A SEQUENTIAL MONTE CARLO APPROXIMATION OF THE HISP FILTER

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ABSTRACT

A formulation of the **h**ypothesised filter for **i**ndependent **s**tochastic **p**opulations (HISP) is proposed, based on the concept of association measure, which is a measure on the set of observation histories. Using this formulation, a particle approximation is introduced at the level of the association measure for handling the exponential growth in the number of underlying hypotheses. This approximation is combined with a sequential Monte Carlo implementation for the underlying single-object distributions to form a mixed particle association model. Finally, the performance of this approach is compared against a Kalman filter implementation on simulated data based on a finite-resolution sensor.

***Index Terms*—** Multi-object filtering; finite-resolution sensor.

1. INTRODUCTION

The **h**ypothesised filter for **i**ndependent **s**tochastic **p**opulations [1], or HISP filter, is a recent solution in the domain of multi-object estimation that naturally provides track estimates through the modelling of distinguishable information, while displaying a low algorithmic complexity. When non-linearity and/or non-Gaussianity has to be modelled, **s**equential **M**onte **C**arlo (SMC) methods need to be considered [2, 3, 4]. In this paper, we show that the HISP filter allows for a clustering-free SMC implementation which maintains the performance of the Kalman filter implementation in linear and Gaussian cases. This approach, in the continuation of [5], bears some similarity with [6], where an association-measure approximation of the probability hypothesis density filter [7] is derived.

Simulations are performed with a finite-resolution sensor which models more faithfully the output of real sensors. This type of sensor are best modelled by using probability theory which is why this formalism is used throughout this article[1]. Let $\mathbf{M}(\mathbf{E})$ (resp. $\mathbf{P}(\mathbf{E})$) stand for the set of finite measures (resp. probability measures) on a given measurable space $(\mathbf{E}, \mathcal{E})$. For any bounded measurable function f on $\mathbf{E}$ and for any measure $\gamma \in \mathbf{M}(\mathbf{E})$, we write $\gamma(f) = \int f(x)\gamma(\mathrm{d}x)$. Let $G : \mathbf{E} \to (0, \infty)$ be a bounded measurable function and let the *Boltzmann-Gibbs transformation* $\Psi_G : \mathbf{M}(\mathbf{E}) \to \mathbf{P}(\mathbf{E})$ be defined as[2]

$$\Psi_G(\gamma)(\mathrm{d}x) \stackrel{\int}{=} \frac{1}{\gamma(G)} G(x)\gamma(\mathrm{d}x),$$

where it is assumed that $\gamma(G) > 0$. Note that if G is a likelihood function and γ is a probability measure then $\Psi_G(\gamma)$ is the corresponding Bayes' posterior law.

2. PROBLEM STATEMENT AND MODELLING

We consider the problem of the estimation of the number and state of individuals in a random and time-varying population, given a sequence of noisy, incomplete and corrupted collections of observations provided by a finite-resolution sensor. The fact that the collections of observations are incomplete is due to the uncertain detection of the individuals, and the fact that they are corrupted is a consequence of the combination of sensor-related noise and of the presence of other individuals that are not part of the population under consideration.

Some parts of the population modelling will be sensor dependent so that we start by introducing the considered model for the sensor before focusing on the population itself. Without loss of generality, the time is indexed by the set $\mathbb{T} \doteq \mathbb{N}$.

2.1. Sensor modelling

We consider a finite-resolution sensor acting, at time $t \in \mathbb{T}$, in the space $\mathbf{Z}_t$ which is assumed to be a closed subset of an Euclidean space. The observations are assumed to be Borel subsets of the observation space $\mathbf{Z}_t$. These subsets correspond to the resolution cells of the sensor that form a partition π_t of $\mathbf{Z}_t$, i.e., it holds that if $A, A' \in \pi_t$ then either $A = A'$ or $A \cap A' = \emptyset$. To each resolution cell $A \in \pi_t$ is associated a unique index z, and the set of all these indices is denoted Z'_t. At every time $t \in \mathbb{T}$, a family $\{A^z_t\}_{z \in Z_t}$ of observations in $\mathbf{Z}_t$ indexed by a set $Z_t \subseteq Z'_t$ is made available by the sensor.

[1] See [8] for an introduction to measure theory in the context of multi-object estimation.

