Adaptive quadratic regularization for baseline wandering removal in wearable ECG devices

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***Abstract*—The electrocardiogram (ECG) is one of the most important physiological signals to monitor the health status of a patient. Technological advances allow the size and weight of ECG acquisition devices to be strongly reduced so that wearable systems are now available, even though the computational power and memory capacity is generally limited. An ECG signal is affected by several artifacts, among which the baseline wandering (BW), i.e., a slowly varying variation of its trend, represents a major disturbance. Several algorithms for BW removal have been proposed in the literature. In this paper, we propose new methods to face the problem that require low computational and memory resources and thus well comply with a wearable device implementation.**

I. Introduction

The development of reliable, low-cost and not intrusive (or wearable) devices for the measurement of physiological signals exhibited in the last decades a huge interest in the field of health-care applications. Such instrumentation enables the monitoring of patients affected by different pathologies to be extend directly at their home; furthermore, it can be used by healthy people to better understand their well-being status in daily activities. Recent technological advances have supported the development of powerful, light-weight, low-consumption devices, using wireless communications to transmit the acquired physiological signals.

Wearable electrocardiogram (ECG) acquisition devices are one example of such a class of instruments. Unfortunately, ECG signals (even when acquired by standard medical equipment) are affected by several artifacts [1], for example, due to electrode contact noise, power line interference, electromyographic noise, etc., that may hinder a correct diagnosis or use of the ECG data. One of the major disturbances is the *baseline wandering* (BW), caused by patient movement and respiration, that appears as a random variation of the signal trend. Thus, a typical ECG diagram can be seen as the superposition of the informative signal, represented by the classical P, QRS, T, U waves, and a baseline variation, characterized by low–frequency components (up to 0.8 Hz). Removing this disturbance is not simple since its spectrum partly overlaps with that of the informative signal [2].

Several methods and tools for solving the baseline wandering problem have been proposed in the literature; some of the most significant ones have been compared in [3]. In [4], [5], notch filters and time-varying filters were proposed for extracting the baseline signal, whereas linear spline and cubic approximations were presented in [6], [7]. Adaptive filters for BW removal were proposed in [8]. A method based on the discrete wavelet transform (DWT) was presented in [9]. In [10], the empirical mode decomposition (EMD) was proposed for both ECG denoising and BW removal.

In this paper, new methods for solving the baseline wander problem in ECG signals are presented. This study has been stimulated by the works in [11], [12], where an algorithm based on *quadratic variation* reduction (QVR) was proposed. A linear time invariant (LTI) implementation approximating the QVR method has also been presented in [13]. According to [11], [14], the QVR method achieves better performances, in comparison to other classical techniques (highpass filtering, median filtering, adaptive filtering, wavelet adaptive filtering, see references in [14]), in achieving a fine BW removal and preserving the shape of the ST segment, which is important for clinical detection of heart diseases. The methods proposed in this study are based on approximating the baseline signal by means of auto-regressive moving-average (ARMA) models, whose parameters are *adaptively* estimated in the framework of QVR optimization. The rationale for introducing ARMA models for BW removal is that of reducing the computational burden and the memory requirements of the QVR method that may hinder its implementation on-board wearable ECG devices characterized by low-cost and low-performance hardware platforms.

The paper is organized as follows. In Section II, the QVR method is reviewed. In Section III, ARMA modeling for the baseline signal is presented and, in Section IV, LMS and RLS approaches for model parameters estimation are proposed. Experimental results obtained by using simulated as well as truly acquired ECG signals are presented in Section V. Some conclusions are drawn in Section VI.

II. BW removal with quadratic regularization

Let $x[k]$, $k = 1, 2, \ldots, n$, be the acquired ECG signal affected by a baseline $q[n]$, $k = 1, 2, \ldots, n$. It is assumed that q is a lowpass signal that introduces slow variations (or trend) into the ECG. The objective of a BW removal algorithm is that of estimating q from x and remove it, so that $x - q$ has the same shape of x and a constant trend.

