Active Noise Control for Pulse Signals by Wave Field Synthesis

Alessandro Lapini*, Massimiliano Biagini*, Francesco Borchi*, Monica Carfagni* and Fabrizio Argenti†

*Department of Industrial Engineering, University of Florence, Via di Santa Marta, 3 – 50139 – Firenze – Italy
Email: {alessandro.lapini, massimiliano.biagini, francesco.borchi, monica.carfagni}@unifi.it
†Department of Information Engineering, University of Florence, Via di Santa Marta, 3 – 50139 – Firenze – Italy
Email: fabrizio.argenti@unifi.it

Abstract—Active noise control (ANC) techniques have been intensively studied and applied for the cancellation of stationary noise. More recently, adaptive solutions for the case of impulsive noise, i.e. stochastic processes for which statistical moments superior to the first are not defined, have been proposed in the literature. Nevertheless, such a model fits a limited class of impulsive disturbances that could be experienced in practice. This paper introduces a preliminary study on a non-adaptive deterministic ANC technique for pulse signals that relies on no statistical assumptions. In particular, the spatial audio rendering framework of Wave Field Synthesis is formally adopted in order to synthesize the cancelling acoustic field. Simulations in free field environment, including the analysis of impairments such as time mismatch and template mismatch, have been carried out, showing promising performances in terms of noise cancellation.

I. INTRODUCTION

Active noise control (ANC) techniques aim at reducing the effect of an acoustic noisy source by means of a cancelling sound wave generated by a suitable array of secondary sources. In the past decades, the interest in the design of ANC systems has considerably grown because of the superior advantages in abating low frequency noise than passive methods [1].

A common feature shared by most of the existing ANC techniques is adaptivity. By means of reference and error microphones, which measure the primary noise and the residual signal (that is, the signal remaining after the application of the cancelling wave), respectively, an adaptive ANC system pilots the array of secondary sources by varying its response in a real-time manner, according to the registered signals. Adaptive algorithms, such as the filtered-x least mean square (FxLMS) [2] and its extensions, are capable to minimize the variance of the residual acoustic signal where the noise can be modeled as a Gaussian stationary process [3]. Although progress has been made, there are still some limitations on the application of ANC systems, such as practical situations where the target noise is impulsive. In the literature, impulsive noise is usually modeled as a symmetric-alpha stable process (SαS), lacking of second-order and higher statistical moments. Therefore, FxLMS-based algorithms might fail to correctly adapt their response, causing a degradation of the cancellation performance and stability problems. The extension of ANC techniques designed for stationary noise to the case of impulsive noise represents a challenging task and it is currently an open research field. Variations of the FxLMS algorithm based on the minimization of the p-norm (1 ≤ p < 2) [4], robust statistics [5] as well as hard thresholding [6] have been proposed. The previous methods generally aim to minimize a mean distortion measure; hence, in presence of a spike noise pulse, they tend to mitigate its effects on the adaptation procedure rather than effectively abate it. In this paper, we address the problem of designing ANC systems for a different scenario. The motivations arise considering practical situations where the typical framework of impulsive noise does not sufficiently fit the experimental setup such as, for example, in shooting ranges, construction sites and puncturing machines. In these cases, we are mainly interested in cancelling the noise spikes and not in minimizing a mean measure of distortion. Hence, we model the disturbance as a deterministic train of short pulses, whose occurrences are unpredictably shaped and distributed across time. With the term ”short” we generic mean that such pulses have a very limited time support and are interleaved by unknown deterministic intervals. For sake of synthesis, we will indicate this characterization as pulse noise in order to distinguish from impulsive noise.

According to the previous definitions, impulsive and pulse noise describe quite different models. Impulsive noise follows a known statistical distribution and it can be exploited to obtain some optimal predictor. In contrast, pulse noise represents a deterministic model and it does not rely on any statistical hypothesis; thus, no adaptation procedure can be performed.

In order to cope with the above issues, a deterministic ANC system for pulse noise based on the Wave Field Synthesis (WFS) is considered in this paper [7]. The idea of using the WFS framework to synthesize a virtual anti-source whose acoustic field is used in a destructive manner has been previously proposed [8], [9]. In this paper we extend the previous works, by formalizing the problem for the case of pulse noise and considering a related preliminary design of a WFS based ANC system operating in free field environment. We make use of simulations in both 2–D and 3–D scenarios in order to assess the cancellation performance.