[2] For any $\mu, \mu' \in \mathbf{M}(\mathbf{E})$, the equality $\mu(\mathrm{d}x) \stackrel{\int}{=} \mu'(\mathrm{d}x)$ refers to: $\mu(f) = \mu'(f)$ for any bounded measurable function f on $(\mathbf{E}, \mathcal{E})$.

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It is assumed that each observation does not correspond to more than one individual, so that, if two individuals have their projection on $\mathbf{Z}_t$ in the same resolution cell, then only one of them can be detected at the same time.

In order to have a complete model of the observation, we consider the empty observation ϕ which corresponds to individuals that have not been detected. The space $\mathbf{Z}_t$ and the set Z_t are accordingly extended to $\bar{\mathbf{Z}}_t \doteq \mathbf{Z}_t \cup \{\phi\}$ and $\bar{Z}_t \doteq Z_t \cup \{\phi\}$ at any time $t \in \mathbb{T}$.

2.2. Population modelling

We assume that there exists a space in which all the individuals of interest can be uniquely characterised so that a population can be understood as being a set of individuals. More details about this approach can be found in [1].

State

Individuals in the population of interest are described at time $t \in \mathbb{T}$ by their respective state in the individual state space $\mathbf{X}_t$, which is assumed to be a closed subset of an Euclidean space. As for the observation space, we define $\bar{\mathbf{X}}_t$ as the extension of the state space $\mathbf{X}_t$ with an empty state ψ which models the individuals that are not part of the population at time t, but which will interact in some way with it at some time $t' \geq t$. Examples of such interactions are: *a*) individuals that are to be born at some later time, and *b*) objects/phenomena that are not part of the population of interest but might interfere in the observation process via the creation of *spurious observations*.

In order to describe the population as a whole, we introduce different ways of indexing laws that might describe individuals of interest. We start with individuals that have already been detected once and can therefore be distinguished by their observation history, or *observation path*. At time $t \in \mathbb{T}$, the set of all possible observation paths can be indexed by the set $\bar{Y}_t \doteq \bar{Z}_0 \times \ldots \times \bar{Z}_t$. This set also contains the empty observation path $\boldsymbol{\phi}_t \doteq (\phi, \ldots, \phi) \in \bar{Y}_t$ and we only consider the set $Y_t \doteq \bar{Y}_t \setminus \{\boldsymbol{\phi}_t\}$ for individuals that have been detected at least once. The symbol "m" is used to refer to these **m**easured individuals. An interval of existence $T \subseteq \mathbb{T}$ of the form $[t', t]$ is conveniently added to the characterisation of individuals.

As far as the undetected individuals are concerned, we assume for the sake of simplicity that they are detected upon **a**ppearance and refer to them via the symbol "a". They are also assumed to be indistinguishable, meaning that there is no specific information available on any of them, so that they are all described by the same law. It is assumed that the number of appearing individuals is driven by a binomial distribution with parameters $a_t \in [0, 1]$ and $n'_t \doteq |Z'_t|$. The spatial distribution of each of these individuals is denoted $p_t^{(\mathrm{a})} \in \mathbf{P}(\bar{\mathbf{X}}_t)$ and verifies $p_t^{(\mathrm{a})}(\{\psi\}) = 0$.

The objects/phenomena that interfere with the observation process, which we now refer to as *spurious-observation generators*, are associated with the symbol "♭" and are almost surely at point ψ, so that the distribution $p_t^{(\flat)} \doteq \delta_\psi$ is associated to them. The individuals in the population of interest are accordingly given the symbol "♯".

We are now in position to build a full index set in which each individual in the extended population, i.e., the one containing the objective population and the spurious-observation generators, is given a unique index. Before the observation update at time t, this index set is defined as

$$\mathbb{I}_t \doteq \mathbb{I}_t^{\mathrm{m}} \cup \{i_t^{\mathrm{a}}, i_t^{\flat}\},$$

where, denoting "$[\cdot, t]$" the abstract time interval ending at time t, $\mathbb{I}_t^{\mathrm{m}} \doteq \{(\sharp, [\cdot, t], y) : y \in Y_{t-1}\}$ corresponds to the detected individuals, where $i_t^{\mathrm{a}} \doteq (\sharp, \{t\}, \boldsymbol{\phi}_t)$ describes newborn individuals, where the spurious-observation generators index is $i_t^{\flat} \doteq (\flat, \emptyset, \boldsymbol{\phi}_t)$.

