from one packet to another according to a stationary and ergodic process. Without loss of generality, we assume that the variance of the random variables $z$ and $H_i$, as well as the AWGN variance, $N_0$, are equal to one. Finally, the individual user rates $R_i$ are defined as the total number of analog symbols per channel use sent to user $i$.

Fig. 1 shows the proposed analog JSCC scheme for the reliable transmission of analog information over the MIMO BC described above. We follow a two step approach where single-user mappings are concatenated with a channel access
method. Such approach is suitable for the transmission of independent analog source streams in multiuser scenarios [10].

3. ANALOG JOINT SOURCE CHANNEL CODING

We consider analog JSCC approaches based on the use of continuous space-filling mappings that directly transform \( M \) consecutive source symbols into \( L \) channel symbols [11, 12]. The \( N \) streams of user symbols are first generated from \( N \) independent and identically distributed (i.i.d.) Gaussian sources with zero mean and variance \( \sigma^2 \), and then they can be directly transmitted with no additional coding or analog encoded with a 2:1 compression rate.

When the source bandwidth exactly matches the channel bandwidth (i.e. \( M = L \)), each user symbol \( s_{ik} \), \( k = 1, \ldots, N \), is first normalized and then input directly to the corresponding channel access scheme. Hence \( x_{ik} = s_{ik}/\sqrt{\sigma^2} \).

In the case of 2:1 compression, the encoding operation comprises three stages: mapping, stretching function and normalization. Mapping is performed with the function \( M_k \) that maps two consecutive source symbols \( (s_{i,2k}, s_{i,2k+1}) \) into one channel symbol \( x_{ik} \). We specifically consider the mapping given by a non-linear doubly interleaved Archimedean spiral [11, 12] defined as

\[
\alpha_i(\theta) = \left[ \text{sign}(\theta), \frac{\theta}{\pi} \sin \theta, \frac{\theta}{\pi} \cos \theta \right]^T, \tag{2}
\]

where \( \theta \) is the angle from the origin to the point \( \alpha = [a_1, a_2]^T \) on the curve, and \( \delta_i \) is the distance between two neighbouring arms of the spiral corresponding to user \( i \).

The mapping operation basically consists in calculating the angle from the origin to the point on the spiral that minimizes the (Euclidean) distance to the 2-D point given by \( s_{ik} = (s_{i,2k}, s_{i,2k+1}) \). Thus,

\[
\hat{\theta}_{ik} = M_{\delta_i}(s_{ik}) = \arg\min_{\theta} ||s_{ik} - \alpha_i(\theta)||^2. \tag{3}
\]

The compressed symbols at the output of the mapping operation are then transformed by using the stretching function \( T_{\delta_i}(\hat{\theta}_{ik}) = (\hat{\theta}_{ik})^{\alpha_i} \). This transformation provides certain degrees of freedom for the optimization of the analog JSCC system. In [11, 12] \( \alpha_i = 2 \) was proposed to simplify the distortion analysis but, system performance can be improved if \( \alpha_i \) is optimized together with \( \delta_i \) [13]. Finally, the coded value is normalized by a factor \( \gamma_i \) to ensure the average transmitted power is equal to one. Hence, the input symbols to the channel access scheme are given by

\[
x_{ik} = \frac{T_{\alpha_i}(M_{\delta_i}(s_{ik}))}{\sqrt{\gamma_i}}. \tag{4}
\]

At the receiver, an estimate of the source symbols is calculated using ML decoding. As shown in [14], this method achieves near optimal performance as long as the received symbols are conveniently filtered prior to decoding.

In general, analog JSCC systems closely approach the optimal distortion-cost tradeoff as long as the encoder parameters \( \delta_i \) and \( \alpha_i \) are conveniently optimized depending on the instantaneous SNR. In a practical setup, the receive SNRs can be estimated at the receivers and sent to the base station over a feedback channel. When the channel information is not available at the encoders, the values for \( \delta_i \) and \( \alpha_i \) corresponding to the expected average SNR can be used.

4. CHANNEL ACCESS SCHEMES

Channel access schemes for broadcast communications can be designed according to diverse objectives such as minimizing the transmission power, ensuring a certain distribution of the user rates or maximizing the transmission reliability. In this work, we focus on the minimization of the total power required to achieve a given distortion (MSE) target at each individual user. In the ensuing sections we propose two access methods for the transmission of analog JSCC symbols depending on whether perfect Channel State Information (CSI) is only available at the users or also at the transmitter: Orthogonal CDMA and Linear MMSE Access.