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The quadratic regularization term used in the QVR method is defined as

$$\mathcal{Q}_q = \sum_{k=1}^{n-1} (q[k] - q[k+1])^2. \quad (1)$$

It is apparent that limiting the quantity $\mathcal{Q}_q$ induces smoothness in the signal q. Thus, the QVR method for baseline estimation can be formulated as follows [11]:

$$\hat{\mathbf{q}} = \arg\min_{\mathbf{q}} \|\mathbf{x} - \mathbf{q}\|^2 \qquad \text{subject to } \mathcal{Q}_q \leq \rho, \quad (2)$$

where $\mathbf{x}$ and $\mathbf{q}$ are n-length column vector representations of x and q, respectively, $\|\cdot\|$ is the l_2 norm of a vector, and ρ is a given constant.

This constrained minimization problem can be reformulated into an unconstrained one by choosing

$$\hat{\mathbf{q}} = \arg\min_{\mathbf{q}} \left[\|\mathbf{x} - \mathbf{q}\|^2 + \lambda \mathcal{Q}_q \right], \quad (3)$$

where λ, which can be related to ρ, can be seen as a parameter that balances the "fidelity" component and the "regularization" component of the function to be minimized. In order to find a smooth baseline q, large values of λ must be chosen. The target function in (3) is quadratic and the solution is linear and easily achievable in a closed form. Let the $(n-1) \times n$ matrix $\mathbf{D}$ be defined as

$$\mathbf{D} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & -1 \end{bmatrix}. \quad (4)$$

By using this notation, we have that $\mathcal{Q}_q = \|\mathbf{D}\mathbf{q}\|^2$ and, thus, the problem can be reformulated as

$$\hat{\mathbf{q}} = \arg\min_{\mathbf{q}} \|\mathbf{x} - \mathbf{q}\|^2 + \lambda \|\mathbf{D}\mathbf{q}\|^2. \quad (5)$$

The solution of (5) is given by

$$\hat{\mathbf{q}} = \left(\mathbf{I}_n + \lambda \mathbf{D}^T \mathbf{D}\right)^{-1} \mathbf{x}, \quad (6)$$

where $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. As can be seen, the baseline estimation is linear in the observed ECG signal, but the storage of long segments of the signal to be filtered is necessary.

In [13], a LTI approximation of the solution in (6) is presented. By taking the kth row of the solution, rewritten as $\left(\mathbf{I} + \lambda \mathbf{D}^T \mathbf{D}\right) \hat{\mathbf{q}} = \mathbf{x}$, for $1 < k < n$ (i.e., excluding the first and last sample) we get

$$-\lambda \hat{q}[k-1] + (1 + 2\lambda)\hat{q}[k] - \lambda \hat{q}[k+1] = x[k]. \quad (7)$$

Thus, the baseline can be estimated by filtering the observed ECG signal by means of the transfer function

$$H(z) = \frac{1}{-\lambda z^{-1} + (1 + 2\lambda) - \lambda z}, \quad (8)$$

which is characterized by a couple of poles, one the inverse of the other, so that the filter can be implemented as a single-pole IIR filter applied once in the forward and once in the backward direction, i.e., in a noncausal fashion.

In order to achieve an on-line implementation of the baseline estimation, new methods are proposed in the next section.

III. ARMA modeling of BW

The quadratic regularization problem has been revisited by assuming that the baseline q can be obtained from the observed ECG signal x by means of an ARMA model. Assume that

$$Q(z) = \frac{B(z)}{A(z)} X(z), \quad (9)$$

where

$$B(z) = \sum_{k=0}^{M} b_k z^{-k}, \qquad A(z) = 1 + \sum_{k=1}^{N} a_k z^{-k}, \quad (10)$$

with M and N the orders of the MA and AR components of the model, respectively, and b_k, $k = 0, 1, \ldots, M$, and a_k, $k = 1, \ldots, N$, their parameters. Thus, the baseline is given by

$$\begin{aligned} q[n] &= \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k q[n-k] \\ &= \boldsymbol{\varphi}_1^T[n] \boldsymbol{\theta}, \end{aligned} \quad (11)$$

where

$$\begin{aligned} \boldsymbol{\varphi}_1[n] &= \left[x[n] \; \ldots \; x[n-M] \; q[n-1] \; \ldots \; q[n-N]\right]^T \\ \boldsymbol{\theta} &= \left[b_0 \; \ldots \; b_M \; a_1 \; \ldots \; a_N\right]^T. \end{aligned} \quad (12)$$

