

Low complexity FRI based Sampling Scheme for UWB Channel Estimation

Tina Yaacoub, Roua Youssef, Emanuel Radoi and Gilles Burel
 Université de Brest, CNRS, UMR 6285 LabSTICC
 6 Avenue Le Gorgeu, 29238 Brest, France

Abstract—In this paper, we propose a low complexity multichannel scheme for Ultra Wideband (UWB) channel impulse response estimation. It is mainly based on the finite rate of innovation (FRI) characteristic of UWB channel impulse response, which allows for a sampling frequency much lower than the Nyquist limit. Since the UWB channel is rich in multipaths, the number of samples required results in an unrealistic number of processing channels. Our approach removes this drawback at the price of a moderate increase of the number of pilot pulses. Compared to other schemes presented in the literature, the one proposed in this paper allows reducing the number of processing channels to values appropriate for practical implementation. Moreover, the same approach is used to further reduce the sampling frequency at each channel. The effectiveness of the proposed approach is demonstrated for IEEE 802.15.3a UWB channel estimation in a coherent reception framework.

I. INTRODUCTION

Impulse Radio Ultra WideBand (IR-UWB) is characterized by the emission of pulses of very short duration (of the order of nanoseconds), with a very low power level. The particular characteristics of IR-UWB signals allow high data rates to be achieved, and present immunity to multipath and high localization potential [1]–[3]. On the other hand, coherent Rake receivers are optimal on a white Gaussian noise channel. However, they require channel estimation, very fine synchronization and a temporal resolution of the order of several tens of picoseconds. These constraints translate into high complexity and energy consumption. In particular, the Nyquist sampling frequency required for IR-UWB channel estimation is of the order of several tens of GHz which pushes the performance of analog to digital converters (ADCs), and sampling systems in general, toward their physical limits.

Our goal is to design a low-complexity and cost sampling scheme for UWB channel estimation. Indeed, the recently proposed concept of the finite rate of innovation (FRI) [4], [5] states that signals having a parametric representation with a finite number of degrees of freedom, can be perfectly reconstructed from a number of samples far below the Nyquist rate. In this context, the authors in [6] proposed a multichannel architecture, called Multi Channel Modulating Waveforms (MCMW) scheme, which consists of modulating the received signal with a set of random waveforms, followed by integrators benches. The scheme allows sampling at the rate of innovation, of infinite streams of delayed and weighted versions of a known pulse shape, while keeping strong stability to noise. The advantage over other FRI single channel schemes, such

as the Sum of Sincs based method [7], is that the sampling frequency at each channel is only equal to the inverse of the signal period, which makes it particularly interesting for UWB signals sampling.

The problem we address in this paper is that the required number of channels in this method is at least equal to twice the number of the UWB channel multipaths. Indeed, an UWB channel may contain up to hundreds of paths [8], thus the real implementation of the scheme becomes impractical. Hence, we propose to reduce the number of MCMW channels, and thus the cost and complexity of the scheme, by increasing the number of pilot pulses used for channel estimation. In addition, we propose to further reduce the sampling frequency at each channel by increasing the number of pilot pulses, and that, for a constant number of channels. Therefore, cheaper ADCs with good resolution may be used.

The remainder of the paper is organized as follows. Section II defines the system model and explains the sampling method for UWB channel estimation. Section III details the approaches proposed to reduce the number of MCMW channels and the sampling frequency. Section IV gives simulation results for UWB channel estimation and coherent reception with Rake receivers.

II. MULTICHANNEL SAMPLING METHOD FOR UWB CHANNEL ESTIMATION AT FINITE RATE OF INNOVATION

In this section, we present the system model then recall the MCMW scheme proposed in [6]. We show its application for UWB channel estimation at the rate of innovation denoted ρ .

A. System model

Let $r(t)$ be an UWB pilot impulse of duration T_r and $h(t) = \sum_{l=1}^L a_l \delta(t - \tau_l)$ the multipath UWB channel where a_l and τ_l are its L gains and delays, respectively. The received signal $x(t)$ is a sum of L delayed and weighted versions of $r(t)$:

$$x(t) = \sum_{l=1}^L a_l r(t - \tau_l) + \lambda(t) \quad (1)$$

$\lambda(t)$ is an additive white gaussian noise with power spectral density $N_0/2$. We consider that $x(t)$ has a limited time support $[0, T]$, i.e., all paths are confined in a duration T :

$$0 \leq \tau_l \leq T - T_r, \quad l = 1, \dots, L \quad (2)$$

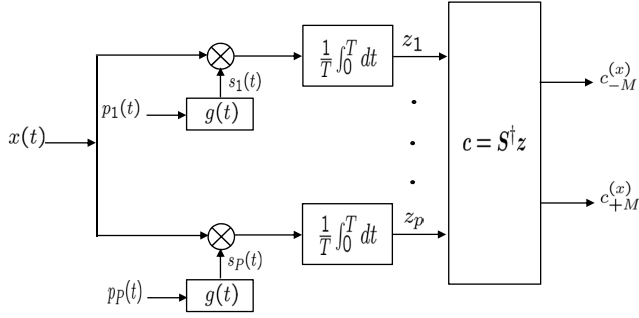


Fig. 1. MCMW sampling scheme.

