NONLINEAR FILTERED-X SECOND-ORDER ADAPTIVE VOLTERRA FILTERS FOR LISTENING-ROOM COMPENSATION

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ABSTRACT

The presence of nonlinearities as well as reverberation effects severely degrades the audio quality in sound reproduction systems. In this context, many adaptive strategies have been developed to compensate for room effects. However, when nonlinear distortion becomes significant, room equalization requires the introduction of suitable solutions to tackle this problem. Linearization of loudspeakers has been deeply investigated but its combination with room equalization systems may not be so straightforward, mainly when the nonlinearities present memory. In this paper, the nonlinear system has been modeled as a Volterra filter that represents the loudspeaker tandemly connected to a linear filter that corresponds to the electroacoustic path including the enclosure and the microphone setup. Based on this structure, we introduce a nonlinear filtered-x second-order adaptive Volterra filter that uses the virtual path concept to preprocess the audio signals. Simulation results validate the performance of the new approach.

Index Terms— Adaptive equalization, Volterra filters, nonlinear distortions, Virtual channel

1. INTRODUCTION

The basic components of sound reproduction systems, such as digital-to-analog (D/A) and analog-to-digital (A/D) converters, amplifiers, loudspeakers and microphones, usually present linear responses, but when they are driven with large amplitude inputs, nonlinear distortions that severely degrade the audio quality can arise. In the context of audio equalization, the main goal is to make the global impulse response of the sound reproduction channel as close as possible to a desired one (see the basic structure of an equalization system in Fig. 1). If the system has a linear behavior, an adaptive linear filter can properly compensate for the distortion (e.g. [1, 2]). However, if the system exhibits non-linearities, a linear filter performs poorly and the nonlinear distortion must be accounted for in the design of the equalization system.

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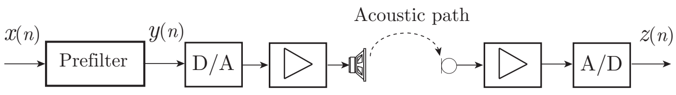

Fig. 1. Acoustic equalization transmission chain.

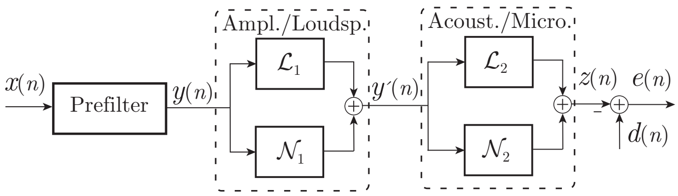

Fig. 2. Nonlinear-parallel model of an acoustic equalization system with two nonlinear filters tandemly connected.

The nonlinear distortion can potentially occur in each of the system components (see Fig. 1). The electroacoustic path that involves the loudspeaker-enclosure-microphone (LEM) setup can be modeled as two nonlinear filters tandemly connected, as it is shown in Fig. 2. Typically, the main sources of nonlinearities occur in the first filter, that represents the D/A converter, the loudspeaker and its amplifier, and are due to the high input signal levels and the loudspeaker physical properties [3, 4]. While the amplifier can be modeled as a memoryless system, the loudspeaker behavior is more properly described as nonlinear distortion with memory [5]. In contrast, the propagation path between loudspeaker and microphone is considered as a linear filter and it is reasonable to expect a linear behavior for the microphone, too. Thus, the second block can be modeled as a linear filter.

Adaptive linearization of loudspeakers has been widely studied using a p-order pre-processor [6], which allows to eliminate the nonlinearities of a system up to the p-order when a p-order filter [7, 8] is implemented. As the pre-processor filter is based on both the perfect identification of the linear and nonlinear filters and the inversion of the linear block, this method is very sensitive to channel estimation misadjustment.