In order to express the difference operation that allows the regularity term to be defined, let $\mathbf{h} = [1 \;\; -1]^T$, so that

$$\begin{aligned} q[n] - q[n-1] &= \mathbf{h}^T \begin{bmatrix} q[n] \\ q[n-1] \end{bmatrix} = \mathbf{h}^T \begin{bmatrix} \boldsymbol{\varphi}_1^T[n] \\ \boldsymbol{\varphi}_1^T[n-1] \end{bmatrix} \boldsymbol{\theta} \\ &= \boldsymbol{\varphi}_2^T[n] \boldsymbol{\theta}, \end{aligned} \quad (13)$$

where (11) has been used and

$$\boldsymbol{\varphi}_2[n] = \left[\boldsymbol{\varphi}_1[n] \; \boldsymbol{\varphi}_1[n-1]\right] \mathbf{h}. \quad (14)$$

Consider now the following cost function that we would like to optimize for estimating $\boldsymbol{\theta}$:

$$\begin{aligned} J[n] = \frac{1}{n} \Bigg[\sum_{k=1}^{n} (x[n] - q[n])^2 \\ + \lambda (q[n] - q[n-1])^2 \Bigg] \end{aligned} \quad (15)$$

where the fidelity and regularization terms can be easily recognized. By using (11) and (13) into (15), we get

$$
\begin{aligned}
J[n] =& \frac{1}{n}\left[\sum_{k=1}^{n}(x[k]-\boldsymbol{\varphi}_1^T[k]\boldsymbol{\theta})^2+\lambda(\boldsymbol{\varphi}_2^T[k]\boldsymbol{\theta})^2\right] \\
=& \frac{1}{n}\sum_{k=1}^{n}x^2[k]-2\left(\frac{1}{n}\sum_{k=1}^{n}x[k]\boldsymbol{\varphi}_1^T[k]\right)\boldsymbol{\theta} \\
&+\boldsymbol{\theta}^T\left(\frac{1}{n}\sum_{k=1}^{n}\boldsymbol{\varphi}_1[k]\boldsymbol{\varphi}_1^T[k]\right)\boldsymbol{\theta} \\
&+\lambda\,\boldsymbol{\theta}^T\left(\frac{1}{n}\sum_{k=1}^{n}\boldsymbol{\varphi}_2[k]\boldsymbol{\varphi}_2^T[k]\right)\boldsymbol{\theta} \\
=& \frac{1}{n}\sum_{k=1}^{n}x^2[k]-2\mathbf{r}_n^T\boldsymbol{\theta}+\boldsymbol{\theta}^T\mathbf{R}_{1,n}\boldsymbol{\theta}+\lambda\,\boldsymbol{\theta}^T\mathbf{R}_{2,n}\boldsymbol{\theta}, \quad (16)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{R}_{1,n} &= \frac{1}{n}\sum_{k=1}^{n}\boldsymbol{\varphi}_1[k]\boldsymbol{\varphi}_1^T[k] \\
\mathbf{R}_{2,n} &= \frac{1}{n}\sum_{k=1}^{n}\boldsymbol{\varphi}_2[k]\boldsymbol{\varphi}_2^T[k] \quad (17) \\
\mathbf{r}_n &= \frac{1}{n}\sum_{k=1}^{n}\boldsymbol{\varphi}_1[k]x[k]
\end{aligned}
$$

The optimal parameter vector that minimizes $J[n]$ is then given by

$$
\hat{\boldsymbol{\theta}} = \left(\mathbf{R}_{1,n}+\lambda\mathbf{R}_{2,n}\right)^{-1}\mathbf{r}_n. \quad (18)
$$

In the proposed metod, the solution is adaptive since (18) can be used to compute a new parameter vector whenever a new sample is available. The complexity is dictated by the order of the matrix to be inverted in (18), which coincides with the number of the parameters of the ARMA model in (10), and by the cost for updating the quantities $\mathbf{R}_{1,n}$, $\mathbf{R}_{2,n}$ and $\mathbf{r}_n$. As to this latter aspect, with reference, for instance, to the updating of $\mathbf{R}_{1,n}$, we have

$$
\begin{aligned}
\mathbf{R}_{1,n+1} &= \frac{1}{n+1}\sum_{k=1}^{n+1}\boldsymbol{\varphi}_1[k]\boldsymbol{\varphi}_1^T[k] \\
&= \frac{1}{n+1}\left(\boldsymbol{\varphi}_1[n+1]\boldsymbol{\varphi}_1^T[n+1]+\sum_{k=1}^{n}\boldsymbol{\varphi}_1[k]\boldsymbol{\varphi}_1^T[k]\right) \\
&= \frac{1}{n+1}\boldsymbol{\varphi}_1[n+1]\boldsymbol{\varphi}_1^T[n+1]+\frac{n}{n+1}\mathbf{R}_{1,n} \quad (19)
\end{aligned}
$$

