THROUGHPUT AND ENERGY OPTIMIZATION OF A COOPERATIVE ARQ SCHEME USING ANALOG NETWORK CODING

Mariem Ayedi, Noura Sellami†, Mohamed Siala*,*

* MEDIATRON laboratory
SUP'COM, Carthage University, Tunisia
mariem.ayedi@supcom.tn
mohamed.siala@supcom.rnu.tn

† LETI laboratory
ENIS, Sfax University, Tunisia
noura.sellami@isecs.rnu.tn

ABSTRACT

In this paper, we propose to improve a Cooperative Automatic Repeat reQuest (C-ARQ) scheme through the optimization of the distribution of the average relay energy cost in a two sources one destination wireless system. We consider a BPSK modulated transmission over Rayleigh fading channels. We propose to use specific relay and destination treatment for different possible cases: both received packets are error free, only one received packet is detected erroneous or both received packets are detected erroneous at the destination. We derive semi-analytical bounds of the throughput and the average relay energy. We find the optimal relay energy distribution which maximizes the throughput by using dichotomy search. Numerical results confirm the accuracy of the derived closed form expressions and show the throughput improvement due to the relay energy optimization compared to the conventional cooperative ARQ scheme.

Index Terms— Cooperative ARQ communication, Analog Network Coding, throughput and energy optimization.

1. INTRODUCTION

Cooperative communication in wireless networks constitutes an interesting diversity technique that offers significant gains in overall throughput and energy efficiency by extending coverage without requiring higher transmission power [1]. In particular, Cooperative Automatic Repeat reQuest (C-ARQ) protocols have been recently investigated to overcome the source destination poor channel conditions by requesting retransmissions from relay nodes whose had overheard the direct transmissions [2]. Different C-ARQ relaying protocols, using relay selection and adaptive techniques have been proposed and have been shown to improve the system performance in terms of throughput [3, 4]. Certainly, in such systems, throughput and capacity are critically constrained by the energy consumption because nodes are typically powered by batteries. The energy efficiency topic has been studied in the conventional ARQ protocols, such as in [5], and in cooperative systems [6], but not enough in C-ARQ. In [7], authors have studied the average transmission power of one source node communicating with one relay node and one destination node in a C-ARQ system while assuming a fixed relay node transmitted power over all retransmissions attempts. In this work, we are interested in a C-ARQ system where an Amplify and Forward (AF) relay node is asked through a feedback channel to retransmit, in case of errors detection, sources packets sent from two sources nodes to one common destination node. We propose to study the relay energy allocation according to two possible error detection events: only one source packet is detected erroneous and both sources packets are detected erroneous. Indeed, the relay amplifies and forwards the received packet in case of one packet detection error or a combination of both received packets, as in Analog Network Coding (ANC) scheme [8], in the case of two packets detection errors. Hence, we propose to allocate to the relay a different energy as the relay transmits one packet or the combination of two packets. We propose to find the optimal relay energy allocation to maximize the overall throughput efficiency constrained by an average relay energy cost. To this end, we give semi-analytical bounds of throughput and average relay energy and use dichotomy search to specify optimal relay energy distribution.

This paper is organized as follows. In Section 2, we present the proposed cooperative ARQ system. In Section 3, we study the relay energy optimization problem. In Section 4, we present numerical results. In Section 5, we give conclusions.