4.1. Orthogonal CDMA

When no channel information is available at the base station, the use of a CDMA scheme based on the use of orthogonal spreading codes allows users transmit their information simultaneously and continuously. The spreading codes are constructed from unitary matrices (e.g. Hadamard matrices) according to given rate constraints. Let us assume that \( R_i = k_i/K, i = 1, \ldots, N \), is the data rate for user \( i \), such that \( \sum_{i=1}^{N} k_i = K \). Thus, user \( i \) should transmit a vector of \( k_i \) analog JSCC symbols \( x_i = [x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}]^T \) along \( K \) channel uses. In this case, the orthogonal code \( C \) will be an \( n_T \times K \) matrix that is constructed by stacking \( n_T \) unitary matrices of order \( K \), i.e.

\[
C = \frac{1}{\sqrt{K}}[U_1 U_2 \cdots U_{n_T}]^T,
\]

where \( T \) represent transpose operation. If we now stack the analog JSCC symbols corresponding to all users into a single vector \( x = [x_1^T, \cdots, x_N^T]^T \), the matrix \( C \) combines the resulting \( K \) symbols to produce a vector of \( n_T \times K \) channel symbols \( \mathbf{z} = \mathbf{Cx} \). These symbols are then sent over the BC using \( n_T \) antennas and \( K \) different channel uses. Notice that in order to ensure that \( E[|\mathbf{z}|^2] = P_t \), the elements of the matrix \( C \) must be normalized by a factor \( 1/\sqrt{K} \) since the base station actually sends a combination of \( K \) symbols at each time instant.
A remarkable property of the proposed CDMA scheme is its flexibility to achieve any distribution of the user rates by properly selecting \( k_i \) and \( K \). In addition, no cooperation among users is required and, therefore, the use of these orthogonal codes provides a simple mechanism to guarantee that all users achieve the target rate.

The CDMA signal received at user \( i \) and time \( k \) is directly obtained from (1) as

\[
y_{ik} = \frac{P_i}{n_T} H_i z_k + n_{ik} = \frac{P_i}{n_T} H_i C_i x + n_{ik}
\]

where \( H_i \) is the block-fading MIMO channel corresponding to user \( i \), \( C_i \) is the \( k \)-th column of the orthogonal code and \( n_{ik} \) is the AWGN at time \( k \). As mentioned in Section 3, an accurate decoding of the received analog JSCC symbols is possible as long as they are previously filtered to reduce the noise distortion.

The BC model given by (5) can be reformulated in a more compact way. If we define the matrix \( \hat{H}_i = \text{blockdiag}\{H_i\}_K \), the equivalent access channel matrix for the \( i \)-th user can be calculated as \( \hat{H}_i = \hat{H}_i C_i \) and the entire block of \( n_{R,K} \) symbols at receiver \( i \) is hence given by

\[
y_i = \sqrt{\frac{P_i}{n_T}} \hat{H}_i x + n_i
\]

where \( n_i = [n_{i1}, n_{i2}, \cdots, n_{iK}]^T \) is the AWGN matrix.

A linear filter \( G_i \) can now be applied to obtain estimates of the transmitted symbols from \( y_i \), i.e. \( \hat{x}_i = G_i y_i \). Since the objective of analog communications is to minimize the MSE between the source and decoded symbols, a natural receiver strategy is the use of MMSE filtering prior to analog ML decoding [14]. In this case, the linear MMSE filter is given by

\[
G_i = (\hat{H}_i^H \hat{H}_i + I)^{-1} \hat{H}_i^H.
\]

### 4.2. Linear MMSE Access

Given that the objective of analog transmissions is to achieve the lowest possible distortion, the channel knowledge at the transmitter can be conveniently exploited to design linear access codes that minimize the MSE distortion between the transmitted and filtered symbols. We continue to assume that the \( i \)-th user transmits at constant rate \( R_i = k_i / K, i = 1, \ldots, N \). Let us also consider that the \( k_i \) analog JSCC symbols to user \( i \) are linearly encoded with an \( n_{R,K} \times k_i \) (possibly) non-orthogonal matrix \( F_i \) to produce the vector of \( n_{R,K} \) channel symbols. The received signal at the \( i \)-th user can hence be expressed as

\[
y_i = H_i F_i x_i + \sum_{j \neq i} H_i F_j x_j + n_i,
\]

where \( n_i \) is an \( n_{R,K} \) vector that represents the AWGN. As in CDMA, the received symbols are filtered before applying the ML decoder. A linear filter \( G_i \) is employed at receiver \( i \) to obtain an estimate of the encoded symbols as \( \hat{x}_i = G_i y_i \). The resulting signal is given by

\[
\hat{x}_i = G_i H_i F_i x_i + \sum_{j \neq i} G_i H_i F_j x_j + G_i n_i.
\]

We can now rewrite as

\[
y = G_i H_i F_i x + n.
\]

BC communications are usually subject to certain quality requirements. In this case, the linear access codes \( F_i \) and the corresponding receive filters \( G_i \) are designed to achieve a given set of MSE targets with minimum transmit power \( P_i \).