Although there are many papers addressing the loudspeaker linearization, only very few papers consider also

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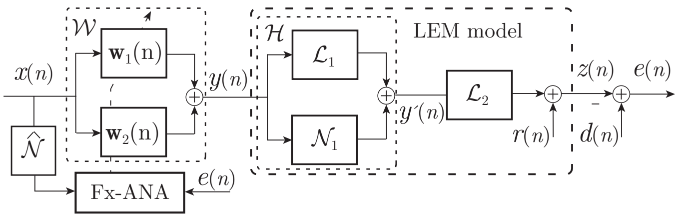


Fig. 3. Adaptive equalization diagram with a filtered-x adaptive nonlinear algorithm (Fx-ANA).

the room compensation problem. Similarly to linear room equalization, adaptive nonlinear room compensation requires filtered-x ad hoc treatment to avoid instability in the filter coefficient update, where the input signal $x(n)$ is filtered through the electroacoustic path. Thus, the filtering of $x(n)$ involves the use of a *nonlinear filtered-x algorithm* as it was introduced in [9]. This structure has been previously used in active noise control (using the so-called virtual secondary path) with adaptive Volterra filters [10,11] and functional-link artificial neural networks (FLANN) [12, 13]. The drawback of the Volterra series is the increasing number of parameters with order. For that reason, only second-order and third-order Volterra filters are usually implemented [8]. Regarding FLANN filter strategy, it does not use products of input samples with different time shifts, thus, its performance can be deteriorated in some situations. To alleviate this problem, different modifications, such as the generalized FLANN (GFLANN) [14] and the Fourier nonlinear (FN) filter [15], have been proposed. However, the nonlinear expansion of the input signal of order P produces $M = 2P + 1$ functions to be filtered through the nonlinear filtered-x algorithm. In [10], a general function expansion is used for both FLANN and Volterra filters, where the nonlinear secondary paths are modeled as nonlinear memoryless systems. To the best of our knowledge, none of the previous works have address the problem of nonlinear distortion with memory in room equalization systems. Motivated by the virtual path model abilities and the good performance of Volterra filters on a nonlinear filtered-x framework, in this work we present a second-order Volterra filter capable of compensating nonlinear distortion with both memory and cross-terms of the input signal $x(n)$, requiring only a single nonlinear filtering of $x(n)$. Fig. 3 illustrates the nonlinear room equalization problem of Fig. 1, where both $\mathcal{L}_1$ and $\mathcal{L}_2$ represent a linear filter, and the nonlinear function with memory is denoted by the symbol $\mathcal{N}_1$.

The paper is organized as follows. A nonlinear adaptive algorithm based on the filtered-x least-mean-square (Fx-LMS) strategy is introduced in Section 2. This approach will be called through the paper as NFx-LMS. Specifically, the virtual nonlinear channel is provided in Section 3. Experimental results are given in Section 4 to confirm the robustness and effectiveness of the developed algorithm. Section 5 summarizes the main conclusions of the paper.

2. ADAPTIVE EQUALIZATION OF NONLINEAR ACOUSTIC SYSTEMS

Assuming that the nonlinear system can be modeled by a Qth-order Volterra series with finite memory, the output signal $z(n)$ (see Fig. 3) can be expressed as

$$\begin{aligned} z(n) &= \mathcal{L}_2\left\{\mathcal{H}[y(n)]\right\} \\ &= \mathcal{L}_2\Bigg\{ \sum_{i_1=0}^{M_1-1} L_1(i_1) y(n-i_1) + \sum_{q=2}^{Q} \Bigg[\sum_{i_1=0}^{M_q-1} \cdots \\ & \sum_{i_q=0}^{M_q-1} N_{1,q}(i_1,\ldots,i_q) y(n-i_1)\cdot\ldots\cdot y(n-i_q) \Bigg]\Bigg\}, \end{aligned} \tag{1}$$

being $\mathcal{H}$ the nonlinear system modeled with the first parallel block filter and M_q is the memory or number of coefficients in the qth-Volterra kernel. Moreover, $L_1(i_1)$ is the i_1-coefficient of the first kernel and $N_{1,q}(i_1,\ldots,i_q)$ refers to the $(i_1,\ldots,i_q)$-coefficient of the qth-kernel, where a symmetric form has been considered for the Volterra kernels [16].