2. COOPERATIVE ARQ SCHEME

We consider a wireless cooperative communication system consisting of two sources $\{S_1, S_2\}$ communicating directly and through the aid of a half-duplex relay node R with one common destination node D as shown in figure 1. In the first time slot, the source node S_1 transmits its data packet to the destination node D. In the second time slot, S_2 transmits its data packet to D. In these two slots, the relay node R has overheard sources transmissions. We adopt a truncated (C-ARQ) protocol where the sources nodes and the relay node perform packets retransmissions until error free detection or a maximum number M of retransmissions (by the sources

978-0-9928626-3-3/15/$31.00 ©2015 IEEE

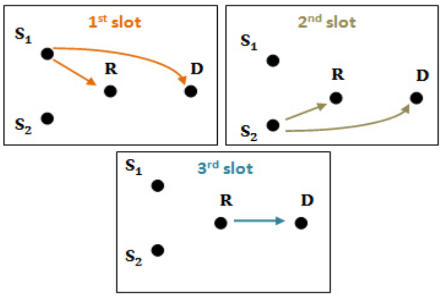

Fig. 1: Three-slots cooperative scheme in case of detected errors.

and the relay) is reached. A feedback channel between nodes is available for positive acknowledgement (ACK) or negative acknowledgement (NACK).

We consider a Rayleigh block fading channel denoted by h_{XY} between each transmitting node $X \in \{S_1, S_2, R\}$ and each receiving node $Y \in \{R, D\}$. We consider a path loss model given by

$$E(|h_{XY}|^2) = (\frac{d_{XY}}{d_0})^{-\alpha}, \qquad (1)$$

where $E(.)$ denotes the expectation, d_0 is a reference distance, d_{XY} is the distance between X and Y and α is the path loss exponent. We assume a perfect Channel State Information (CSI) knowledge of h_{XY} at the receiver node Y, $Y \in \{R, D\}$. Perfect error detection based on Cyclic Redundancy Check (CRC) is adopted and Binary Phase Shift Keying (BPSK) modulation is used by transmitting nodes (sources and relay). In the time slot $i \in \{1, 2\}$, the received signals at nodes R and D broadcasted by S_i are respectively

$$\mathbf{y}_{S_iR} = \sqrt{E_S} h_{S_iR} \mathbf{x}_{S_i} + \mathbf{n}_{S_iR}, \qquad (2)$$

$$\mathbf{y}_{S_iD} = \sqrt{E_S} h_{S_iD} \mathbf{x}_{S_i} + \mathbf{n}_{S_iD}, \qquad (3)$$

where E_S is the transmitted energy per symbol for each source, $\mathbf{x}_{S_i}$ is the transmitted packet by the node S_i and $\mathbf{n}_{S_ij}$ where $j \in \{R, D\}$ is the zero-mean complex additive white Gaussian noise vector with variance $\sigma_n^2 = N_0$.

For next slots, three different cases are possible :

- D decodes successfully the two transmitted packets of S_1 and S_2. In this case, R remains silent. The Signal to Noise Ratio (SNR) at the destination of each direct transmission denoted by Γ_{S_iD}, $i \in \{1, 2\}$ is expressed as

  $$\Gamma_{S_iD} = \frac{E_S|h_{S_iD}|^2}{N_0}. \qquad (4)$$

  In the two next slots, the two sources transmit their new packets as in the previous slots.

- D decodes successfully one of the two transmitted packets. In this case, R amplifies and forwards only the erroneous packet indexed i with $i \in \{1, 2\}$, then the received signal at D is

  $$\mathbf{y}_{RD} = \beta_i \sqrt{E_{R_1}} h_{RD} \mathbf{y}_{S_iR} + \mathbf{n}_{RD}, \qquad (5)$$

  where E_{R_1} is the relay node energy spent to retransmit one data packet and β_i is the amplification factor given by

  $$\beta_i = \frac{1}{\sqrt{E_S|h_{S_iR}|^2 + N_0}}. \qquad (6)$$

  At the destination node D, the Maximum Ratio Combining (MRC) is used. The decision variable is expressed as follows

  $$\tilde{\mathbf{x}}_i = \frac{\sqrt{E_S}h^*_{S_iD}}{N_0}\mathbf{y}_{S_iD} + \frac{\beta_i\sqrt{E_{R_1}E_S}h^*_{RD}h^*_{S_iR}}{N_0(\beta_i^2 E_{R_1}|h_{RD}|^2+1)}\mathbf{y}_{RD}. \qquad (7)$$