Focusing on the individuals in the objective population that have been distinguished, i.e., the ones with index in $\mathbb{I}_t^{\mathrm{m}}$, we define the measure γ_t on $\bar{\mathbf{X}}_t \times \mathbb{I}_t^{\mathrm{m}}$ as

$$\gamma_t(\mathrm{d}(x, i)) \stackrel{\int}{=} \alpha_t(\mathrm{d}i) p_t^{(i)}(\mathrm{d}x). \tag{1}$$

The measure α_t on $\mathbb{I}_t^{\mathrm{m}}$ is referred to as an *association measure* [9, 6, 5], and characterises the probability for a law $p_t^{(i)}$ with a given index $i = (\sharp, T, y)$, where y is an observation path, to represent an individual in the objective population.

Observation

We first introduce the sub-σ-algebra $\mathcal{A}_t \subset \mathcal{B}(\mathbf{Z}_t)$ generated by the countable measurable partition π_t and its extended counterpart $\bar{\mathcal{A}}_t$ on the space $\bar{\mathbf{Z}}_t$. Events in $\bar{\mathcal{A}}_t$ are exactly the ones that we are interested in since they describe all the possible outputs of the observation process. The observation of the measured individuals in the objective population can then be described concisely at time $t \in \mathbb{T}$ by a single Markov kernel L_t^{m} on $\bar{\mathbf{X}}_t \times \bar{\mathcal{A}}_t$ defined as follows: for any $x \in \mathbf{X}_t$ and any $A \in \mathcal{A}_t$:

a) $L_t^{\mathrm{m}}(x, A)$ gives the probability for an object that has state x to be detected in the Borel set A,

b) $L_t^{\mathrm{m}}(x, \{\phi\})$ gives the probability for the detection of an object that has state x to fail.

We also assume that $L_t^{\mathrm{m}}(\psi, \{\phi\}) = 1$ since individuals in the objective population that are not yet in the scene cannot trigger an observation, i.e., $L_t^{\mathrm{m}}(\psi, A) = 0$ for any $A \in \mathcal{A}_t$.

A collection of likelihoods need to be defined for the spurious-observation generators: let $\{L_t^z\}_{z \in Z'_t}$ be a collection of Markov kernels on $\{\psi\} \times \bar{\mathcal{A}}_t$ such that $L_t^z(\psi, A_z)$ is the probability of finding a spurious observation in A_z and

$$L_t^z(\psi, A_z \cup \{\phi\}) = 1, \qquad \forall z \in Z'_t.$$

In this work, we consider that appearing individuals are almost surely detected, so that no estimation of the undetected individuals is required. This assumption implies that the kernel L_t^{m} cannot be used for appearing individuals. We thus introduce another Markov kernel L_t^{a} on $\bar{\mathbf{X}}_t \times \bar{\mathcal{A}}_t$ such that $L_t^{\mathrm{a}}(x, \{\phi\}) = 0$, which is defined as

$$L_t^{\mathrm{a}}(x, A) = \frac{L_t^{\mathrm{m}}(x, A)}{L_t^{\mathrm{m}}(x, \mathbf{Z}_t)}, \qquad \forall A \in \mathcal{A}_t,$$

if $L_t^{\mathrm{m}}(x, \mathbf{Z}_t) \neq 0$ and as $L_t^{\mathrm{a}}(x, \cdot) = 0$ otherwise.

Overall, when updating a population described by the index set $\mathbb{I}_t$, the following correspondence is used: L_t^{m} is used to update individuals in $\mathbb{I}_t^{\mathrm{m}}$, L_t^z is used for individuals with index $i_t^\flat$, and L_t^{a} is used for appearing individuals.

Motion

The motion of individuals from time $t \in \mathbb{T}$ to time $t+1$ is characterised by a Markov kernel M_t from $\bar{\mathbf{X}}_t$ to $\bar{\mathbf{X}}_{t+1}$ such that, for any $x \in \mathbf{X}_t$ and any Borel set $B \in \mathcal{B}(\mathbf{X}_{t+1})$:

a) $M_t(x, B)$ gives the probability for an object that had state x at time t to persist to time $t+1$ and to be in B,

b) $M_t(x, \{\psi\})$ gives the probability for an object that had state x at time t to disappear from the scene,

We also assume that $M_t(\psi, \{\psi\}) = 1$, which can be seen as a modelling choice implying that disappeared individuals cannot enter the scene again. It also holds that spurious-observation generators cannot move away from state ψ since they will never be part of the objective population.