IV. LMS and RLS solutions

In order to reduce the computational cost of the previous solution, adaptive least mean square (LMS) and recursive least squares (RLS) solutions can also be devised. To formulate such approaches in the context of quadratic regularization, define the following vectors:

$$
\mathbf{y}[k] = \left[x[k]\ 0\right]^T \quad (20)
$$

$$
\boldsymbol{\varphi}[k] = \left[\boldsymbol{\varphi}_1[k]\ \sqrt{\lambda}\boldsymbol{\varphi}_2[k]\right] \quad (21)
$$

$$
\mathbf{e}[k] = \mathbf{y}[k]-\boldsymbol{\varphi}^T[k]\boldsymbol{\theta}. \quad (22)
$$

A. LMS solution

The LMS solution [15] coincides with the following parameter vector updating:

$$
\hat{\boldsymbol{\theta}}[n] = \hat{\boldsymbol{\theta}}[n-1]-\frac{\mu}{2}\nabla\|\mathbf{e}[n]\|^2 \quad (23)
$$

where the time index has been added to $\hat{\boldsymbol{\theta}}$, μ is the updating gain and $\nabla\|\mathbf{e}[n]\|^2$ is the gradient of squared norm of the last error given by

$$
\nabla\|\mathbf{e}[n]\|^2 = \nabla\|\mathbf{y}[n]-\boldsymbol{\varphi}^T[n]\hat{\boldsymbol{\theta}}[n-1]\|^2 = -2\boldsymbol{\varphi}[n]\mathbf{e}[n], \quad (24)
$$

where $\hat{\boldsymbol{\theta}}[n-1]$ is used for the computation of $\mathbf{e}[n]$. By substituting (24) into (23) yields

$$
\hat{\boldsymbol{\theta}}[n] = \hat{\boldsymbol{\theta}}[n-1]+\mu\boldsymbol{\varphi}[n]\mathbf{e}[n] \quad (25)
$$

B. RLS solution

Consider the following RLS cost function

$$
J_{\mathrm{RLS}}[n] = \sum_{k=1}^{n}\alpha^{n-k}\|\mathbf{e}[k]\|^2, \quad (26)
$$

where the forgetting factor α has been added to the least squares cost in (15) and the factor $1/n$ has been omitted. The normal equations that allow the estimate $\hat{\boldsymbol{\theta}}[n]$ to be achieved are given by

$$
\mathbf{R}[n]\hat{\boldsymbol{\theta}}[n] = \mathbf{d}[n], \quad (27)
$$

where

$$
\begin{aligned}
\mathbf{R}[n] &= \sum_{k=1}^{n}\alpha^{n-k}\boldsymbol{\varphi}[k]\boldsymbol{\varphi}^T[k] \\
\mathbf{d}[n] &= \sum_{k=1}^{n}\alpha^{n-k}\boldsymbol{\varphi}[k]\mathbf{y}[k]. \quad (28)
\end{aligned}
$$

The RLS approach avoids solving the system in (27) by updating $\hat{\boldsymbol{\theta}}[n]$ from $\hat{\boldsymbol{\theta}}[n-1]$ and by computing $\mathbf{P}[n]=\mathbf{R}^{-1}[n]$ from $\mathbf{P}[n-1]$ by using an updating formula [16]. In fact, it can be shown that

$$
\begin{aligned}
\hat{\boldsymbol{\theta}}[n] &= \hat{\boldsymbol{\theta}}[n-1]+\mathbf{K}[n](\mathbf{y}-\boldsymbol{\varphi}^T[n]\hat{\boldsymbol{\theta}}[n-1]) \\
&= \hat{\boldsymbol{\theta}}[n-1]+\mathbf{K}[n]\mathbf{e}[n] \quad (29)
\end{aligned}
$$

where

$$
\mathbf{K}[n] = \alpha^{-1}\mathbf{P}[n-1]\boldsymbol{\varphi}[n](\mathbf{I}_2+\alpha^{-1}\boldsymbol{\varphi}^T[n]\mathbf{P}[n-1]\boldsymbol{\varphi}[n])^{-1} \quad (30)
$$

and

$$
\mathbf{P}[n] = \alpha^{-1}\mathbf{P}[n-1]-\mathbf{K}[n]\boldsymbol{\varphi}^T[n]\alpha^{-1}\mathbf{P}[n-1] \quad (31)
$$