  As a result, the output SNR is expressed as the sum of the individual instantaneous SNRs and is given by

  $$\begin{aligned}\Gamma_{MRC} &= \Gamma_{S_iD} + \Gamma_{S_iRD} \\ &= \frac{E_S|h_{S_iD}|^2}{N_0} + \frac{\beta_i^2 E_{R_1} E_S |h_{RD}|^2 |h_{S_iR}|^2}{N_0(\beta_i^2 E_{R_1}|h_{RD}|^2 + 1)}.\end{aligned} \qquad (8)$$

  If D fails again in decoding the data packet, since channels are independent from slot to slot and for complexity and buffer size reasons, we propose to drop received signals in the two previous slots. Then, we refer to the source node S_i to retransmit the packet. The process of retransmitting the packet alternatively by the source node and the relay node will continue until successful decoding or reaching the maximum number of retransmissions.

- D fails in decoding both transmitted packets. Then, R amplifies and forwards the combination of both packets it receives from the sources. The received signal at D is

  $$\mathbf{y}_{RD} = G\sqrt{E_{R_2}} h_{RD} [\mathbf{y}_{S_1R} + \mathbf{y}_{S_2R}] + \mathbf{n}_{RD}, \qquad (9)$$

  where E_{R_2} is the relay node energy spent to retransmit two data packets and G is the amplification factor given by

  $$G = \frac{1}{\sqrt{E_S|h_{S_1R}|^2 + E_S|h_{S_2R}|^2 + 2N_0}}. \qquad (10)$$

  The decoding scheme here is based on successive interference cancellation using MRC. The destination D starts with the decoding of the data packet which has the best chance to be decoded correctly, i.e., which has the best Signal to Interference Noise Ratio (SINR) at the output of the MRC by treating the other packet as noise. In the case of successful decoding of the packet, its contribution is removed from the signal y_{RD}. Then, D continues the decoding of the other packet using the MRC. If D fails in decoding the packet, received packets during the two previous slots are dropped and the corresponding source is asked to retransmit its packet. In case of decoding failure at D, R retransmits it alternatively with the source until successful decoding or reaching the maximum number of retransmissions. In case of failure of decoding of both packets, received signals are dropped and both source nodes are asked to retransmit their data packets and the treatment is repeated from the beginning with the three possible cases.

3. RELAY ENERGY OPTIMIZATION

3.1. Optimization problem

We aim to optimize the relay node energy per symbol spent in the case where the relay retransmits one packet denoted by E_{R1}, and in the case where it retransmits the combination of two packets denoted by E_{R2}, so as to maximize the system throughput for a given constraint on the average relay energy cost per symbol E_{Ravg}:

$$\max_{E_{R1} \leq E_{R1max}, E_{R2} \leq E_{R2max}} Throughput \quad \text{subject to } E_{Ravg} \leq E_{Rtarget}, \tag{11}$$

where $E_{Rtarget}$ is the total target relay energy cost, E_{R1max} and E_{R2max} are the maximum allowed relay energies for transmitting one and two packets respectively, and

$$Throughput = \frac{k}{k+n_p} \times \frac{1-PPE}{E[T_r]}, \tag{12}$$

where k is the number of information bits per packet, n_p is the number of parity bits added, $E[T_r]$ is the average number of transmissions and PPE is the probability of packet erasure after M retransmissions.

In the following, we propose to derive closed bounds of $E[T_r]$, PPE and E_{Ravg}. We consider the case where $M = 6$. The generalization of the study to a greater number of retransmissions can be easily done.