3. PARTICLE ASSOCIATION MEASURE

We first express the HISP filter [1] as a function of its association measure, before devising approximations based on this concept. Some additional notations are required: for any $j = (s, T, y) \in \mathbb{I}_t$ and any $z \in \bar{Z}_t$, the index $(s, T, (y, z)) \in \mathbb{I}_t$ will be denoted $j \cdot z$. Also, introduce φ as an additional empty observation corresponding to the case where the underlying association path does not represent an actual object, and define the measures $\mathcal{I}_t$ and $\mathcal{Z}_t$ as

$$\mathcal{I}_t \doteq \sum_{i \in \mathbb{I}_t} \delta_i, \qquad \text{and} \qquad \mathcal{Z}_t \doteq \sum_{z \in Z_t} \delta_z.$$

The measures $\mathcal{I}_t^{\mathrm{m}}$, $\mathcal{I}_t^\sharp$ are defined as restrictions of $\mathcal{I}_t$ to $\mathbb{I}_t^{\mathrm{m}}$ and $\mathbb{I}_t^\sharp$ respectively. Similarly, $\bar{\mathcal{Z}}_t$ and $\tilde{\mathcal{Z}}_t$ are defined as extensions of $\mathcal{Z}_t$ to $\bar{Z}_t$ and $\tilde{Z}_t \doteq \bar{Z}_t \cup \{\varphi\}$ respectively. We also define the index set

$$\mathbb{I}_{t,t} \doteq \{(i, \mathrm{m}, z) | i \in \mathbb{I}_t^{\mathrm{m}}, z \in \tilde{Z}_t\} \cup \{(i^{\mathrm{a}}, \mathrm{a}, z) | z \in \tilde{Z}_t\} \cup \{(i_t^\flat, z, z) | z \in Z_t'\},$$

which indicates which likelihood should be used with which particular combination of predicted indices and observations.

Proposition 1. *The measure $\gamma_{t+1} \in \mathbf{M}(\bar{\mathbf{X}}_{t+1} \times \mathbb{I}_{t+1}^{\mathrm{m}})$ which characterises the predicted hypotheses at time $t+1$ is:*

$$\gamma_{t+1}(\mathrm{d}(x, i)) \stackrel{\int}{=} \alpha_{t+1}(\mathrm{d}i) p_{t+1}^{(i)}(\mathrm{d}x)$$

where, introducing $(j, s, z) \in \mathbb{I}_{t,t}$ such that $i = j \cdot z$, the predicted law $p_{t+1}^{(i)}$ is characterised by

$$p_{t+1}^{(i)}(B) = \Psi_{L_t^s(\cdot, A_t^z)}\big(p_t^{(j)}\big)(M_t(\cdot, B)), \qquad \forall B \in \mathcal{B}(\bar{\mathbf{X}}_{t+1}),$$

and where the association measure α_t can be expressed, for any $(j, z) \in \mathbb{I}_t^\sharp \times Z_t$, as

$$\alpha_{t+1}(\mathrm{d}(j \cdot z)) \stackrel{\int}{=} \mathcal{Z}_t(\mathrm{d}z) \frac{\mathcal{I}_t^\sharp(\mathrm{d}j) w_{\mathrm{ex}}(j, z) w_t^{(j,z)}}{\int \mathcal{I}_t(\mathrm{d}k) w_{\mathrm{ex}}(k, z) w_t^{(k,z)}} \tag{2}$$

or, for any $(j, z) \in \mathbb{I}_t^{\mathrm{m}} \times \bar{Z}_t$, as

$$\alpha_{t+1}(\mathrm{d}(j \cdot z)) \stackrel{\int}{=} \mathcal{I}_t^{\mathrm{m}}(\mathrm{d}j) \frac{\bar{\mathcal{Z}}_t(\mathrm{d}z) w_{\mathrm{ex}}(j, z) w_t^{(j,z)}}{\int \tilde{\mathcal{Z}}_t(\mathrm{d}z') w_{\mathrm{ex}}(j, z') w_t^{(j,z')}} \tag{3}$$

where, extending α_t to $\mathbb{I}_t^\sharp$ by adding the term $a_t \delta_{i_t^{\mathrm{a}}}$,

$$w_t^{(j,z)} = \begin{cases} \alpha_t(\{j\}) p_t^{(j)}(L_t^s(\cdot, A_t^z)) & \text{if } z \in \bar{Z}_t \\ 1 - \alpha_t(\{j\}) & \text{if } z = \varphi. \end{cases}$$