V. Experimental results

In this section, some experimental results obtained by applying the proposed methods are presented. In the first set of tests, synthetic baseline-free ECG signals are used. A known pseudo-random baseline is generated and added to the synthetic ECG signals, so that BW removal performance of different algorithms can be quantified. In the second set of experiments, real ECG signals acquired by using a prototype of wearable ECG device developed in our laboratory are processed and the algorithms are visually compared.

A. Experimental tests with synthetic ECG signals and baseline

The algorithm presented in [17] (a Matlab implementation of which is available in PhysioNet [18]) has been used to generate synthetic baseline-free ECG signals. The data were generated by setting the heart rate to 60 bpm, with a sampling frequency $f_s = 256$ Hz and an additive Gaussian noise with standard deviation $\sigma_n = 0.01$. The output is an ECG-like signal normalized between -0.4 and 1.2 mV. Then, a synthetic pseudo-random baseline was added to the ECG signal. The baseline was generated by filtering a white Gaussian process with a fourth-order Butterworth filter with a 3-dB cutoff frequency set to a given value f_t. The amplitude of the output baseline was adjusted so that its standard deviation σ_b was equal to 0.5 mV.

The BW removal algorithms proposed in this paper were compared with the QVR methods presented in [11] and [13]. All methods have been implemented in Matlab. The estimation of the ARMA parameters, obtained from inverting the normal equations (i.e., by using (18)), and from either the LMS or RLS approaches, are denoted in the following as NE, LMS, and RLS, in that order. The order of the ARMA model was $M = 0$ and $N = 3$, whereas the value of λ was set to 800. Such parameters were selected according to the outcome of experimental tests. In the RLS method, the forgetting factor α was set to $\exp(\log(0.5)/5000)$, such that the weight in (26) is halved after 5000 samples. The method in [11] was implemented on a block basis, that is the baseline is estimated on nonoverlapping blocks, having a length of 8192 samples. For both methods in [11] and [13], denoted in the following as QVR and QVR-LTI, respectively, the value of λ was set to 10^4 (as suggested in the original papers). The methods are compared in terms of the mean square error (MSE), defined as

$$MSE = \frac{\sum_n (q[n] - \hat{q}[n])^2}{N_q}, \quad (32)$$

where q is the synthetically generated baseline, $\hat{q}$ is its estimation obtained from the BW removal algorithms, and N_q is their length. Adaptive algorithms are evaluated at their convergence, i.e., the head portion of the baseline was discarded.

In Table I, the values of MSE obtained with different values of f_t and averaged over 50 realizations of the pseudo-random baseline are reported. The table shows that the proposed methods seem able to better estimate the BW when the variation of the baseline becomes more rapid (i.e., for higher f_t). Among the proposed algorithms, the RLS method outperforms the LMS one for every choice of f_t and the NE method for the lowest f_t. In Fig. 1, the BW removal obtained with the QVR-LTI and the RLS methods (the others are not shown for the sake of plots' clarity) are presented for one realization of the synthetic signals in the case $f_t = 0.4$ Hz.

Table I
MSE values (averaged over 50 realizations of the baseline).