In the BC model, given a set of MSE targets, \( \epsilon_{BC} = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_N\} \), we can define the achievable region of transmit powers \( \mathcal{R}_P(\epsilon_{BC}) \) as the set of those powers that ensure the information corresponding to each user \( i \) is recovered with distortion \( \epsilon_i \), i.e.

\[
\mathcal{R}_P(\epsilon_{BC}) = \{ P_i \mid \epsilon_i \leq \epsilon_i \ \forall i \},
\]

where \( \epsilon_{BC} = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_N\} \) represents the effective MSEs after decoding the user information. The objective is hence to find the matrices \( F_i \) (and implicitly \( G_i \)) with the minimum transmit power such that \( \text{tr}\{FF^H\} \leq \mathcal{R}_P(\epsilon_{BC}) \). This minimization problem can be formally defined as

\[
\min_{F,G} \text{tr}\{FF^H\} \quad \text{s.t.} \quad \forall i \ \epsilon_i \leq \epsilon_i.
\]

Such matrices can be found following the per-user MSE constrained power minimization solution presented in [15]. In particular, we propose to employ the algorithm referred to as Algorithm 5 which exploits the MSE duality between the downlink and the virtual uplink channels, i.e. conversion formulas that allow to exchange from the BC to the dual MAC preserving both the total transmit power and the user MSEs. This way an alternating optimization of the MMSE filters can be performed by switching between the BC and the dual MAC. The MMSE receive filters \( G \) are computed in the BC whereas the MMSE linear filters \( F \) are calculated in the MAC. Moreover, the dual MAC formulation allows to write the MSE in matrix form so that the power allocation is computed efficiently as a matrix product.

### 4.2.1. Feasibility Region

An interesting question is whether the set of MSE targets, \( \epsilon_{BC} \), is simultaneously achievable for all users. Let us consider the joint channel and precoders response in the dual MAC, i.e., \( \Theta_i = H_i^H P_i \in \mathbb{C}^{n_{R,K} \times k_i} \). Then, when we stack previous matrices in \( \Theta = [\Theta_1, \ldots, \Theta_N] \), the MSE for the \( i \)-th user, using the indexes \( a = \sum_{j=1}^{i-1} k_j + 1 \) and \( b = \sum_{j=1}^{i} k_j \), reads as

\[
\xi_i = \text{tr}\left\{ I_K - \Theta^H \left( \Theta \Theta^H + \sigma^2 I_{n_{R,K}} \right)^{-1} \Theta \right\}_{a,b,a,b}.
\]
where \( \sigma^2 \) is the receivers noise variance. Following a procedure similar to that from [16], the sum-MSE is, accordingly,

\[
\sum_{i=1}^{N} \xi_i = K - n_T K + \text{tr} \left\{ I_{n_T K} + \sigma^{-2} \Theta^H \Theta \right\}. \tag{13}
\]

Thus, when we increase the power unlimitedly we obtain a lower bound for the sum-MSE. Since the matrix \( \Theta^H \Theta \) is positive semidefinite, the right side of (13) is lower bounded by \( n_T K - \text{rank}(\Theta) \), and then

\[
\sum_{i=1}^{N} \xi_i \geq K - \text{rank}(\Theta), \tag{14}
\]

which provides a test to check the feasibility of the MSE targets \( \epsilon_{BC} \).

### 5. Optimal Performance Theoretically Attainable (OPTA)

The performance of analog communications is upper bounded by the optimal cost-distortion tradeoff curve. The cost function is usually measured as the transmit power required to achieve a certain distortion. In the literature, this bound is referred to as the Optimum Performance Theoretically Attainable (OPTA).

As mentioned in the introduction, source-channel separation is not necessarily optimum for multiuser communications [2, 3, 5]. However, given that we consider independent information and no cooperation between users, the OPTA curve can be calculated by equating the separate regions corresponding to the rate distortion function and the channel capacity [17].