The term $w_{\mathrm{ex}}(j, z) \in [0, 1]$ is the probability for all the individuals except (the) one with index j to be associated with the observations in $Z_t \setminus \{z\}$, i.e., w_{ex} assesses the compatibility between the time-predicted law and the collection of observations **ex**cluding the/an individual with index j and the observation z. The expression of w_{ex} has not been detailed here, but its calculation would be computationally demanding when performed exactly [1]. To reduce this cost, the following sparsity-type assumption is considered for subsets I and Z of the sets $\mathbb{I}_t^{\mathrm{m}}$ and Z_t respectively.

$$\big(\mathbf{S}(I, Z)\big) \;\; \forall j \in I, \forall z, z' \in Z, \qquad w_t^{(j,z)} w_t^{(j,z')} \approx 0.$$

Assuming $\mathbf{S}(I, Z)$ is equivalent to considering that two observations in Z are unlikely to be identified with the same individual representation in I. Using this assumption on particular subsets of $\mathbb{I}_t^{\mathrm{m}}$ and Z_t, the weight w_{ex} can be re-expressed as follows.

Proposition 2. *For any $(j, z) \in \mathbb{I}_t \times \bar{Z}_t$, the term $w_{\mathrm{ex}}(j, z)$ factorises when assuming $\mathbf{S}\big(\mathbb{I}_t^{\mathrm{m}} \setminus \{j\}, Z_t \setminus \{z\}\big)$ as*

$$w_{\mathrm{ex}}(j, z) \approx C_t'(j, z) \prod_{k \in \mathbb{I}_t^{\mathrm{m}} \setminus \{j\}} \left[w_t^{(k,\phi)} + \sum_{z' \in Z_t \setminus \{z\}} \frac{w_t^{(k,z')}}{C_t^{\mathrm{a},\flat}(z')} \right]$$

where, denoting "a" and "$\flat$" the indices i_t^{a} and $i_t^\flat$,

$$C_t^{\mathrm{a},\flat}(z) = \frac{w_t^{(\mathrm{a},z)}}{w_t^{(\mathrm{a},\phi)}} + \frac{w_t^{(\flat,z)}}{w_t^{(\flat,\phi)}},$$

$$C_t'(j, z) = \frac{\big[w_t^{(\mathrm{a},\phi)} w_t^{(\flat,\phi)}\big]^{n_t'}}{\big[w_t^{(\mathrm{a},\phi)}\big]^{\mathbf{1}_{\mathrm{a}}(j)} \big[w_t^{(\flat,\phi)}\big]^{\mathbf{1}_\flat(j)}} \left[\prod_{z \in Z_t \setminus \{z\}} C_t^{\mathrm{a},\flat}(z) \right].$$

The approximation of w_{ex} brings the complexity down to linear in the number of observation and in the number of hypotheses. Yet, this means that the number of hypotheses will still be multiplied by the number of observations at each time step, quickly leading to an unmanageable number of them. In order to introduce an empirical-measure approximation of the association measure α_t, an expression of it as a process composed of a mass and a probability measure needs to be introduced, as described in [9, Sect. 6.4]. First, the recursive expressions (2) and (3) of α_t can be expressed in a more concise way as

$$\alpha_{t+1}(\mathrm{d}(j \cdot z)) \stackrel{\int}{=} \big(\alpha_t \otimes \bar{\mathcal{Z}}_t\big)\big(\mathrm{d}(j,z)\big) w_{\mathrm{ex}}^{(\gamma_t)}(j,z) p_t^{(j)}\big(F_{t,\gamma_t}^{(j,z)}\big), \tag{4}$$

where the potential function $F_{t,\gamma}^{(j,z)}$ is defined, for any observation z in $\bar{Z}_t$ and any measure γ in $\mathbf{M}(\bar{\mathbf{X}}_t \times \mathbb{I}_t^{\mathrm{m}})$, as

$$F_{t,\gamma}^{(j,z)}(x) \doteq \begin{cases} \dfrac{L_t^s(x, A_t^z)}{\int \mathcal{I}_t(\mathrm{d}k) w_{\mathrm{ex}}^{(\gamma)}(k,z) w_t^{(k,z)}} & \text{if } z \in Z_t \\ \dfrac{L_t^s(x,\phi)}{\int \tilde{\mathcal{Z}}_t(\mathrm{d}z') w_{\mathrm{ex}}^{(\gamma)}(j,z') w_t^{(j,z')}} & \text{if } z = \phi, \end{cases}$$

