AN ADAPTIVELY FOCUSING MEASUREMENT DESIGN FOR COMPRESSED SENSING BASED DOA ESTIMATION

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***Abstract* – In this paper we propose an adaptive design strategy for the measurement matrix for applying Compressed Sensing (CS) to Direction Of Arrival (DOA) estimation with antenna arrays. Instead of choosing the coefficients of the compression matrix randomly, we propose a systematic design methodology for constructing a measurement matrix that focuses the array towards a specific area of interest and thereby achieves a superior DOA estimation performance. The focusing is performed in a sequential manner, i.e., we start with a uniform measurement design from which regions of interest can be extracted that the subsequent measurements then focus on. By continuously updating these target regions, gradual movement of the sources can also be tracked over time. Numerical results demonstrate that the focused measurements possess a superior SNR leading to significantly enhanced DOA estimates.**

Keywords: *Compressive Sensing, DOA Estimation, Measurement Design*

1. INTRODUCTION

Direction of arrival (DOA) estimation is a task required for various applications, including biomedical imaging, communications, channel modeling, tracking and surveillance in radar, and many others [1]. Recent advances in the field of Compressed Sensing (CS) [2, 3, 4] have given new research focus to the field of sparse recovery algorithms. This has led to the exploration of a strong link between DOA estimation and sparse recovery based on the fact that a superposition of planar wavefronts admits a sparse representation [5]. Based on this idea, sparse recovery has been considered as a tool for DOA estimation in applications like localization of the transmitting sources [6], channel modeling [7], tracking and surveillance in radar [8], and many others. Many powerful sparsity-based DOA estimation algorithms have been proposed in recent years [9, 10, 11, 12]. Compared to existing parameter estimation algorithms, sparsity-based DOA estimation techniques may provide some advantages, such as, being insensitive to source correlation, allowing arbitrary array geometries, working with a single snapshot, and providing certain guarantees for obtaining a global optimum in polynomial time [13].

The fact that the underlying RF signals possess a sparse representation suggests that CS can be applied for their acquisition [14, 15]. To implement the CS paradigm in the spatial domain, linear combinations of M passive antenna elements are formed via an analog combining network which reduces the antenna to $m < M$ channels that are actively sampled and digitized. In so doing, the hardware complexity is comparable to an m-element antenna array while being able to cover a much larger aperture than a traditional $\lambda/2$-spaced array. If the measurement kernel is appropriately chosen, the signal can be recovered from $m < M$ measurements due to its sparse representation in the angular domain.

Concerning the choice of the measurement kernels $\mathbf{\Phi}$ for CS-based DOA estimation, existing papers suggest a random choice, e.g., drawing its elements from Gaussian or Bernoulli distributions [15]. These are popular choices in the CS context as they allow to prove certain probabilistic theorems on the uniform support recovery of the signals. However, in our recent paper [16] we have shown that from an array processing perspective such a random design is not the optimal choice for the DOA estimation task since it may result in the effective array having certain blind spots (i.e., angles from which the energy is severely attenuated) or high sidelobes (which could be mistaken for spurious paths). We have proposed a design of $\mathbf{\Phi}$ that avoids these effects by optimizing the resulting effective beam pattern. Mathematically, this problem shows striking similarities to the beamspace design problem in array processing [17, 18, 19, 20]. However, besides for a different motivation, we also consider different criteria, e.g., a narrow autocorrelation with a controlled sidelobe level for achieving a high resolution. In fact, our results demonstrate that the optimized design leads to more favorable spatial correlation functions and a significantly improved DOA estimation performance.