The paper is organized as follows: Section II summarizes the concepts of WFS. The formalization of WFS based ANC systems is proposed in Section III. Section IV reports the results obtained in the simulations as well as the discussion of some impairments. Finally, conclusions and remarks are presented in Section V.
II. WAVE FIELD SYNTHESIS

The theory of WFS directly arises from the Kirchhoff-Helmholtz integral [7], which states that the pressure field inside a volume \( V \) generated by a distribution of primary acoustic sources can be reproduced by a continuous distribution of secondary elementary sources placed over the boundary of \( V \), i.e., the surface \( S \). Mathematically, it is equivalent to

\[
P(\mathbf{r}, \omega) = -\frac{1}{4\pi} \int_{S} \left[ G(\mathbf{r}_{S}|\mathbf{r}, \omega) \nabla_{r_{S}} P(\mathbf{r}_{S}, \omega) \right. \\
\left. - P(\mathbf{r}_{S}, \omega) \nabla_{r_{S}} G(\mathbf{r}_{S}|\mathbf{r}, \omega) \right] \cdot \mathbf{n}_{r_{S}} dS.
\]

(1)

In eq. (1), \( P(\mathbf{r}, \omega) \) denotes the pressure field at point \( \mathbf{r} \in V \) and having frequency \( \omega \); \( \mathbf{r}_{S} \in S \) is a point on the surface \( S \); \( \mathbf{n}_{r_{S}} \) represents the surface normal in \( \mathbf{r}_{S} \) pointing inwards \( V \); \( \nabla_{r_{S}} \) is the gradient evaluated in \( \mathbf{r}_{S} \) and \( G(\mathbf{r}_{S}|\mathbf{r}, \omega) \) is the appropriate Green function for the considered \( N-D \) space.

The above relation allows a virtual source (or more virtual sources) to be synthesized for a listener inside \( V \), by knowing the values of the pressure field \( P(\mathbf{r}_{S}, \omega) \) and the gradient \( \nabla_{S} P(\mathbf{r}_{S}, \omega) \) that it induces on the surface \( S \); such values are used to excite monopole \( G(\mathbf{r}_{S}|\mathbf{r}, \omega) \) and dipole \( \nabla_{S} G(\mathbf{r}_{S}|\mathbf{r}, \omega) \) secondary sources.

In order to be practically useful, eq. (1) is usually arranged under some convenient geometrical configurations. A typical example is the Rayleigh I integral. Indeed, for a 2-D space, it can be shown that the pressure in the entire half-plane not containing the primary sources can be synthesized by an infinitely extended linear array of elementary monopoles laying on a line \( L \):

\[
P(\mathbf{r}, \omega) = -\frac{1}{2\pi} \int_{L} G(\mathbf{r}_{L}|\mathbf{r}, \omega) \nabla_{r_{L}} P(\mathbf{r}_{L}, \omega) \cdot \mathbf{n}_{r_{L}} dL.
\]

(2)

An analogous formulation for a 3-D space is

\[
P(\mathbf{r}, \omega) = -\frac{1}{4\pi} \int_{A} G(\mathbf{r}_{A}|\mathbf{r}, \omega) \nabla_{r_{A}} P(\mathbf{r}_{A}, \omega) \cdot \mathbf{n}_{r_{A}} dA,
\]

(3)

which states that the pressure can be reproduced by an infinitely extended planar array of elementary monopoles laying on a plane \( A \).

