ABSTRACT
In this paper, a new subspace method based on spatio-
temporal structure of data is presented for estimation of
directions-of-arrival (DOAs) of sources impinging on an
array of sensors. Firstly, the signals received on the different
sensors are processed independently sensor by sensor to es-
timate the Times Of Arrival (TOAs) of multipaths. Then, the
obtained TOAs are post-processed to estimate the DOA of
each ray path. Simulation results show that the performance
of the proposed method is similar to those of HR methods,
with an advantage in its ability to cope with the situation
where the number of multipaths is much larger than that of
antenna sensors, which arises in many practical situations.

Index Terms— Localization, High Resolution, Time of
arrival, ray path association

1. INTRODUCTION
Array processing methods are interested for a long time to the
study of spatio-temporal signals, sampled in time and space
by a network of sensors. This allows to obtain information on
both the emitting sources, as their range to the antenna or the
signal DOA, and the medium properties, through the study of
TOA of the signals on the sensors. The performances of these
methods degrade severely with the high correlated multipath
signals encountered in target low angle tracking as it is the
case in many applications such as geophysics, communica-
tions, underwater acoustics...

In such a case, it becomes difficult to separate and extract
efficiently the parameters of each ray path. The first experi-
ments conducted in [1]: a point to point configuration, failed
correctly separating too close ray paths. Consequently,
beamforming has been applied to a point to array configu-
ration and adding the DOA as a discrimination parameter.
More recently, double-beamforming method has been pro-
posed in a array to array configuration [2–4]. To overcome
the low resolution ability of the beamforming, a high resolu-
tion (HR) processing has also been proposed. The HR meth-
ods (or subspace-based methods), include the classical MU-
SIC or ESPRIT, use eigenvector decomposition of the cross-
spectral density matrix. They were first proposed to estimate
the DOA and arrival times, both separately and then jointly,
using a combination of the active large band MUSIC (MU-
SICAL) with spatio-frequential smoothing processing method
[5]. All these HR methods perform well with a linear equi-
spaced array including at least one more sensor than the ra-
diating sources, a white noise spatially uncorrelated and un-
correlated sources signals; otherwise a spatial or frequential
smoothing is applied.

In this paper, we present in a synthetic way the perfor-
manences of an incoherent method based on HR processing and
frequential smoothing, in which the signal impinging on the
array is processed independently sensor by sensor. This al-
 lows to overcome the constraints concerning correlated sig-
nals and number of sensors.

In the following, the superscripts T, *, and H stand for trans-
pose, conjugate, and transpose conjugate. \( \cdot \) stands for the
modulus and \( \cdot \) for the Euclidean norm.

2. SIGNAL MODEL
Let us consider a wave field composed of \( P \) ray paths of a
known, large band signal \( s \) impinging on an linear array of \( N \)
sensors. The temporal signal received on the \( n \)-th sensor \( x_n \) is
modeled as:

\[
x_n(t) = \sum_{p=1}^{P} c_{p,n} s(t - \tau_{p,n}) + n_n(t)
\]  

(1)

where: \( s(t) \) is the transmitted signal, \( c_{p,n} \) is the complex
amplitude of the \( p \)-th ray path on the \( n \)-th sensor, \( n_n(t) \) is an
additive noise.

\( \tau_{p,n} \), the TOA of the \( p \)-th ray path on the \( n \)-th sensor, can be
expressed as:

\[
\tau_{p,n} = T_p + t_n(\theta_p)
\]  

(2)

where \( T_p \) represent the TOA of the \( p \)-th ray path on the first
sensor and \( t_n(\theta_p) \) is the delay between the first sensor and the
\( n \)-th sensor. \( t_n(\theta_p) \) is a function of \( \theta_p \) which is the DOA of the
ray path and is usually express in far field assumption as:

\[
t_n(\theta_p) = \frac{(n-1)d}{v} \sin(\theta_p)
\]  

(3)

\[
e^{j2\pi(\tau_{p,n}v/d)}
\]  

(4)

where \( d \) is the inter element spacing.
with $d$ the distance between to consecutive sensor and $v$ the celerity of the considered wave.