To implement the adaptive compensation prefilter, a second-order Volterra filter $\mathcal{W}$ has been used in Fig. 3 to remove nonlinearities up to $p = 2$ order. The relationship between input and output of the adaptive filter is given by

$$\begin{aligned} y(n) = \mathcal{W}[x(n)] &= \sum_{i_1=0}^{N_1-1} w_1(i_1;n) x(n-i_1) \\ &+ \sum_{i_1=0}^{N_2-1}\sum_{i_2=0}^{N_2-1} w_2(i_1,i_2;n) x(n-i_1) x(n-i_2), \end{aligned} \tag{2}$$

where N_p is the number of coefficients of the pth-Volterra kernel ($p = 1, 2$) and $w_p(i_1,\ldots,i_p;n)$ is the specific coefficient at time n.

The error signal $e(n)$ is computed as the difference between the signal $z(n)$ measured at the microphone (1) and the desired signal $d(n)$, which corresponds to the input signal with a proper time delay (τ)

$$e(n) = d(n) - z(n) = x(n-\tau) - z(n). \tag{3}$$

The filter coefficients are updated by applying an stochastic gradient algorithm that uses the instantaneous estimate of the gradient of $E\{e^2(n)\}$ with respect to the filter coefficients as in the classical LMS [9, 17] that leads to

$$\begin{aligned} w_1(i_1;n) &= w_1(i_1;n-1) - \frac{\mu_1}{2}\frac{\partial e^2(n)}{\partial w_1(i_1;n)} \\ &= w_1(i_1;n-1) + \mu_1 e(n)\frac{\partial e(n)}{\partial w_1(i_1;n)}, \end{aligned} \tag{4}$$

$$\begin{aligned} w_2(i_1,i_2;n) &= w_2(i_1,i_2;n-1) - \frac{\mu_2}{2}\frac{\partial e^2(n)}{\partial w_2(i_1,i_2;n)} \\ &= w_2(i_1,i_2;n-1) + \mu_2 e(n)\frac{\partial e(n)}{\partial w_2(i_1,i_2;n)}, \end{aligned} \tag{5}$$

where μ_1 and μ_2 are the step size parameters.

If the system takes the structure shown in Fig. 3, we can derive

$$\frac{\partial e(n)}{\partial w_1(i_1;n)} = \sum_{m=0}^{M-1} \frac{\partial z(n)}{\partial y(n-m)} \cdot \frac{\partial y(n-m)}{\partial w_1(i_1;n)}, \quad (6)$$

$$\frac{\partial e(n)}{\partial w_2(i_1,i_2;n)} = \sum_{m=0}^{M-1} \frac{\partial z(n)}{\partial y(n-m)} \cdot \frac{\partial y(n-m)}{\partial w_2(i_1,i_2;n)}, \quad (7)$$

where M is the memory size of the LEM system, which is given by $M = \max(M_1, M_2) + M_{L_2} - 1$, being M_{L_2} the length of the second linear block $\mathcal{L}_2$.

Moreover, when the step sizes are small enough to allow slow variations of the filter coefficients, from (2) it can be written

$$\frac{\partial y(n-m)}{\partial w_1(i_1;n)} \approx x(n-m-i_1), \quad (8)$$

$$\frac{\partial y(n-m)}{\partial w_2(i_1,i_2;n)} \approx x(n-m-i_1)x(n-m-i_2). \quad (9)$$

For simplicity, we are using the concept of virtual path as in [9, 10] to refer to the derivative of the nonlinear system defined in (1) with respect to the delayed inputs

$$\widehat{N}(m;n) = \frac{\partial z(n)}{\partial y(n-m)}, \quad m = 0, \ldots, M-1. \quad (10)$$