In [16] our target was a static measurement matrix design that yields an array with uniform sensitivity, which is a good choice if no prior knowledge of the targets is available. In this paper, we extend [16] towards an adaptive design of the measurement matrix that uses the fact that for a slowly changing scene, the estimates from the previous snapshot provide prior information about the source locations in the next snapshot. This fact can be utilized for building an adaptive measurement matrix design that focuses the array's sensitivity towards regions of interest where the targets are expected. In so doing, the SNR and the effective resolution in these areas can be improved, resulting in a superior DOA estimation performance. Based on this idea, we propose a sequential measurement strategy which starts with a measurement matrix designed for uniform sensitivity (e.g., using the one proposed in [16]) and then gradually refines it towards the regions of interest that have been identified in the collected observations. Numerical simulations demonstrate that the focusing design results in a significant performance improvement compared to the uniform design.

2. DATA MODEL

We consider a system where an M-element antenna array is recording a sequence of snapshots of its received RF signals with the goal to find and monitor sources of RF transmissions (which could be actively transmitting sources in a communication-type scenario, reflecting sources in a surveillance/Radar-type scenario, or multipath components). One particular snapshot is modeled as the superposition of K planar wavefronts impinging from the directions of arrival (DOA) θ_k, $k = 1, 2, \ldots, K$. Mathematically, the observed output signal at the M antenna ports can be expressed as

$$\boldsymbol{x}(t) = \sum_{k=1}^{K} \boldsymbol{a}(\theta_k) \cdot s_k(t) + \boldsymbol{w}(t), \tag{1}$$

where $\boldsymbol{a}(\theta) \in \mathbb{C}^{M\times 1}$ is the array manifold as a function of the azimuth angle, $s_k(t)$ denotes the transmit signal of the k-th source, and $\boldsymbol{w}(t)$ represents the additive measurement noise. Note that due to target movement, the DOAs may change from snapshot to snapshot. However, this change is expected to be gradual so that the estimate from the current snapshot can be used as prior information in the next snapshot. For simplicity, we consider an M-element half-wavelength spaced uniform linear array (ULA) such that $\boldsymbol{a}(\theta) = [1, \mathrm{e}^{\jmath\mu}, \ldots, \mathrm{e}^{\jmath(M-1)\mu}]^{\mathrm{T}}$ for $\mu = \pi \cdot \cos(\theta)$. In order to be able to resolve sources with closely spaced angles, a large antenna array aperture is required. However, since the spatial sampling theorem allows sensors to be spaced no more than half a wavelength apart, this leads to a large required number of sensor elements M. Sampling many antenna ports is costly, since it requires many RF chains with costly components such as amplifiers, filters, and A/D converters.

In order to reduce the number of channels that have to be sampled actively without any loss in aperture, it has therefore been suggested to apply the Compressed Sensing (CS) framework to this setting [15]. The application of CS is based on the sparsity of the signal in the angular domain. In fact, we can rewrite (1) into a sparse formulation given by

$$\boldsymbol{x}(t) = \boldsymbol{A} \cdot \boldsymbol{s}(t) + \boldsymbol{w}(t), \tag{2}$$

where $\boldsymbol{A} = \left[\boldsymbol{a}(\theta_1^{(\mathrm{G})}) \quad \ldots \quad \boldsymbol{a}(\theta_N^{(\mathrm{G})})\right] \in \mathbb{C}^{M\times N}$ is the array manifold sampled on a prespecified N-point sampling grid and $\boldsymbol{s}(t) \in \mathbb{C}^{N\times 1}$ is K-sparse, provided that the actual DOAs θ_k are on the sampling grid. For details on the sampling grid, the reader is referred to [5, 16].

Due to the sparse model (2) we know that $\boldsymbol{x}(t)$ can be recovered from $m < M$ linear measurements $\boldsymbol{y}(t) = \boldsymbol{\Phi} \cdot \boldsymbol{x}(t) \in \mathbb{C}^{m\times 1}$ if the measurement kernel $\boldsymbol{\Phi} \in \mathbb{C}^{m\times M}$ is suitably selected. In practice, this operation can for instance be realized by an analog combining network with adjustable phase shifters and amplifiers. The application of $\boldsymbol{\Phi}$ transforms our data model (1) into