In the frequency domain, (1) can be expressed as:

$$\hat{x}_n(\omega) = \sum_{p=1}^{P} c_{p,n} \hat{s}(\omega) e^{-2\pi f \tau_{p,n}} + \bar{h}_n(\omega)$$  \hspace{2cm} (4)

3. HIGH RESOLUTION METHODS FOR TOA ESTIMATION

Most of the HR methods [20,21], initially developed for DOA of narrowband signal estimation, are based on a $N \times N$ covariance matrix of received signal on an antenna by assuming that the signal is a mixture of $P$ plane waves with DOA $\theta_1, \cdots, \theta_P$ and a white Gaussian noise of power $\sigma$.

One can use most of these methods for TOA estimation, by changing the spatial dimension into a frequential dimension for a mixture of $P$ echoes of a known wide-band signal ($M$ frequencies) impinging on a sensor. This is why, inspired by this method, we propose in this paper a new way to characterize fully correlated sources in presence of spatially correlated noise [22], without constraint on the number of sensors. Considering the vector $y_n$ obtained by concatenating the signal received on the $n$th sensor at the different frequencies:

$$y_n = [\hat{x}_n(f_1), \cdots, \hat{x}_n(f_M)]^T$$  \hspace{2cm} (5)

The covariance matrix is given by:

$$\Gamma_n = E[y_n y_n^H] = ACC^H + \sigma' \Phi_M$$  \hspace{2cm} (6)

where for a given sensor receiving $P$ echoes: $C$ is a $P \times P$ source covariance matrix and $A$ is a $M \times P$ matrix of steering vectors:

$$A = [a(\tau_1), \cdots, a(\tau_P)]$$  \hspace{2cm} (7)

with $a(\tau) = [e^{-i f_1 \omega(\tau)}, \cdots, e^{-i f_M \omega(\tau)}]^T$ and $\omega(\tau) = 2\pi \tau$.

$\sigma'$ is defined by $\sigma' = \frac{\sigma}{\sigma(f)}$

Let $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_P$ be the $P$ largest eigenvalues of $\Gamma_n$, with the corresponding set of eigenvectors $V = \{v_1, \cdots, v_P\}$ and $\Phi = \Phi_M - VV^H$ a projector onto the orthogonal subspace.

MUSIC : MUltiple SIgnal Classification [6] is the standard orthogonal subspace method that can exploit some estimate $\hat{\Phi}$ of the orthogonal subspace projector $\Phi$ so that the TOA parameters are given by solving the maximization problem in $\tau$ using a pseudo-spectrum function:

$$\phi(\tau) = \frac{1}{a(\tau)^H \Phi a(\tau)}$$  \hspace{2cm} (8)

In order to evaluate the accuracy of this TOA estimation algorithm, one can express using some straight-forward calculations [7,8] the unified TOA estimation errors:

$$\Delta \tau_p = \frac{\mathcal{R}(\phi(\tau) \Delta \Gamma_n(\tau_p))}{\tau_p}$$  \hspace{2cm} (9)

where $\Delta \Gamma_n = \Gamma_n - \Gamma_n$, $\alpha_p = C^{-1} a(\tau_p)$, $\beta_p = \Pi \hat{a}(\tau_p)$ and $\gamma_p = \omega(\tau_p) \hat{a}(\tau_p) \Pi \hat{a}(\tau_p)$ with:

$$\hat{a}(\tau) = [f_1 e^{-i f_1 \omega(\tau)}, \cdots, f_M e^{-i f_M \omega(\tau)}]^T$$

the first order derivative of $a$, and $\omega(\tau) = 2\pi f d \cos(\gamma)$. Note that all the HR methods are based on the assumption that the matrix of the sources signal or equivalently the matrix $C$ is full rank. However, when the sources are correlated, which is the case most of the time, the source covariance matrix $C$ is not full rank and, as a consequence, so isn’t the covariance matrix $AC^H$. To face this issue, many methods have been developed over the years to artificially increase the rank of $C$ (Rank $(C) = P$). Most of them exploit the information on sub-arrays or/and sub-bands [9–17] to estimate a smoothed covariance matrix.