3.2. Analysis of the throughput and the average relay energy

We denote by

- P_{S_iD} the probability of the event: the received packet at D through the direct link S_iD, $i \in \{1, 2\}$ contains detected errors.
- P_{1S} the probability of the event: only one received packet at D from one source node contains detected errors.
- P_{2S} the probability of the event: both received packets at D from source nodes contain detected errors.
- $P_{1S,1R}$ the probability of the event: only one received packet at D from one of the two sources nodes contains detected errors and after the retransmission of this packet by R, the estimated packet at D obtained using MRC contains also detected errors.
- $P_{2S,1R}$ the probability of the event: both received packets at D from sources nodes contain detected errors and after the retransmission by R, only one estimated packet at D obtained using MRC interference cancellation contains also detected errors.
- $P_{2S,2R}$ the probability of the event: both received packets at D from sources nodes contain detected errors and after the retransmission by R, both packets are still detected erroneous after MRC interference cancellation.

In the following, we first give the expressions of these different probabilities, then we analyse the throughput and the average relay energy.

The probabilities P_{1S} and P_{2S} can be written respectively as

$$P_{1S} = P_{S_1D}(1-P_{S_2D}) + P_{S_2D}(1-P_{S_1D}), \tag{13}$$

$$P_{2S} = P_{S_1D}P_{S_2D}, \tag{14}$$

where for a BPSK modulation, P_{S_iD} is given by

$$P_{S_iD} = 1 - \int \left[1 - \frac{1}{2}\text{erfc}(\sqrt{u})\right]^N p_{\Gamma_{S_iD}}(u)du, \tag{15}$$

where N is the size of the data packet and $p_{\Gamma_{S_iD}}$ is the Probability Density Function (PDF) of Γ_{S_iD} which follows a chi-square distribution with 2 degrees of freedom (exponential distribution) for Rayleigh fading channel. The PDF $p_{\Gamma_{S_iD}}$ is then given by

$$p_{\Gamma_{S_iD}}(u) = \frac{1}{\bar{\Gamma}_{S_iD}} e^{\frac{-u}{\bar{\Gamma}_{S_iD}}}, \tag{16}$$

where $\bar{\Gamma}_{S_iD} = E(\Gamma_{S_iD})$.

In the case of equal direct channels powers, we can write that $P_{S_1D} = P_{S_2D} = P_S$.

To derive the expression of $P_{1S,1R}$, we need to determine the statistical characterization of the MRC output SNR given in (8). In cooperative systems with AF relay, an upper bound of the equivalent SNR Γ_{S_iRD} is given by [9] as a function of the SNRs of the two hops Γ_{S_iR} and Γ_{RD}

$$\Gamma_{S_iRD} \prec \Gamma^{up}_{S_iRD} = \frac{\Gamma_{S_iR}\Gamma_{RD}}{\Gamma_{S_iR} + \Gamma_{RD}}, \tag{17}$$

where Γ_{S_iR} and Γ_{RD} are respectively given by

$$\Gamma_{S_iR} = \frac{E_S|h_{S_iR}|^2}{N_0} \text{ and } \Gamma_{RD} = \frac{E_{R_1}|h_{RD}|^2}{N_0}. \tag{18}$$

${\Gamma_{S_iRD}}^{up}$ follows a chi-square distribution of 2 degrees of freedom with a PDF given by

$$p^{up}_{\Gamma_{S_iRD}}(u) = \frac{1}{\bar{\Gamma}^{up}_{S_iRD}} e^{\frac{-u}{\bar{\Gamma}^{up}_{S_iRD}}}, \tag{19}$$

where

$$\bar{\Gamma}^{up}_{S_iRD} = \frac{\bar{\Gamma}_{S_iR}\bar{\Gamma}_{RD}}{\bar{\Gamma}_{S_iR} + \bar{\Gamma}_{RD}} \tag{20}$$

and $\bar{\Gamma}_{XY} = E(\Gamma_{XY})$.
Since Γ_{MRC} is the sum of two random variables each of them is following a chi-square distribution of 2 degrees of freedom with different variances, the PDF expression of Γ_{MRC} is given by [10] as