$$\boldsymbol{y}(t) = \sum_{k=1}^{K} \tilde{\boldsymbol{a}}(\theta_k) \cdot s_k(t) + \tilde{\boldsymbol{w}}(t), \tag{3}$$

where $\tilde{\boldsymbol{w}}(t) = \boldsymbol{\Phi} \cdot \boldsymbol{w}(t)$ and $\tilde{\boldsymbol{a}}(\theta) = \boldsymbol{\Phi} \cdot \boldsymbol{a}(\theta)$. Consequently, the CS operator $\boldsymbol{\Phi}$ has effectively transformed the M-element array into an m-port "CS-array" with a beam pattern given by $\tilde{\boldsymbol{a}}(\theta)$.

Therefore, the elements of $\boldsymbol{\Phi}$ give us control over the effective beam pattern of the CS-array. In our previous paper [16], we have designed $\boldsymbol{\Phi}$ to provide a CS-array with a uniform sensitivity, which is desired for a generic direction finder with no prior information. In this paper, we extend our previous work by considering the adaptation of $\boldsymbol{\Phi}$ over the snapshots in order to adapt the beam pattern of the array to the prior knowledge of the scene. As we show such a design allows the array to sequentially focus on certain regions of interest and therefore provide a superior SNR and resolution in these areas.

3. MEASUREMENT DESIGN

In this section we introduce the proposed adaptive design of the measurement matrix $\boldsymbol{\Phi}$. It is based on the effective array manifold $\tilde{\boldsymbol{a}}(\theta)$ that is introduced in Section 2 and depends on $\boldsymbol{\Phi}$ via $\tilde{\boldsymbol{a}}(\theta) = \boldsymbol{\Phi} \cdot \boldsymbol{a}(\theta)$. The main idea is to design $\boldsymbol{\Phi}$ such that the spatial correlation function $r(\theta_1, \theta_2) \doteq \tilde{\boldsymbol{a}}(\theta_1)^{\mathrm{H}} \cdot \tilde{\boldsymbol{a}}(\theta_2)$ follows as close as possible to a prespecified target $T(\theta_1, \theta_2)$, i.e., a matrix $\boldsymbol{\Phi}$ that minimizes

$$e(\boldsymbol{\Phi}, \theta_1, \theta_2) = |r(\theta_1, \theta_2) - T(\theta_1, \theta_2)| \tag{4}$$

The target $T(\theta_1, \theta_2)$ is adapted to the current knowledge of the scene. A uniform target function is used when no prior knowledge is available. When regions of interest have been specified (e.g., via an estimate of the angular power spectrum or a previous reconstructed scene), the target can be adapted to focus on these regions in order to provide a superior estimate (e.g., improved SNR and/or resolution).

To this end, an ideal uniform target function can be described by

$$T_{\mathrm{uni}}(\theta_1, \theta_2) = \begin{cases} \text{const} & \theta_1 = \theta_2 \\ 0 & \theta_1 \neq \theta_2 \end{cases}, \tag{5}$$

where the first condition guarantees that the array gain is constant for all angles (to make the array uniformly sensitive in all directions) and the second condition asks for good cross-correlation properties to tell signals from different directions apart.

On the other hand, a target function that focuses in an interval Θ is given by

$$T_{\Theta}(\theta_1, \theta_2) = \begin{cases} \text{const} & \theta_1 = \theta_2 \in \Theta \\ 0 & \theta_1 \neq \theta_2 \end{cases}, \tag{6}$$

where the interval Θ can for instance be describe by a center c_θ and a width w_θ via $\Theta = \left[c_\theta - \frac{w_\theta}{2}, c_\theta + \frac{w_\theta}{2}\right]$.