Method	$f_t = 0.2$ Hz	$f_t = 0.4$ Hz	$f_t = 0.6$ Hz
QVR	0.0128	0.0295	0.0539
QVR-LTI	0.0127	0.0296	0.0544
NE	0.0221	0.0261	0.0356
LMS	0.0230	0.0349	0.0477
RLS	0.0183	0.0267	0.0356

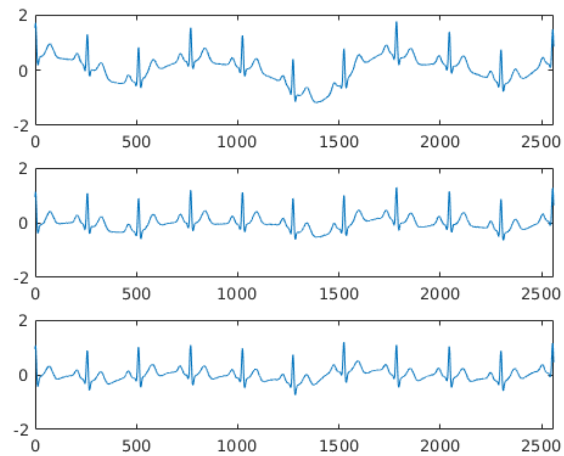

Figure 1. Sinthetic ECG with synthetic baseline generated with $f_t = 0.4$ (top plot) and BW removal results obtained with the QVR-LTI method (middle plot) and the RLS method (bottom plot).

As to the computational burden of the proposed methods, the cost of the RLS and LMS approaches is $O(L^2)$ and $O(L)$, respectively, where $L = M + N + 1$ is the number of ARMA parameters to be estimated. Since L is low, the cost (even for the NE method, where a symmetric matrix needs to be inverted) is limited.

B. Real ECG data

A prototype of ECG acquisition device, shown in Fig. 2, has been developed in our laboratory. Its features are the following: acquisition of 3 ECG bipolar derivations (DI, DII, DIII) and 1 pre-cordial derivation (V1), by using 5 standard electrodes; analogic front-end and ADC at 24 bit (Texas Instruments ADS1293), sampling frequency up to 25.6 ksps; micro-controller ARM STM32F411; storage onto microSD; transmission of ECG signals in real time by means of wireless Bluetooth 4.0 Low Energy (Nordic Semiconductor nRF8001) or by means of USB connection; accelerometer on-board (Analog Device ADXL363); PCB dimension of 44x60 mm; long duration battery with capacity of 1300 mAh; standard

ECG connectors DIN, diameter 1.5mm. In Fig. 3, an example of a real ECG signal acquired with the prototype device is shown as well as the results of the QVR-LTI and RLS BW removal algorithms (the samples amplitude is not expressed in voltage, but as integer values). The methods were run in Matlab after importing the data (a porting of the algorithms on-board the device is under development).

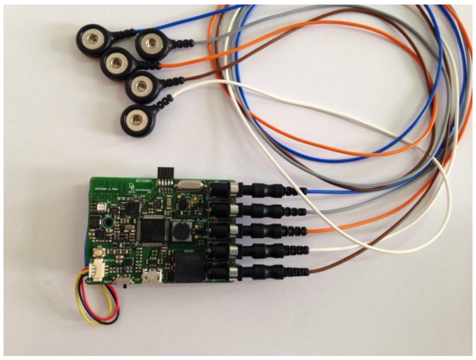

Figure 2. Prototype of ECG acquisition device.

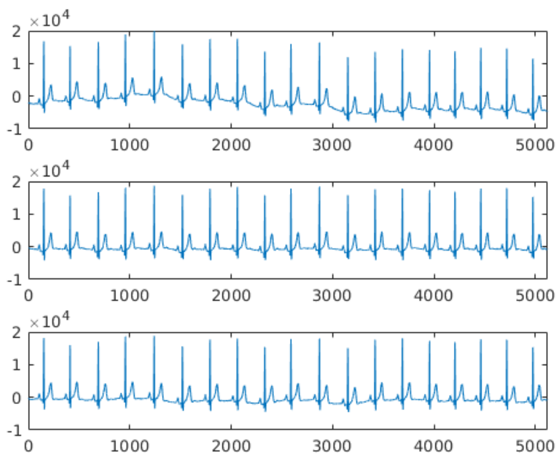

Figure 3. ECG aquired with the laboratory prototype (top plot) and BW removal results obtained with the QVR-LTI method (middle plot) and the RLS method (bottom plot).

VI. Conclusion

In this paper, we have presented methods for baseline wandering removal in ECG signals based on quadratic regularization and ARMA modeling. The validity of the proposed algorithms has been assessed by using both synthetic and real ECG signals and by comparisons with known algorithms. The proposed methods are characterized by limited computational burden and memory requirements so that its implementation on wearable ECG devices is feasible.

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