4. INCOHERENT SENSOR BY SENSOR PROCESSING : ISSP METHOD

In this section we describe the different steps of our original method, processing the signal sensor by sensor, using its spectral diversity instead of the spatial one. We suggest in this case how to evaluate the performances concerning the TOA and DOA estimation.

4.1. Ray path separation step

One can use high resolution algorithm for TOA estimation [18], to separate the ray paths on each sensor $n$ and estimate the intermediate TOA $\hat{\tau}$ given in (2). The observation can be pre-processed by a frequential smoothing method [19] to ensure full rank of the covariance matrix while keeping an appropriate time resolution. Let the $M$ frequencies band $[f_1, \cdots, f_M]$ where $f_m = f_1 + (m-1) \Delta f$ for all $m \in \{1, \cdots, M\}$ be divided into $K_h$ overlapping sub-bands of $L$ frequencies, with an inter-frequency distance of $h \Delta f$ so that the $k$th sub-band is made of the $\{f_k \cdot h\}_{k=0}^{L-1}$ frequencies, yielding the relation between $M$, $L$, $h$ and $K_h$:

$$M = h(L-1) = K_h$$  \hspace{2cm} (10)

A modified covariance matrix $\Gamma_n$ is estimated by the $L \times L$ matrix:

$$\Gamma_n = \frac{1}{K_h} \sum_{k=1}^{K_h} \Gamma_n^k + J \Gamma_n^k J$$  \hspace{2cm} (11)

where $\Gamma_n^k$ is the covariance matrix of the modified observations $y_n^k$ in the $k$th sub-band and $J$ is the exchange matrix, whose elements are define such that: $J(i,j) = 1$ if $j = l - i + 1$ and $J(i,j) = 0$ if $j \neq l - i + 1$, $l = 1, \cdots, L$. This persymmetric approach has been shown to be give better
performances than redundancy averaging for most subspace-based techniques [8].

However, for all \( n \in \{1, \ldots, N\} \), the \( \{\bar{\tau}_{p,n}\}_{P=1}^{P} \) estimated correspond to an unknown permutation of the \( \{\tau_{p,n}\}_{P=1}^{P} \), that is, for each sensor \( n \in \{1, \ldots, N\} \) it exists a permutation \( \sigma_n \) so that, for all \( p \in \{1, \ldots, P\} \):

\[
\bar{\tau}_{p,n} = \tau_{\sigma(n),p} + \Delta(\tau_{\sigma(n),p})
\]  

(12)

were \( \Delta(\tau_{p,n}) \) is an estimation error, which unified expression has been given in (9).

4.2. Ray path association and characterization steps

In order to estimate the TOA and the DOA of each ray path, one must know the permutation \( \sigma_n \) that link \( \{\bar{\tau}_{p,n}\}_{P=1}^{P} \) and \( \{\tau_{p,n}\}_{P=1}^{P} \) for each \( n = 1, \ldots, N \). By considering (2) and (3), one can see that:

\[
\forall n \in \{1, \ldots, N - 1\} \left\{ \begin{array}{l}
\tau_{p,n+1} - \tau_{p,n} = \frac{d}{v} \sin(\theta_p) = \alpha_p \\
\tau_{p,1} = T_p
\end{array} \right.
\]  

(13)

To associate each TOA with a given source, we propose algorithm 1, based on (13): if we consider a given set of estimated TOA \( \{\tilde{\tau}_{n,n}\}_{n=1}^{N}, \{k_n\}_{n=1}^{N} \in \{1, \ldots, N\} \) for each \( n = 1, \ldots, N - 1 \) we can compute the value:

\[
v_n(k_{n+1}, k_n) = \tilde{\tau}_{k_{n+1},n+1} - \tilde{\tau}_{k_n,n} = \tau_{\sigma(n+1),n+1} - \tau_{\sigma(n),n} + \Delta(\tau_{\sigma(n+1),n+1}) - \Delta(\tau_{\sigma(n),n})\Delta(n_{k_{n+1},n})
\]  

(14)