In a homogeneous 3D scenario, if the primary source is a monopole located in \( \mathbf{r}_{0} \) having excitation \( S(\omega) \) and if we are mainly interested in the acoustic field on a plane \( B \) passing through \( \mathbf{r}_{0} \), a simplified solution can be also considered. The 2D\(^{1/2} \) Rayleigh I integral, proposed in [10], states that the wave field synthesis can be approximate by means of a linear array \( L \) laying on \( B \):

\[
P(\mathbf{r}, \omega) \approx -g_{0} \int_{L} G(\mathbf{r}_{L}|\mathbf{r}, \omega) \\
\cdot \langle S(\omega) \rangle \sqrt{\frac{j\omega |\mathbf{r}_{L} - \mathbf{r}_{0}|}{2\pi c}} \cos \varphi_{inc} G(\mathbf{r}_{0}|\mathbf{r}_{L}, \omega) dL,
\]

(4)

where \( c \) is the speed of sound in the medium; \( \varphi_{inc} \) in the angle between the normal to the array line \( L \) laying on \( B \) and the line passing through \( \mathbf{r}_{0} \) and \( \mathbf{r} \). In order to define \( g_{0} \), please consider the plane \( C \) normal to \( B \) and passing through \( L \); then

\[
g_{0} = \sqrt{\Delta_{C}\mathbf{r}/(\Delta_{C}\mathbf{r} + \Delta_{C}\mathbf{r}_{0})},
\]

being \( \Delta_{C}\mathbf{r} \) and \( \Delta_{C}\mathbf{r}_{0} \) the distance between \( \mathbf{r} \) and \( C \) and between \( \mathbf{r}_{0} \) and \( C \), respectively. Since \( g_{0} \) depends on \( \mathbf{r} \), the approximation given by the 2D\(^{1/2} \) Rayleigh I integral is more accurate for listeners on the plane \( B \) and placed at a specific distance from \( L \). Moreover, the approximation becomes increasingly poorer also when \( \mathbf{r} \) moves farther form the plane \( B \).

III. FORMALIZATION OF WFS BASED ANC FOR PULSE SIGNALS IN FREE FIELD

In the considered practical scenario, the primary noise source is represented by an acoustic monopole placed in a homogeneous medium at \( \mathbf{r}_{0} \) and excited by a signal \( \hat{s}(t) \)

\[
\hat{s}(t) = \sum_{i} s_{i}(t)
\]

(5)

where each \( s_{i}(t) \) is a compactly supported pulse in a corresponding temporal interval \( \tau_{i} \) that is disjoint from the other ones, i.e., \( s_{i}(t) = 0 \) \( \forall t \notin \tau_{i} \) and \( \tau_{i} \cap \tau_{j} = \emptyset \) \( \forall i \neq j \). Since no assumptions are made on \( s_{i}(t) \), we can consider the analysis of single pulse \( s_{0}(t) \) without loss of generality.

Due to the limited duration, \( s_{0}(t) \) is generally a wideband signal having Fourier transform \( S_{0}(\omega) \) and inducing a pressure field on \( \mathbf{r} \) given by

\[
p_{0}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} P_{0}(\mathbf{r}, \omega)e^{j\omega t} d\omega,
\]

(6)

Our goal is to reproduce a cancelling pressure field \( \hat{p}(\mathbf{r}, t) = -\hat{s}(t) \) in an \( N-D \) half-space not containing the source. For the 2-D scenario, according to eq. (6) and the Rayleigh I integral, a cancelling linear array on the line \( L \) provides

\[
\hat{p}(\mathbf{r}, t) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ \int_{L} G(\mathbf{r}_{L}|\mathbf{r}, \omega) \nabla_{r_{L}} P(\mathbf{r}_{L}, \omega) \cdot \mathbf{n}_{r_{L}} dL \right] e^{j\omega t} d\omega.
\]

(7)

For practical purposes, discrete sources must be considered. Considering a uniform sampling of \( L \) with interval \( \Delta_{L} \), eq. (7) can be rearranged as

\[
\hat{p}(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{i} G(\mathbf{r}_{i}|\mathbf{r}, \omega) \frac{\Delta_{L}}{2\pi} \nabla_{r_{i}} P(\mathbf{r}_{i}, \omega) \cdot \mathbf{n}_{r_{i}} e^{j\omega t} d\omega,
\]

(8)

where \( \mathbf{r}_{i} \) represents the position of the monopoles on the sampling grids over \( L \). Equality in the previous relation strictly holds if \( \Delta_{L} \leq c\tau_{i}/\omega_{\text{max}} \), being \( \omega_{\text{max}} \) the maximum frequency such that \( S(\omega) \neq 0 \); otherwise, anti-aliasing strategies have to be considered [10]. By comparison with eq. (6), eq. (8) states that the \( i \)-th monopoles belonging to the cancelling array has frequency excitation \( \hat{S}_{i}(\omega) \):

\[
\hat{S}_{i}(\omega) = \frac{\Delta_{L}}{2\pi} \nabla_{r_{i}} P(\mathbf{r}_{i}, \omega) \cdot \mathbf{n}_{r_{i}}.
\]