$$p_{\Gamma_{MRC}}(u) = \frac{\alpha_{S_iD}}{\bar{\Gamma}_{S_iD}} e^{\frac{-u}{\bar{\Gamma}_{S_iD}}} + \frac{\alpha_{S_iRD}}{\bar{\Gamma}_{S_iRD}} e^{\frac{-u}{\bar{\Gamma}_{S_iRD}}}, \qquad (21)$$

where

$$\alpha_{S_iD} = \frac{\bar{\Gamma}_{S_iD}}{\bar{\Gamma}_{S_iD} - \bar{\Gamma}_{S_iRD}} \text{ and } \alpha_{S_iRD} = \frac{\bar{\Gamma}_{S_iRD}}{\bar{\Gamma}_{S_iRD} - \bar{\Gamma}_{S_iD}}. \qquad (22)$$

The error probability $P_{1S,1R}$ is

$$P_{1S,1R} = 1 - \int \left[1 - \frac{1}{2}\mathrm{erfc}(\sqrt{u})\right]^N p_{\Gamma_{MRC}}(u)du, \qquad (23)$$

where $p_{\Gamma_{MRC}}$ is given in (21).
In the case where both received packets at D contain detected errors, due to the complexity of the analytical derivation of the equivalent SNR and its PDF, we propose to calculate the probabilities $P_{2S,2R}$ and $P_{2S,1R}$ numerically through simulations.
Since error events are dependent, the average number of transmissions of a packet can be lower bounded by

$$\begin{aligned} E[T_r] \geq\ & \frac{1}{2}[2 + P_{1S} + P_{2S} \\ & + P_{1S}P_{1S,1R} + 2P_{2S}P_{2S,2R} + P_{2S}P_{2S,1R} \\ & + P_{1S}(P_{1S,1R}P_S + P_{2S,2R}P_{2S}) \\ & + P_{2S}(P_{2S,2R}P_{2S} + 2P_{2S,1R}P_S)]. \qquad (24) \end{aligned}$$

The probability of packet erasure can be lower bounded by

$$\begin{aligned} PPE \geq\ & P_{1S}P_{1S,1R}P_SP_{1S,1R} \\ & + P_{2S}P_{2S,2R}[P_{1S}P_{1S,1R} + P_{2S}P_{2S,2R} + P_{2S}P_{2S,1R}] \\ & + P_{2S}P_{2S,1R}P_SP_{1S,1R}. \qquad (25) \end{aligned}$$

The average relay energy can be lower bounded by

$$\begin{aligned} E_{Ravg} \geq\ & \frac{1}{2}[E_{R1}[P_{1S}(1 + P_{1S,1R}P_S) \\ & + P_{2S}(P_{2S,2R}P_{1S} + P_{2S,1R}P_S)] \\ & + E_{R2}[P_{2S}(1 + P_{2S,2R}P_{2S})]]. \qquad (26) \end{aligned}$$

3.3. Optimization methodology

We recall here that probabilities $P_{2S,1R}$ and $P_{2S,2R}$ depend on E_{R1} and E_{R2} which lead to the non linearity and complexity of the optimization problem (11). Hence, we resort to a numerical dichotomy search of energies portions E_{R1} and E_{R2}. We quantize the energies E_{R1} and E_{R2} and start our search with a first search loop of energies which maximize the throughput using a moderate quantization footstep. Subsequently, we pursue the search with a second loop using a refined footstep around the values of E_{R1} and E_{R2} found in the first loop. We repeat the process as many times as needed in order to achieve a target quantization precision.

3.4. Relay energy optimization during the retransmissions

We propose here to optimize the relay energy distribution during the retransmissions. Thus, when $M = 6$, there are four energy values to optimize namely $E_{R1}^{(1)}$, $E_{R2}^{(1)}$, $E_{R1}^{(2)}$ and $E_{R2}^{(2)}$. The exponent is relative to the relay retransmission number. It is easy to extend the study of sections 3.1, 3.2 and 3.3 to this case. However, the dichotomy search becomes more complex since the number of energy values to optimize is higher.

4. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we provide analytical and simulation results illustrating the performance of the proposed C-ARQ approach. We consider data packets composed of $k = 90$ information bits and $n_p = 10$ parity bits modulated with BPSK. We consider that the relay node R is closer to the destination D than sources nodes. The path loss exponent α is set to 4. The quantity $E_{Rtarget}$ is set to 1.5, maximum allowed relay energies E_{R1max} and E_{R2max} are set to 1 and sources nodes energy E_s is also set to 1. The maximum number of retransmissions is $M = 6$.
Figure 2 compares semi-analytical and simulation results for lower bounds of $E[T_r]$, PPE and E_{Ravg}. The figure confirms the accuracy of the semi-analytical derived expressions (24), (25) and (26) especially at medium and high SNR. At low SNR, semi-analytical bounds are affected by the dependency of errors events.
Figure 3 illustrates simulation results of the throughput performance of the optimized relay energy proposed C-ARQ scheme compared to the proposed C-ARQ scheme with fixed relay energy and the conventional C-ARQ scheme. For fixed relay energy proposed C-ARQ scheme, we adopt equal relay energy allocation set to the maximum allowed energy, $E_{R1} = E_{R1max}$ and $E_{R2} = E_{R2max}$. For conventional C-ARQ scheme, in the case of two erroneous packets at D, the relay node amplifies and forwards the two erroneous source packets alternatively, thus, it needs one more time slot compared to the scheme using network coding. Equal relay energy allocation set to the maximum allowed energy is also used in this scheme. We propose to evaluate the throughput performance as a function of E_{avg}/N_0, where E_{avg} is the average energy of the relay and the sources spent per symbol. Compared to the conventional C-ARQ scheme, the proposed C-ARQ scheme with fixed relay energy provides a better throughput for the same total energy cost. Indeed, the proposed treatment in the case of both packets errors detection improves efficiently the system throughput. The throughput is improved when the relay energy distribution is optimized according to the number of retransmitted packets (green curve) corresponding to two energies values (E_{R1} and E_{R2}). It is further improved when the optimization is also done during the retransmissions as proposed in section 3.4 (pink curve) corresponding to four energy values ($E_{R1}^{(1)}$, $E_{R2}^{(1)}$, $E_{R1}^{(2)}$ and

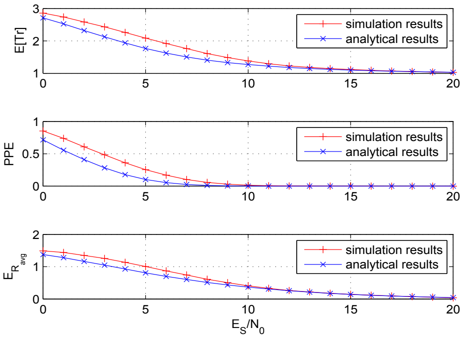

Fig. 2: Semi-analytical and numerical results of the average number of transmissions, the probability of packet erasure and the average relay energy.

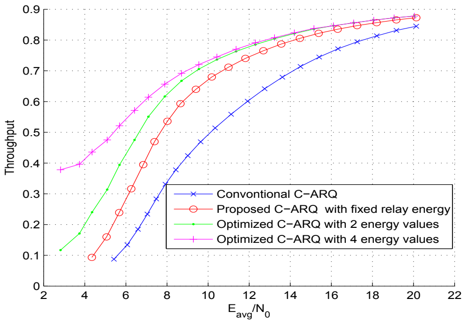

Fig. 3: Throughput for proposed schemes compared to conventional C-ARQ.

$E_{R2}^{(2)}$). The improvement is more significant at low E_{avg}/N_0 since in this case, the probability of packets retransmissions by the relay increases.