Finally, combining (6 - 10) into (4) and (5), we obtain the update equations of the NFx-LMS algorithm

$$w_1(i_1;n) = w_1(i_1;n-1) + \mu_1 e(n) \sum_{m=0}^{M-1} \widehat{N}(m;n) x(n-m-i_1), \quad (11)$$

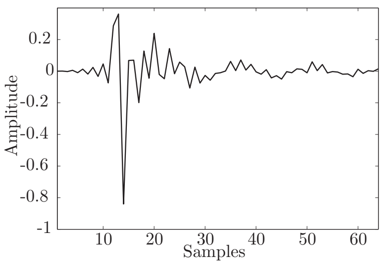
$$w_2(i_1,i_2;n) = w_2(i_1,i_2;n-1) + \mu_2 e(n) \sum_{m=0}^{M-1} \widehat{N}(m;n) x(n-m-i_1) x(n-m-i_2). \quad (12)$$

Notice the similarity between (11) and the conventional filtered-x LMS (Fx-LMS) algorithm used for linear applications [18]. Since the implementation of (11) and (12) is not straightforward, next section will provide the specific implementation of the virtual channel for this listening-room compensation approach.

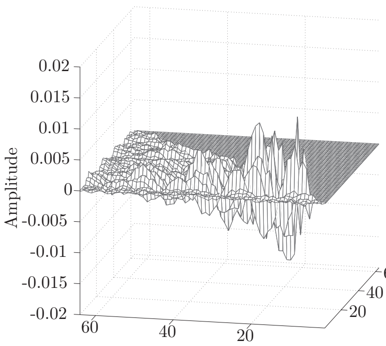
3. VIRTUAL CHANNEL DESCRIPTION

The virtual channel defined in (10) is a time-varying filter of M-length whose coefficients depend on the input signal $y(n)$. From the block diagram in Fig. 3, (10) can be expressed as

$$\widehat{N}(m;n) = \frac{\partial z(n)}{\partial y(n-m)} = \sum_{l=0}^{M_{L_2}-1} \frac{\partial z(n)}{\partial y'(n-l)} \cdot \frac{\partial y'(n-l)}{\partial y(n-m)}$$
$$= \mathbf{L}_2^T \left[\frac{\partial y'(n)}{\partial y(n-m)} \quad \frac{\partial y'(n-1)}{\partial y(n-m)} \quad \frac{\partial y'(n-M_{L_2}+1)}{\partial y(n-m)} \right]^T, \quad (13)$$



(a) Linear distortion



(b) Nonlinear distortion

Fig. 4. Linear (a) and quadratic (b) kernels of the system $\mathcal{H}$.

where the derivative of the output of the nonlinear system $\mathcal{H}$ is filtered through the enclosure/microphone linear block, modeled as a FIR filter defined by vector $\mathbf{L}_2$.

In order to obtain the derivative coefficients as in [9], we should also take into account the linear filter $\mathbf{L}_2$ to relate the input samples $y(n-m)$ to the loudspeaker output signal for different time delays $y'(n-l)$. Thus, the coefficients can be expressed as

$$\frac{\partial y'(n-l)}{\partial y(n-m)} = L_1(m-l) + 2 \sum_{i=0}^{M_2-1} N_{1,2}(m-l,i) y(n-i-l), \quad (14)$$

when $0 \leq (m-l) < \max(M_1, M_2)$. In other cases, the coefficients are 0. Finally, substituting (14) into (13), and (13) into (11) and (12), we will obtain the adaptive algorithm for the prefilter coefficients of the system $\mathcal{W}$.

Note, that if $\mathcal{H}$ is linear ($\mathcal{N}_1 = 0$), only the linear coefficients in (14) are considered and the overall virtual channel is the linear convolution of the linear components ($\mathcal{L}_1$ and $\mathcal{L}_2$), which leads in (11) to the conventional Fx-LMS algorithm.