Given a set of distortion targets \( D = \{D_1, D_2, \ldots, D_N\} \), the rate-distortion region \( \mathcal{R}(D) \) is defined by the rate-distortion functions corresponding to the individual sources, i.e. \( R_i^D(D_i) \), \( i = 1, \ldots, N \), and by the sum-rate distortion function \( R_{\text{sum}}^D(D) \). Notice that \( R_{\text{sum}}^D(D) \) determines the minimum number of bits required to achieve the distortions in \( D \) when the \( N \) sources are jointly sent.

In the particular case of memoryless complex-valued Gaussian sources and the Mean Square Error (MSE) distortion metric, the rate distortion function is

\[
R_i^D(\text{SDR}) = \max[\log_2(\text{SDR}), 0], \tag{15}
\]

where the Signal-to-Distortion Rate (SDR) is directly calculated as \( \text{SDR}_i = \sigma_i^2 / D_i \). Because of the independence assumption between the data streams corresponding to different users, the sum-rate distortion function can be computed as the sum of the individual source rates, i.e.

\[
R_{\text{sum}}^D(D_1, \ldots, D_N) = \sum_{i=1}^{N} R_i^D(D_i). \tag{16}
\]

Similarly, the capacity region \( \mathcal{R}(C) \) is defined by the capacities corresponding to the individual channels, i.e. \( C_i(\mathbf{H}_i) \), and by the BC sum-capacity \( C_{\text{sum}}(\mathbf{H}_1, \ldots, \mathbf{H}_N) \). The expression for the individual capacities when using linear filters is specified at the top of the next page and the sum-capacity can be computed as the sum of these capacities.

A reliable transmission of analog information is possible over the BC as long as the regions corresponding to the rate-distortion and capacity satisfy that

\[
\mathcal{R}(D) \subseteq \mathcal{R}(C). \tag{17}
\]

The optimal situation is represented by the perfect matching between the rate-distortion region and the capacity region. We can hence calculate an optimal cost-distortion region from (17) by simply equating both regions. However, since we focus on the individual distortions of the BC users, we can determine the OPTA bound for each user from the equality

\[
M k_i \log_2(\text{SDR}_{\text{opt}}) = K C_i(\mathbf{H}_i), \tag{18}
\]

where \( M \) is the compression factor, while \( k_i \) and \( K \) are given by the rate of the \( i \)-th user. The term \( \text{SDR}_{\text{opt}} \) represents the minimum distortion user \( i \) may achieve or, equivalently, the optimal performance of the analog JSCC system for such user.

### 6. Simulation Results

In this section, the results of several computer simulations are presented to illustrate the performance of the two proposed BC access schemes: orthogonal CDMA and linear MMSE. We consider the case where the base station transmits i.i.d. Gaussian symbols using\( n_T = 2 \) antennas to two users equipped with \( n_{R_i} = 2 \) antennas. Let us also assume the source information is sent with constant rates \( k_1 = 3/8 \) and \( k_2 = 5/8 \) to users 1 and user 2, respectively.

We specifically focus on the symmetrical situation, i.e. the source information is assumed to be recovered with the same average distortion at both BC users. Fig. 2 and Fig. 3 show the performance of the two considered access schemes when the analog samples are either uncoded (\( M = 1 \)) or compressed with rate 2:1 (\( M = 2 \)), respectively. System performance is measured in terms of SDR in dB, which is computed as \( \text{SDR}_i = 10 \log_{10}(\sigma_i^2 / D_i) \), with respect to the average SNR over all channel realizations. In these figures, we also represent the theoretical performance (OPTA) which is calculated according to (18). Notice that in the symmetrical situation, the same curves are obtained for the two users.

As observed in both figures, the overall performance of analog JSCC systems is substantially improved when the channel information is exploited to obtain the linear MMSE access codes. The SDR curve corresponding to CDMA remains about 2 dB and almost 4 dB below the performance curve of the linear MMSE access for uncoded transmission and 2:1 compression, respectively. Indeed, the performance of the analog system with MMSE access closely approaches the OPTA in both cases, although the performance loss of the 2:1 system with respect to the OPTA is motivated by the compression operation. Another remarkable result is the capacity of CDMA to provide satisfactory performance when the CSI is not available at the base station.
7. CONCLUSIONS

In this work, we have proposed two access schemes for the transmission of discrete-time continuous-amplitude information symbols over a block-fading BC using analog JSCC: Orthogonal CDMA and linear MMSE. Both access methods are specially suitable for BC systems with per-user rate constraints because they provide a flexible framework to obtain any distribution of the individual user rates. Computer simulation shows that CDMA provides good performance for the whole SNR region in spite of not using CSI. The obtained results also confirm that the system performance is substantially improved when linear MMSE access codes are employed.

REFERENCES