Figure 4 shows the probability of packet erasure performance versus E_{avg}/N_0 of the different proposed schemes. We notice that the performance in terms of PPE is improved when the relay energy is optimized according to the number of retransmitted packets and further improved when optimization is also done through retransmissions.

5. CONCLUSIONS

In this paper, we studied the optimization problem of relay energy distribution in a C-ARQ system using Analog Network Coding. The optimization is done according to the number of packets retransmitted by the relay and during the retransmissions. The aim is to maximize the throughput under an average relay energy constraint. We derived semi-analytical bounds of the average number of transmissions, the probability of packet erasure and the average relay energy and proposed to solve the optimization problem via a dichotomy search. Numerical results showed the performances enhancement of both throughput and probability of packet erasure rate using optimized approach compared to the proposed approach with fixed relay energy and conventional C-ARQ approach.

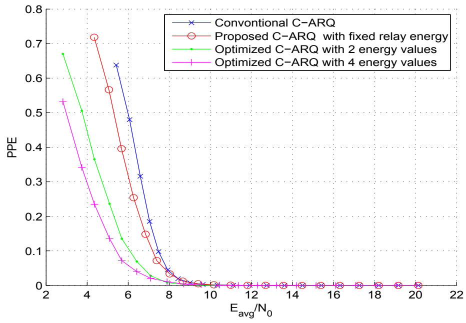

Fig. 4: Probability of packet erasure for proposed schemes compared to conventional C-ARQ.

REFERENCES

[1] J Nicholas Laneman, David NC Tse, and Gregory W Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.

[2] Ernesto Zimmermann, Patrick Herhold, and Gerhard Fettweis, “On the performance of cooperative relaying protocols in wireless networks,” *European Transactions on Telecommunications*, vol. 16, no. 1, pp. 5–16, 2005.

[3] Guanding Yu, Zhaoyang Zhang, and Peiliang Qiu, “Cooperative arq in wireless networks: Protocols description and performance analysis,” in *IEEE International Conference on Communications*, Jun. 2006, vol. 8, pp. 3608–3614.

[4] Lin Dai and Khaled B Letaief, “Throughput maximization of ad-hoc wireless networks using adaptive cooperative diversity and truncated arq,” *IEEE Transactions on Communications*, vol. 56, no. 11, pp. 1907–1918, 2008.

[5] Bessem Kaddour, Mohamed Siala, and Hatem Boujemâa, “Improving the performance of arq protocols over awgn channels using an optimized energy allocation,” in *IEEE International Conference on Communications and Networking*, Nov. 2010, pp. 1–6.

[6] Zhong Zhou, Shengli Zhou, Jun-Hong Cui, and Shuguang Cui, “Energy-efficient cooperative communication based on power control and selective single-relay in wireless sensor networks,” *IEEE Transactions on Wireless Communications*, vol. 7, no. 8, pp. 3066–3078, 2008.

[7] Sangkook Lee, Weifeng Su, Dimitrios A Pados, and John D Matyjas, “The average total power consumption of cooperative hybrid-arq on quasi-static rayleigh fading links,” in *IEEE Global Telecommunications Conference*, Dec. 2010, pp. 1–5.

[8] Sachin Katti, Shyamnath Gollakota, and Dina Katabi, “Embracing wireless interference: analog network coding,” in *ACM SIGCOMM Computer Communication Review*, 2007, vol. 37, pp. 397–408.

[9] Mazen O Hasna and M-S Alouini, “End-to-end performance of transmission systems with relays over rayleigh-fading channels,” *IEEE Transactions on Wireless Communications*, vol. 2, no. 6, pp. 1126–1131, 2003.

[10] Sihem Chaabouni, Noura Sellami, and Mohamed Siala, “Mapping optimization for a map turbo detector over a frequency-selective channel,” *IEEE Transactions on Vehicular Technology*, vol. 63, no. 2, pp. 617–627, 2014.