In order to find a matrix $\boldsymbol{\Phi}$ that minimizes (4), we utilize a mechanism introduced in [16] which we restate here for convenience. It is based on the following steps: First, to eliminate the continuous variables θ_1 and θ_2, we consider the N-point sampling grid $\theta_n^{(\mathrm{G})}, n = 1, 2, \ldots, N$ used for CS and define the $N \times N$ matrices $\boldsymbol{R}$ and $\boldsymbol{T}$ according to $\boldsymbol{R}_{(i,j)} = r(\theta_i^{(\mathrm{G})}, \theta_j^{(\mathrm{G})})$ and $\boldsymbol{T}_{(i,j)} = T(\theta_i^{(\mathrm{G})}, \theta_j^{(\mathrm{G})})$. Note that $\boldsymbol{R}$ can be written as $\boldsymbol{R} = \boldsymbol{A}^{\mathrm{H}} \cdot \boldsymbol{\Phi}^{\mathrm{H}} \cdot \boldsymbol{\Phi} \cdot \boldsymbol{A}$. The deviation between the sampled spatial correlation function $\boldsymbol{R}$ and its target $\boldsymbol{T}$ can then be measured via a suitable norm of the error matrix $\boldsymbol{E} \doteq \boldsymbol{R} - \boldsymbol{T}$. A closed-form solution for $\boldsymbol{\Phi}$ can be obtained if we choose the Frobenius norm of $\boldsymbol{E}$. In particular, if we let

$$\boldsymbol{\Phi}_{\mathrm{opt}} = \arg\min_{\boldsymbol{\Phi}} \|\boldsymbol{E}\|_{\mathrm{F}}^2 . \tag{7}$$

we can obtain $\boldsymbol{\Phi}_{\mathrm{opt}}$ via the following procedure: Let $\boldsymbol{S} = \boldsymbol{A} \cdot \boldsymbol{T} \cdot \boldsymbol{A}^{\mathrm{H}}$ and let $\boldsymbol{S}_m$ be a rank-m-truncated version of $\boldsymbol{S}$ obtained by setting its $N - m$ smallest eigenvalues to zero. Then every square-root factor of $\boldsymbol{S}_m$ (i.e., any $\boldsymbol{\Phi}$ satisfying $\boldsymbol{\Phi}^{\mathrm{H}}\boldsymbol{\Phi} = \boldsymbol{S}_m$) is an optimal solution to (7) [16].

Since $\boldsymbol{\Phi}_{\mathrm{opt}}$ can be obtained in closed form with a very low computational complexity, it is feasible to adapt it during the observations, i.e., the target can be refined to the current knowledge of the scene. We propose to apply the following adaptation mechanism:

1. Begin by scanning the scene with a matrix $\boldsymbol{\Phi}$ designed for a uniform target T_{uni}.

2. Identify regions of interest based on, e.g., an estimate of the angular power spectrum or a full reconstruction of the scene based on the initial observations(s).
3. Define a focusing region Θ as the union of all regions of interest.
4. Modify $\boldsymbol{\Phi}$ by solving (7) for a target designed for the focusing region Θ.
5. As the sources are assumed to change their position gradually, track sources by repeating steps (2) to (4) sequentially, moving the regions of interest along with the currently identified source locations.
6. Every P snapshots, rescan the scene with a matrix $\boldsymbol{\Phi}$ designed for a uniform target T_{uni} in order to detect newly appearing targets. If new sources are found, incorporate their location into the set Θ.

The parameter P represents a design parameter that determines how quickly the system reacts to targets appearing outside the current region of interest. Note that this adaptation mechanism allows for many degrees of freedom, e.g., in terms of the rate of adaptation of $\boldsymbol{\Phi}$ or the definition of the focusing regions.

4. NUMERICAL RESULTS

In this section we present some numerical results to demonstrate the advantage of using the focusing measurement matrix design according to our proposed methodology. To this end, we consider a $M = 12$ element ULA that is reduced to $m = 8$ channels via an 8×12 compression/focusing matrix $\boldsymbol{\Phi}$. We sample the spatial space using an $N = 64$ point uniform sampling grid, i.e., $\mu_{1,2} = (n_0 \pm d/2) \cdot \Delta$ where $n_0 \in [1, N]$ and d is the inter-source spacing in grid points.