4. EXPERIMENTAL RESULTS

In this section, the robustness and effectiveness of the NFx-LMS algorithm is evaluated in nonlinear equalization scenarios. A normalized adaptation of NLMS type has been used in the NFx-LMS, that leads to the NFx-NLMS algorithm.

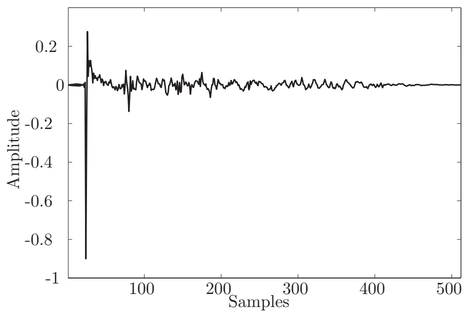


Fig. 5. Impulse response of the acoustic path including the microphone response.

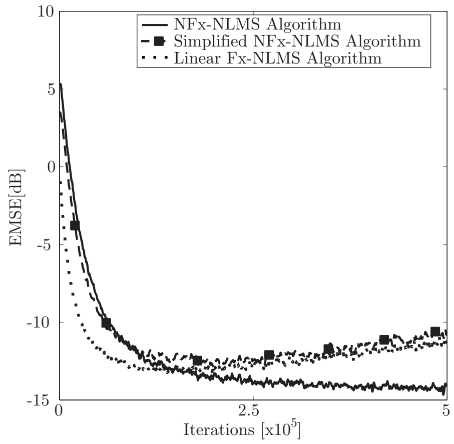


Fig. 6. EMSE evolution of the NFx-NLMS algorithm (solid line), the simplified NFx-NLMS algorithm (dashed line and square markers) and linear Fx-NLMS algorithm (dotted line).

To model the nonlinear LEM scheme, the system $\mathcal{H}$, that includes a loudspeaker, has been measured in low-reverberant conditions at a sampling frequency of 8 kHz using a second-order Volterra filter with $M_1 = M_2 = M^{'} = 64$ coefficients. The linear and quadratic kernels are shown in Fig. 4. As the second-order kernel presents a symmetric behavior [16], a triangular representation has been used in Fig. 4 (b). The impulse response of the acoustic path and the microphone ($\mathcal{L}_2$ system) has been also measured with 8 kHz and 512 samples, within a room with a reverberation time of $T_{60} = 170$ ms, see Fig. 5.

Different Linear-to-NonLinear Ratio (LNLR) setups will be considered. The LNLR level is defined as the ratio between the powers of the linear and nonlinear components ($y_l^{'}(n)$ and $y_n^{'}(n)$, respectively). To modify the LNLR value, an α parameter is used as

$$y^{'}(n) = y_l^{'}(n) + \alpha y_n^{'}(n) = \mathbf{y}^T(n)\mathbf{L}_1 + \alpha \mathbf{y}^T(n)\mathbf{N}_{1,2}\mathbf{y}(n), \quad (15)$$

where $\mathbf{L}_1$ and $\mathbf{N}_{1,2}$ represent the linear and quadratic kernels of size $M^{'} \times 1$ and $M^{'} \times M^{'}$, respectively. Moreover, $\mathbf{y}(n)$

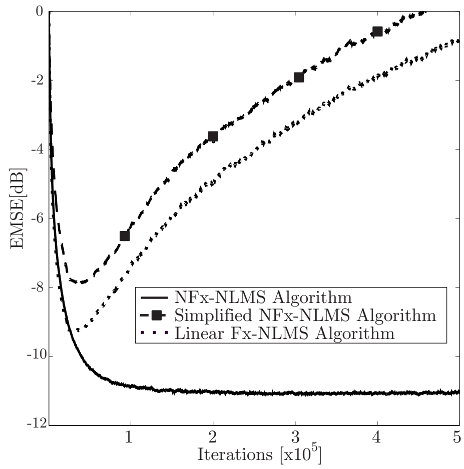


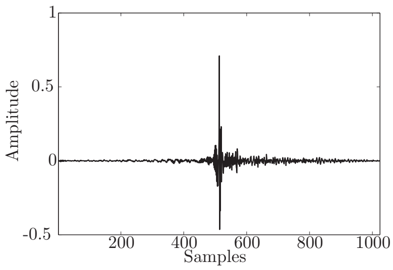
Fig. 7. EMSE evolution of the NFx-NLMS algorithm and the simplified and linear versions, with LNLR= 0 dB ($\alpha = 7$).

is a column vector that contains the last $M^{'}$ samples of $y(n)$.