(9)

The excitation of a monopole in a cancelling planar array for the 3-D space can be derived as an extension of the previous case, yielding

\[
\hat{S}_{iq}(\omega) = \frac{\Delta_{L} \Delta_{r}}{4\pi} \nabla_{r_{iq}} P(\mathbf{r}_{iq}, \omega) \cdot \mathbf{n}_{r_{iq}},
\]

(10)
where $i, q$ are sampling indexes in the orthogonal directions of the plane $A$, having sampling intervals equal to $\Delta_l$ and $\Delta_q$, respectively.

Finally, for the cancelling linear array in the 3–D space given by the $2D^{1/2}$ Rayleigh I integral, the excitations are given by

$$
\hat{S}_i(\omega) = \Delta_L g_0 S_0(\omega) \cos \phi_{inc} G(r_i, \omega).
$$

IV. RESULTS

All the simulations presented in this paper have been carried out by using the k-Wave [11] MATLAB toolbox.

A. Simulations setup

The real shot of a competition shotgun has been preliminary recorded by a measurement microphone and sampled at 51.2 kHz. The related pulse and frequency response are reported in Figures 1a and 1b, respectively. The signal has been successively resampled to $f_s = 5.12$ kHz for reducing the computational complexity, preserving about 85% of the original energy, as it can be noted by the cumulative energy plot reported in Figure 1c. Finally, such a signal has been used to feed the primary source.

Figure 2a depicts the simulation scenario for a WFS based ANC system in a 2–D space. In a homogeneous medium constituted by air, the WFS array is placed $sa = 4$ m far from the primary source. The array has a variable aperture $a$; it is composed by discrete elementary monopoles excited according to eq. (9) and sampled at $da = \lambda/2 \approx 67$ mm, being $\lambda = c_0/(f_s/2)$ the shorter wavelength associated to the primary acoustic field and $c_0 = 343$ m/s the speed of sound in the air. Such a value of $da$ is the upper bound on sampling spacing and prevents from spatial aliasing effects on the synthesized field.

The sound energy flowing in the lower half plane is measured by a linear microphone array located at $sm = 17$ m from the primary source and having a variable $\beta$ angular aperture. The performance of the ANC system is assessed by means of the sound energy attenuation

$$
A_{db} = 10 \log_{10} \frac{E_{\beta}}{\tilde{E}_{\beta}},
$$

where $\tilde{E}_{\beta}$ and $E_{\beta}$ are the overall sound energy measured by the microphone array having $\beta$ angular aperture, in the case of the ANC system switched on and off, respectively.

Simulations in the 3–D space are straightforward extensions of the previous case. As depicted in Figure 2b, $a_y$ and $a_z$ are the horizontal and vertical apertures of the planar WFS array, respectively. The array is still placed $sa = 4$ m far from the primary source. Elementary monopoles are distributed over a rectangular grid spaced by $da_y = da_z \approx 67$ mm and excited according to eq. (10). A planar microphone array is located at $sm = 12$ m from the primary source and is characterized by variable horizontal and vertical angular apertures, $\beta_y$ and $\beta_z$, respectively. Thus, the attenuation is measured as

$$
A_{db} = 10 \log_{10} \frac{E_{\beta_y,\beta_z}}{\tilde{E}_{\beta_y,\beta_z}},
$$

being the overall energies a function of both horizontal and vertical angular aperture of the microphone array.

For the linear array in 3–D space, a setup analogous to the case of planar array has been considered (specifically $da$ is equal to $da_y$ of the planar array). The scenario is depicted in Figure 2c.