To construct the uniform matrix ensemble $\boldsymbol{\Phi}_{\text{uni}}$ we solve the optimization problem (7) to obtain the closed form solution. As a target we set $\boldsymbol{T}_{\text{uni}} = \boldsymbol{I}_N$ which is the ideal uniform target function described by (5). More details about the performance of $\boldsymbol{\Phi}_{\text{uni}}$ can be found in [16]. We will only consider it here as an initial estimator of the regions of interest towards which the main beam is focused afterwards.

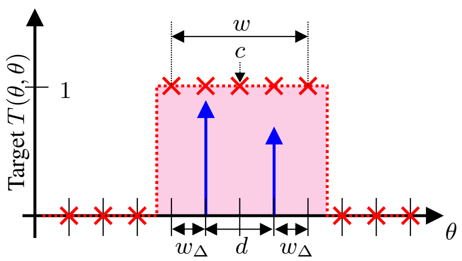

Fig. 1. Definition of the focusing window: the blue arrows indicate the estimated DOAs with a distance of d grid points. The window is centered at grid point c, located in the middle of the targets, and has a width of w grid points, where $w = d + 2w_\Delta$.

The focusing measurement design $\boldsymbol{\Phi}_{\text{foc}}$ is obtained by modifying the target according to (6) where $\boldsymbol{T}_{\text{focus}}$ is an $N \times N$ matrix that contains the identity matrix in the focusing region and nulls otherwise. The focusing region is identified based on an initial estimate of the source locations, e.g., by a reconstruction of the scene based on a first measurement carried out with $\boldsymbol{\Phi}_{\text{uni}}$. Figure 1 shows how to define the target for two closely spaced sources, in which case we define one interval containing both as the region of interest. The blue arrows represent the (estimated) source positions. The focusing interval Θ is described as mentioned earlier by a center grid point c which we place in the middle of the two sources and a width w. Naturally, we have $w = d + 2w_\Delta$, where w_Δ is the number of extra grid points we allow to both sides of the identified sources (in Figure 1 we have $w_\Delta = 1$) and d is the distance between the sources estimated from the uniform measurement initialization step. In general, the width w represents a design parameter where smaller values indicate a more narrow focus. If the initial estimate of the regions of interest is not very reliable, w should be chosen larger to allow for some deviations of the source position estimate in the refocused measurements. A concrete strategy for the choice of w is discussed below. Note that if more than two sources are present, the focusing interval Θ can be found by the union of several intervals, each centered around the middle of a cluster of identified sources.

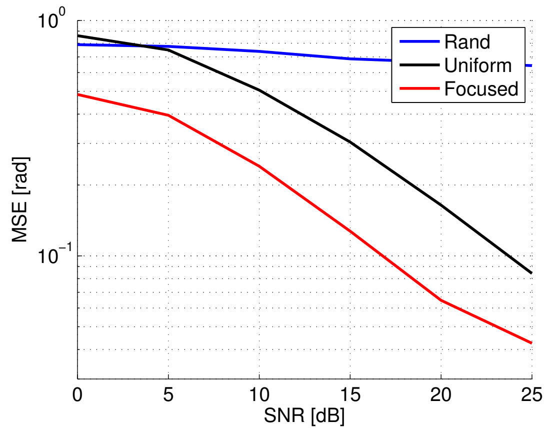

Fig. 2. MSE versus SNR for two sources using the random, uniform, and focusing design where the source separation d is varied randomly. The focusing interval is defined according to the knowledge about the region of interest obtained from the first uniform measurement with a window width given by $w_\Delta = 6$.