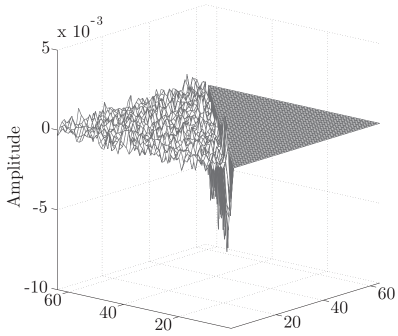
The algorithm performance is evaluated in terms of convergence speed and steady-state error by inspecting the excess mean-square-error, EMSE $= E\left\{[e(n) - r(n)]^2\right\}$ that has been estimated by averaging over 100 independent runs of the algorithm. The input signal $x(n)$ is a white Gaussian noise with zero mean and unit variance. Moreover, an uncorrelated noise signal $r(n)$, with zero mean and signal-to-noise ratio (SNR) of 40 dB, has been added to the microphone signal. The adaptive filters have been designed to have $N_1 = 1024$ and $N_2 = 64$ coefficients for the first and second kernel, respectively. The delay of the desired signal has been chosen closed to $N_1/2$, specifically $\tau = 549$.

Fig. 6 illustrates the EMSE evolution of the NFx-NLMS algorithm (solid line) with $\alpha = 1$ (LNLR= 20 dB). The performance of the algorithm is compared to the linear Fx-NLMS algorithm (dotted line), which implies that the nonlinear component ($\mathcal{N}_1$) is not considered and the virtual channel is only obtained from linear component filters ($\mathcal{L}_1$ and $\mathcal{L}_2$). Also a simplified approach derived from the NFx-NLMS algorithm has been implemented (named as simplified NFx-NLMS, in dashed line and with square markers), which uses a virtual channel which depends only on these linear components. The step sizes are set to $\mu_1 = \mu_2 = 0.1$. Although the three adaptive filters exhibit a similar convergence rate, that is even faster for the Fx-NLMS, only the NFx-NLMS approach has a stable behavior. Thus, a suitable design of the virtual path is essential in this context.

To study the behavior of the NFx-NLMS algorithm with a high degree of nonlinearities, $\alpha = 7$ has been used to provide a LNLR of 0 dB. Fig. 7 shows the evolution of EMSE, with $\mu_1 = \mu_2 = 0.01$. As it can be observed, the NFx-NLMS filter exhibits a stable behavior reaching a steady-state EMSE of approximately -11 dB. Furthermore, the simplified and



(a) Linear adaptive filter



(b) Triangular representation of the second-order adaptive filter

Fig. 8. Adaptive linear and second-order Volterra filters.

linear algorithms are also shown, which exhibit a bad performance as expected. The adaptive filter coefficients obtained for the NFx-NLMS algorithm at steady state are shown for each kernel in Fig. 8.

5. CONCLUSIONS

A novel nonlinear filtered-x adaptive algorithm has been proposed for room equalization to compensate both room reverberation and nonlinear distortion with memory. The new approach is based on the development of a novel time-varying virtual filter that avoids problems of instability due to filter delays.

The effectiveness and robustness of the NFx-NLMS algorithm has been evaluated in terms of the EMSE for different LNLR values. The proposed approach outperforms the linear Fx-NLMS type algorithm even with high LNLR conditions. Moreover, it exhibits a good performance for low LNLR.

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