where $(j,s,z) \in \mathbb{I}_{t,t}$ and where the dependency of w_{ex} on the considered measure γ, from which the terms $w_t^{(j,z)}$ can be recovered, is underlined through the use of a superscript. Defining, for any $t \in \mathbb{T}$, the probability measure $\beta_t \in \mathbf{P}(\mathbb{I}_t^{\mathrm{m}})$ as the normalised association measure at time t and $A_t \in \mathbb{R}_+$ as the total mass in α_t, i.e., $A_t \doteq \alpha_t(\mathbb{I}_t^{\mathrm{m}})$ and $\beta_t \doteq \alpha_t / A_t$, the recursion (4) for the association measure α_t can be translated into a recursion for the normalised association measure β_t as

$$\beta_{t+1} = \Pi_t(A_t, \beta_t),$$

where Π_t describes the update of β_t and is such that

$$\begin{aligned} \Pi_t : \mathbb{R}_+ \times \mathbf{P}(\mathbb{I}_t^{\mathrm{m}}) &\to \mathbf{P}(\mathbb{I}_{t+1}^{\mathrm{m}}) \\ (A, \beta) &\mapsto \Psi_{G_{A,\beta}}(\beta \otimes \bar{\mathcal{Z}}_t), \end{aligned}$$

where the potential function $G_{A,\beta}$ is defined as

$$G_{A,\beta}(j,z) \doteq w_{\mathrm{ex}}^{(\mu)}(j,z) p_t^{(j)}\big(F_{t,\mu}^{(j,z)}\big), \qquad \forall j \in \mathbb{I}_t^{\mathrm{m}}, \forall z \in \bar{Z}_t,$$

with $\mu = A\beta\big(p_t^{(\cdot)}\big)$. Describing the recursion of the association measure through the transformation of a probability measure allows for considering recursive approximations as follows: The normalised version $\eta_{t+1} \in \mathbf{P}(\bar{\mathbf{X}}_{t+1} \times \mathbb{I}_{t+1}^{\mathrm{m}})$ of γ_{t+1} is approximated by

$$\eta_{t+1}^N(\mathrm{d}(x,i)) \stackrel{\int}{=} \beta_{t+1}^N(\mathrm{d}i) p_{t+1}^{(i)}(\mathrm{d}x),$$

where β_{t+1}^N is an empirical measure based on N i.i.d. random variables with common law $\Pi_t(A_t, \beta_t^N)$, and it appears that

$$\eta_{t+1}^N \underset{N\uparrow\infty}{\approx} \eta_{t+1}.$$

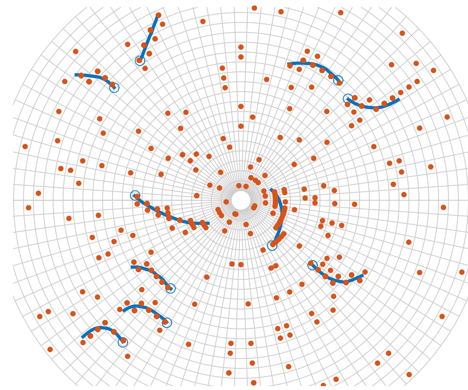


Fig. 1: Trajectories and observations with $(20\mathrm{m}, 4°)$ cells.

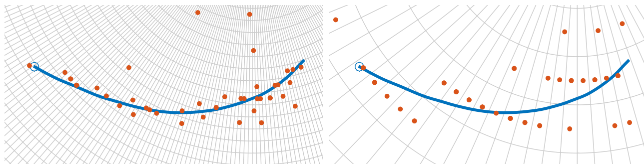
This approximation allows for managing the number of propagated hypotheses. Yet, several of these hypotheses might be statistically close and bring a limited diversity to the estimation; they should therefore be dealt with. As there is no statistical distance between empirical measures that are sufficiently well behaved for the purpose, a way of detecting close hypotheses has to be devised. The first step is to analyse the properties of the single-object filter which explain statistical similarity between two posterior laws, that is: *a*) the stability, which refers to the reduced effect of past observations on the current distribution, and *b*) the robustness, which relates to the fact that a small change in the observation path only induces small modifications of the distribution. For instance, if a distance d is available for observations in $\bar{\mathbf{Z}}_{t'}$ at any time $t' \in \mathbb{T}$, then an example of distance on observation paths up to time $t \in \mathbb{T}$ would take the form

$$\boldsymbol{d}(y,y') = \sum_{t' \le t} \exp\big(-c(t-t')\big) d\big(y_{t'}, y'_{t'}\big),$$

where $c \in \mathbb{R}_+$ is a coefficient that controls the duration required for the effect of large deviations of observations to become negligible. The fact that $\boldsymbol{d}$ is a distance follows directly.