If \( \sigma_n(k_n) = \sigma_{n+1}(k_{n+1}) = p \), we have:

\[
v_n(k_{n+1}, k_n) = \alpha_p + \Delta_n(k_{n+1}, k_n)
\]  

(15)

Equation (14) will be used to compute the mean square error:

\[
\bar{v}(k_1, \ldots, k_N) = \frac{1}{N-2} \sum_{n=1}^{N-1} v_n(k_{n+1}, k_n)^2
\]

- \( N - 2 \sum_{n=1}^{N-1} v_n(k_{n+1}, k_n)^2 \)

\[
\bar{v}(k_1, \ldots, k_N) = \frac{N-1}{N-2} \left( \sum_{n=1}^{N-1} v_n(k_{n+1}, k_n)^2 \right)
\]

(17)

This way, the number of combinations to be tested is limited to \( P^2 \).

Then, one can use the sorted \( \{\bar{\tau}_{p,n}\}_{P=1}^{P} \) \( \{\bar{\tau}_{p,n}\}_{n=1}^{N} \) by algorithm 1 to estimate \( \{\overline{T}_p\}_{p=1}^{P} \) and \( \{\overline{\theta}_p\}_{p=1}^{P} \) using algorithm 2.

Algorithm 1 Sort-\( \tau_{p,n} \) by supposed source

**Require:** \( N, P, \{\bar{\tau}_{p,n}\}_{P=1}^{P} \)

1. for \( p = 1 \) to \( P \) do
2. \( \bar{\tau}_{p,1} \leftarrow \bar{\tau}_{p,1} \)
3. for \( p_1 = 1 \) to \( P \) do
4. \( v_{p_1,1} \leftarrow \bar{\tau}_{p_1,2} - \bar{\tau}_{p_1,1} \)
5. \( \sigma_{p_1,1} \leftarrow p_1 \)
6. for \( n = 2 \) to \( N - 1 \) do
7. Find \( k \in \{1, \ldots, P\} \) so that \( \bar{\tau}_{k,n+1} - \bar{\tau}_{p_1,1} \)
8. \( v_{p_1,n} \leftarrow \tilde{\tau}_{k,n+1} - \tilde{\tau}_{p_1,n+1} \)
9. \( \sigma_{p_1,n} \leftarrow k \)
10. end for
11. end for
12. Find \( p_2 \in \{1, \ldots, P\} \) so that \( \bar{v} = \frac{1}{N-2} \sum_{n=1}^{N-1} v_{p_2,n}^2 - \frac{N-1}{2} \sum_{n=1}^{N-1} v_{p_2,n}^2 \) is minimum.
13. for \( n = 1 \) to \( N - 1 \) do
14. \( \bar{\tau}_{p,n+1} \leftarrow \tilde{\tau}_{p_2,n+1} \)
15. end for
16. end for
17. return \( \{\bar{\tau}_{p,n}\}_{P=1}^{P} \) \( \{\bar{\tau}_{p,n}\}_{n=1}^{N} \)

Algorithm 2 Straightforward TOA-DOA estimation

**Require:** \( d, v, N, P, \{\bar{\tau}_{p,n}\}_{P=1}^{P} \)

1. for \( p = 1 \) to \( P \) do
2. \( \bar{T}_p \leftarrow \bar{\tau}_{p,1} \)
3. \( \theta_p \leftarrow \arcsin(\frac{v}{d} \alpha_p) \)
4. \( \bar{\theta}_p \leftarrow \bar{\tau}_{p,1} \)
5. end for
6. return \( \{\bar{T}_p\}_{p=1}^{P} \) and \( \{\bar{\theta}_p\}_{p=1}^{P} \)

To evaluate the performances of the process, using the unified performance expression (9) and algorithm 2, for all \( p \in \{1, \ldots, P\} \), we have

\[
\Delta\theta_p = \frac{\frac{d}{v} \Delta\alpha_p}{1 - (\frac{d}{v} \alpha_p)^2} = \frac{\frac{d}{v} \Delta\alpha_p}{1 - \sin^2(\theta_p)}
\]  

(19)

\[
\Delta T_p = \Delta\tau_{p,1} \quad , \quad \Delta\alpha_p = \frac{\Delta\tau_{p,N} - \Delta\tau_{p,1}}{N - 1}
\]  

(20)

The proposed method is summarized in fig. 1. In the first step (in red), a frequential smoothing pre-processing ensure the full rank of the covariance matrix, then in a second step (in blue) HR algorithm for TOA estimation is used to separate the ray paths on each sensor. Each TOA must then be
associated to a given source (step 3 in purple), and finally in the 4th step (green one on the Figure 1), straightforward algorithm is proposed to jointly estimate DOA and TOA, with error estimation respectively given by (19) and (20).