B. Linear array in a 2–D space

The simulations in the case of a cancelling WFS linear array in a 2–D space have been carried out considering two different array apertures, $a = 20$ m and $a = 30$ m, respectively. Furthermore, a finer sampling space $da = \lambda/4 = 33.5$ mm has been also tested in order to verify the robustness of the antialiasing assumption. The results are presented in Table I. The effect of different array apertures are noticeable for $\beta = 150^\circ$, for which the configuration $a = 20$ m exhibits a loss of about 12–13 dB. As to the impact of spatial sampling of the array, accordingly to the theoretical argument, there is no appreciable advantage on decreasing the value of $da$.

It is interesting to observe the local attenuations introduced by the ANC system. Figures 3a–3b depicts the attenuations measured in the frequency-angle plane for an aperture $a = 20$ m and $a = 30$ m, respectively. The effect of finite array apertures are clearly visible in correspondence with the horizontal cutoffs. Note that the attenuation loss measured around 1700 Hz corresponds to a local attenuation in the original shot signal.

1) Time mismatch: In the previous experimental setup it has been implicitly assumed a perfect synchronization of the secondary sources w.r.t. the incoming primary noise field. Nevertheless, it is interesting to relax this hypothesis due to the very short time support of the noisy pulse, i.e., 4 ms. The effect of imperfect synchronization has been simulated by introducing a temporal mismatch on the array monopoles, i.e., a delay, $\Delta t$. The sound energy attenuations obtained with array aperture $a = 30$ m and setting different values for $\Delta t$ are reported in Table II. A performance degradation of about 12 dB is measured starting from $\Delta t = 0.03$ ms. Furthermore,
the ANC system introduces a gain on the overall sound energy for $\Delta t = 0.5$ ms.

2) Template mismatch: Another source of impairments derives form the synthesis of a pulse different from that one of the primary source. Indeed, in the previous simulations, it has been assumed that the actual pulse signal is perfectly known to the monopole sources. Nevertheless, a possible alternative scenario is represented by the usage of a different template to the monopole sources. Indeed, in the previous simulations, it has been simulated by using two different horizontal apertures, $a = 10$ m and $a = 20$ m, and two different vertical apertures, $a_z = 1$ m and $z = 3$ m, respectively. The sampling space $da = \lambda/2 = 67$ mm has been set. The results are presented in Table IV. No appreciable discrepancies emerge by changing the horizontal aperture $a_y$ for the tested measurement angles. On the contrary, performances are strongly influenced by the vertical aperture $a_z$. Since the array has shorter vertical aperture w.r.t. the horizontal one, the attenuation gradually decreases as $\beta_z$ grows.

D. Linear array in a 3–D space

As to the scenario adopting a cancelling WFS linear array in a 3–D, simulations have been carried out by using two different horizontal apertures, $a = 10$ m and $a = 20$ m. The sampling space $da = \lambda/2 = 67$ mm has been kept. The results are presented in Table V. Even if no appreciable discrepancies appear vs. the horizontal aperture $a$ for the tested measurement angles, a sudden loss of attenuation is experienced as $\beta_z$ increases due to the fact that the array lacks of vertical aperture.
In this paper, the problem of active noise control (ANC) for pulse signals has been considered. According to some realistic scenarios, an alternative free field modelling of the problem that relies on no statistical assumptions has been proposed. The adoption of the Wave Field Synthesis as a deterministic non-adaptive framework for the synthesis of the cancelling acoustic field has been formalized; specifically, the excitations of discrete monopoles constituting linear and planar cancelling arrays have been explicitly provided according to the Rayleigh I integral. In absence of impairments, simulations of both 2–D and 3–D scenarios have shown that attenuation greater than 10–15 dB of sound energy are achievable in the central regions, whose extensions depend on the aperture(s) of the cancelling array.

V. Conclusion

In this paper, the problem of active noise control (ANC) for pulse signals has been considered. According to some realistic scenarios, an alternative free field modelling of the problem that relies on no statistical assumptions has been proposed. The adoption of the Wave Field Synthesis as a deterministic non-adaptive framework for the synthesis of the cancelling acoustic field has been formalized; specifically, the excitations of discrete monopoles constituting linear and planar cancelling arrays have been explicitly provided according to the Rayleigh I integral. In absence of impairments, simulations of both 2–D and 3–D scenarios have shown that attenuation greater than 10–15 dB of sound energy are achievable in the central regions, whose extensions depend on the aperture(s) of the cancelling array.

TABLE IV

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