As a first step, we compare the performances of the three measurement designs of $\boldsymbol{\Phi}$: the random design advocated in the earlier papers [14, 15], the uniform we proposed in [16] and the focusing design proposed in this paper. The latter uses the estimate of the uniform design as an initialization, i.e., its first measurement is carried out with $\boldsymbol{\Phi}_{\text{uni}}$, the scene is reconstructed, and then used to identify the regions of interest to find $\boldsymbol{\Phi}_{\text{foc}}$ for the second measurement. For this experiment, we choose $w_\Delta = 6$, i.e., a window width of $w = d + 12$ grid points as the focusing region. Figure 2 shows the mean square error (MSE) versus the signal to noise ratio (SNR) for a scenario with two sources that are located on the grid and d grid points apart. The MSE is averaged over randomly drawn distances d and noise vectors $\boldsymbol{w}$ (cf. (2)) drawn from a zero mean circularly symmetric complex Gaussian distribution. For each trial, the fast orthogonal matching pursuit (OMP) [21] is used for the DOA estimation process and then the mean square error $\text{MSE} = \frac{1}{2}\sum_{k=1}^{2}(\mu_k - \hat{\mu}_k)^2$ is calculated for the three designs. As depicted in the figure, the random measurement design shows the worst performance as expected (see [16] for more details). The results show that the focusing design provides a significant improvement in terms of the SNR.

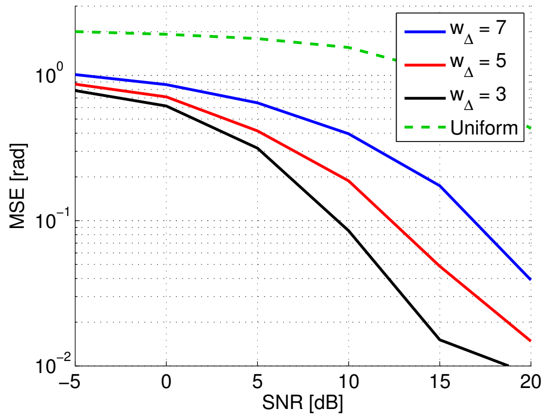

Fig. 3. MSE versus SNR for two sources separated by four grid points using different focusing interval lengths. A smaller window size leads to a more narrow focus, resulting in an improved resolution.

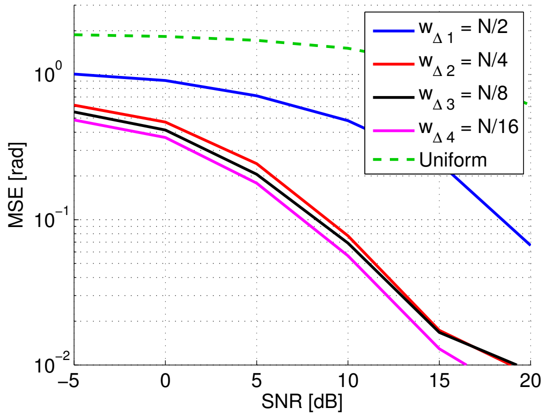

Fig. 4. Same scenario as shown in Figure 3 but this time the focusing is done sequentially with w_Δ being halved at each step.

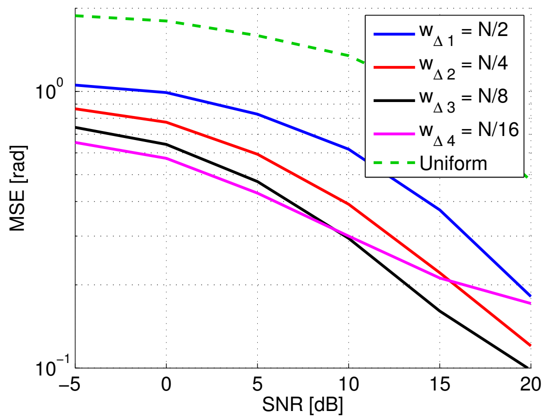

Fig. 5. Same scenario as shown in Figure 4 but this time the sources are only $d = 2$ grid intervals apart.