4. SIMULATION RESULTS

We consider a finite-resolution range-bearing sensor with range between 20 and 500 meters in two different configurations for the resolution cells, with a cell size of $(5\mathrm{m}, 1°)$, and of $(20\mathrm{m}, 4°)$. The sensor is located at the centre of the coordinate system. Observations are acquired synchronously every 0.1s and are generated as follows: each object is assumed to have an extension modelled by a Gaussian of standard deviation 2m in each direction, is detected with probability 0.8, and the corresponding resolution cell is selected randomly according to the amount of probability mass that is induced in each cell by the Gaussian distribution modelling the extension. An average of 5 spurious observations is triggered at each time step.



(a) $5\text{m}, 1^\circ$ (b) $20\text{m}, 4^\circ$

Fig. 2: Accumulated observations and trajectory of an object with different cell sizes.

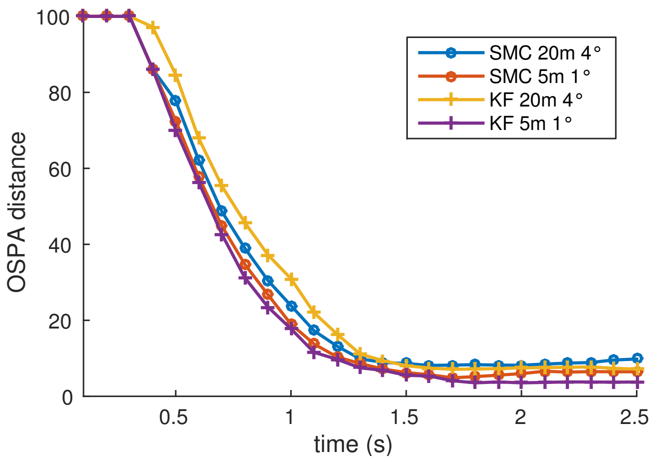


Fig. 3: OSPA distance over 25 time steps, averaged over 100 Monte Carlo runs.

There are 10 objects in the field of view of the sensor evolving in the 2-dimensional Cartesian plane, with trajectories as shown in Figure 1. The motion model of these objects is described by a known and constant turn of $\omega = 1/5$ with a Gaussian noise driven by a non-zero acceleration with 0 mean and standard deviation 10m.s^{-2}. Figure 2 shows one of the objects with the two different resolution-cell sizes in the background.

Two versions of the HISP filter are compared, one is the proposed SMC implementation and the other is based on the **K**alman **f**ilter (KF). Since the considered observation model cannot be used directly in a Kalman filter, we represent resolution cells by a Gaussian centred on the cell and with a standard deviation equal to $1/4$ of the size of the cell in each direction. For the SMC implementation, 2500 particles are used for each hypothesis, and 20 particles are used for modelling the extension. The same HISP parameters are used for both implementations, with probability of disappearance of $1 - 10^{-4}$ and an hypothesis confirmation threshold of 0.9.

Figure 3 compares the two implementations with different cell sizes in terms of OSPA distance [10], which is the distance between the multi-object estimate and the ground truth, with a 2-norm and a cutoff of 100. The performance of the two implementation is similar, confirming that the SMC implementation does not bring down the efficiency while allowing for more diverse types of models to be used. The SMC version also have faster initialisation for larger resolution cells, but tends to be less accurate in the longer term.

5. CONCLUSION

A mixed particle association model has been derived for the HISP filter and demonstrated on a simulated scenario using a finite-resolution sensor with various resolution-cell sizes. Under these conditions, and using a linear motion model, it has been shown to perform as well as a Kalman filter implementation thus showing the efficiency of the considered approach. The choice of a finite-resolution sensor also demonstrates the possibilities offered when using probability theory to model events and uncertainties.

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