Fig. 1. Incoherent Sensor by Sensor Processing

5. PERFORMANCES STUDY ON SIMULATED DATA

In this section, we propose to evaluate the performances of ISSP method, considering that the signals impinging on the array is the mixture of \( P \) = 2 echoes of a same known wide-band signal, that is, assuming matrix \( C = I_P \). These two plane waves are respectively set with DOA \( 8^\circ \) and \( 9.3^\circ \) and with TOA \( 0.5 \times 10^{-3}s \) and \( 2.5 \times 10^{-3}s \). The velocity of the wave is set to \( v = 1460m/s \) and the lower frequency of the band is set to \( f_1 = 150kHz \) and the inter-frequency distance \( \Delta f = 75Hz \). We realized 500 simulations, for which 500 snapshots are computed. We defined the signal-to-noise ratio (SNR) as \( SNR_{dB} = 10 \log_{10} \left( \frac{\sigma_{signal}}{\sigma_{noise}} \right) \), where \( \sigma_{signal} \) (resp. \( \sigma_{noise} \)) is the power of the signal (noise) fixed to 10dB.

For ISSP method, we first have to choice the number of frequency \( M \), which will impact on the estimation of all the intermediate estimates TOA. In theory, to successfully separate the 2 echoes, we should have \( M \geq P + 1 = 3 \). Figure 2 shows the average RMSE [23] of the two closed echoes TOA versus the number of frequency \( M \). One can see that as soon as \( M \geq 6 \), the TOA RMSE is minimal (lower than \( 10^{-6}s \)). So in the following, we will use \( M = 6 \).

For the chosen experimental set, \( d = \frac{\lambda}{2f_1} \) (necessary in classical HR method to fulfill Nyquist Shannon sampling theorem) Figure 3 shows for ISSP method the TOA RMSE versus \( N \) with \( d = \frac{\lambda}{2f_1} \) and for a fixed width antenna of 0.07m and \( d = \frac{0.07}{N-1} \). The TOA is always well-estimated with an error smaller than \( 3 \times 10^{-6}s \). Figure 4 shows that for a fixed width antenna, the DOA RMSE is smaller than for a fixed \( d \) antenna. When \( N \geq 15 \) the ISSP method with both antennas estimates the DOA with an error smaller than \( 0.1^\circ \). One can also remark from Figure 4 that in the same conditions with MUSIC to separate the 2 echoes, we need to ensure at least \( N \geq 14 \) and \( N \geq 17 \) to obtain error around \( 0.1^\circ \).

Fig. 2. TOA RMSE versus \( M \)

![Fig. 2. TOA RMSE versus \( M \)](image)

Fig. 3. TOA RMSE versus \( N \), obtained by ISSP with \( d = \frac{\lambda}{2f_1} \) and \( d = \frac{0.07}{N-1} \)

![Fig. 3. TOA RMSE versus \( N \)](image)

Fig. 4. DOA RMSE versus \( N \), obtained by ISSP with \( d = \frac{\lambda}{2f_1} \) and \( d = \frac{0.07}{N-1} \), and by MUSIC

![Fig. 4. DOA RMSE versus \( N \)](image)
is diagonal so this method shall not be impacted by the noise spatial correlation.

6. CONCLUSION

This paper has proposed a method that does not depend on the array geometry, which is not constrained by the number of sources to be localized, and which is influenced neither by the correlation of the sources or by the spatially noise correlation. This method shows very good separation performance and usability. Its capabilities will be evaluated more accurately by a statistical study of its performances and tests on real data in a forthcoming work.

REFERENCES