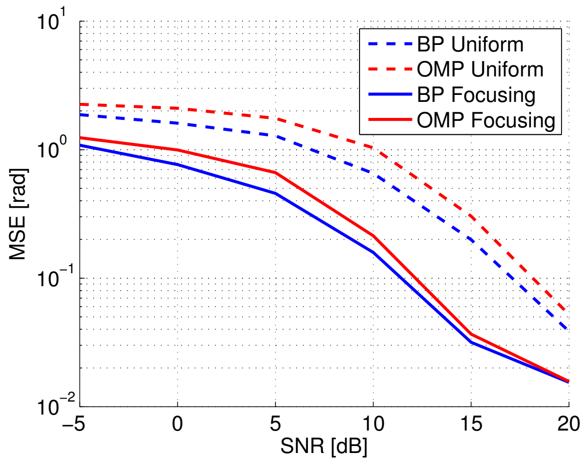

Fig. 6. Same scenario as before but instead using BP for initialization

To investigate the effect of the focusing interval width, we have repeated the experiment from Figure 2 with different focusing widths while fixing the sources' spacing to $d = 4$ which corresponds to 0.75 Rayleigh distances (i.e., they are closely spaced). Figure 3 compares the resulting MSE for a focusing width parameter $w_\Delta = 3$, 5, and 7, respectively. We observe that a more narrow focus leads to significantly reduced MSEs and thus a superior resolution of the two closely spaced sources compared to the unfocused, uniform design.

We now turn our attention towards a concrete example of a possible implementation of our focusing design without any special prior knowledge about the scene. Our idea is to perform the focusing sequentially, starting with an unfocused, uniform design and then gradually narrowing the focus by sequentially reducing the window size w. Each of the sequential measurements provides an improved estimate of the scene (as we have seen in Figure 3) which can be used to update the center of the window c and thus make sure that the focus is put in the correct direction. Figure 4 shows such a process where we investigate a scenario with two sources $d = 4$ grid points apart and we keep decreasing the window size parameter w_Δ from $N/2$ to $N/4$ to $N/8$ to $N/16$. The curves labeled $w_{\Delta,i}$ for $i = 1, 2, 3, 4$ correspond to the i-th sequential measurement which positions the target window according to the estimate from the $(i-1)$-th measurement and sets the window size to $N/2^i$, as indicated in the legend of the figure. The results show that each of the sequential measurements provides a more narrow focus which leads to a lower MSE, although the change from $N/8$ to $N/16$ does not improve the MSE significantly anymore.

Figure 5 depicts the result for the same scenario with the sources only $d = 2$ grid points apart. Here the fourth measurement $w_{\Delta,i} = N/16$ shows a worse performance than the third measurement using $w_{\Delta,3} = N/8$. This suggests that over-focusing might result in a worse performance, e.g., if the focus center c is not placed exactly in the correct direction.

So far, all the numerical results were based on the OMP algorithm for the sparse recovery step. To demonstrate that our proposed measurement matrix design can be applied to any sparse recovery algorithm, Figure 6 shows the same scenario as Figure 3, comparing the basis pursuit (BP) [22] with the OMP algorithm. The results show that as expected, the convex optimization based BP algorithm outperforms OMP, however, both algorithms benefit in a similar way from our proposed adaptive measurement matrix design.

5. CONCLUSIONS

In this paper, we have discussed a focusing design of the compression matrix for applying Compressive Sensing (CS) to the Direction of Arrival (DOA) problem. The main idea is to apply measurements in a sequential fashion: first, we measure with a uniform design that is equally sensitive in all directions and thus allows us to obtain a good estimate for the region(s) of interest in the angular domain. Then, the measurements are iteratively focused towards these regions. We have demonstrated that the focusing design results in a significant performance improvement compared to the uniform design. In particular, our numerical results demonstrated that a more narrow focus, leads to an improved SNR and resolution. We have demonstrated that the width of the focusing region is a parameter that can be used to control the degree of focus depending on the reliability of the estimate of the regions of interest and provided a sequential focusing strategy as a concrete example.

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