For channel estimation, $r(t)$ is known at the receiver thus $x(t)$ is totally characterized by $2L$ parameters, namely the L gains a_l and the L delays τ_l . Therefore, according to [4] and [5], $x(t)$ can be fully reconstructed from only $2L$ samples every T , or in other terms, sampled at the rate of innovation $\rho = \frac{2L}{T}$.

Indeed, the ratio of the Fourier transforms of $x(t)$ and $r(t)$ is written as:

$$\frac{X(\nu)}{R(\nu)} = \sum_{l=1}^L a_l e^{-j2\pi\nu\tau_l} + I(\nu) \quad (3)$$

where $I(\nu) = \frac{\Lambda(\nu)}{R(\nu)}$ and $\Lambda(\nu)$ is the Fourier transform of $\lambda(t)$. Given the vector $\{\frac{X(\nu_k)}{R(\nu_k)}\}_{k=1,\dots,K}$, algorithms like Prony, Matrix Pencil or MUSIC, are able to estimate the channel parameters $\{a_l, \tau_l\}_{l=1,\dots,L}$ provided that $K \geq 2L$. The frequencies positions k must be chosen to be consecutive or uniformly spaced, and it is better to choose them from the -3 dB spectrum of $R(\nu)$, so that to avoid dividing by $R(\nu_k)$ values close to zero in (3). We note that since $x(t)$ is limited to T , we will equivalently use in the following the spectrum values $X(\nu_k)$ and the Fourier series expansion coefficients $c_k^{(x)} = \frac{1}{T} X(\nu_k) = \frac{1}{T} \int_0^T x(t) e^{-j2\pi\frac{k}{T}t} dt$.

Vector $\{\frac{X(\nu_k)}{R(\nu_k)}\}_{k=1,\dots,K}$ can be obtained digitally by a simple Fast Fourier Transform (FFT). However, this requires sampling the signal $x(t)$ at the Nyquist rate which is very high for UWB signals (all values $R(\nu_k)$ are known at the receiver). An alternative is to use the MCMW method proposed in [6] which allows to obtain a combination of K Fourier coefficients $c_k^{(x)}$ with a sampling frequency equal to $\frac{1}{T}$ at each channel, far below the Nyquist limit.

B. Multi Channel Modulating Waveform for UWB signals

Figure 1 depicts the MCMW sampling scheme with P channels. At each channel i , the received signal $x(t)$ is modulated by a specific waveform $s_i(t)$ then integrated and sampled every T so that every sample z_i is a combination of P Fourier coefficients $c_k^{(x)}$. For practical implementation issues due to the filtering stage, K must be an odd number, thus, it is set to be equal to $2M + 1$ where $M \geq L$, and $k \in \{-M, \dots, +M\}$. The number of channels P must be at least equal to K ($P \geq K$), to efficiently get the Fourier coefficients from their combinations.

One easy way to obtain the modulating waveforms $s_i(t)$, is

by low-pass filtering random binary T -periodic sequences of (± 1) $\{p_i(t)\}_{i=1,\dots,P}$ [6]. There are exactly P pulses during each period, with unitary amplitudes and random phases. The simplest choice is to consider only one such sequence, e.g., $p_1(t)$, and obtain the remaining $P - 1$ random sequences by a circular shifting of $(i - 1)T/P$ of $p_1(t)$ at each channel. The sequences being T -periodic, they can be expanded in a Fourier series as follows:

$$p_i(t) = \sum_{k \in \mathbb{Z}} c_k^{(p_i)} e^{j2\pi\frac{k}{T}t}, \quad i = 1, \dots, P \quad (4)$$

The low-pass filter $g(t)$ allows to keep only K zero-centered, consecutive and $1/T$ -spaced spectral coefficients $P_i(\nu_k)$, $k \in \{-M, \dots, +M\}$. Its transfer function is given by:

$$G(\nu) = \begin{cases} \neq 0 & \text{if } \nu \in \{-\frac{M}{T}, \dots, \frac{M}{T}\} \\ = 0 & \text{if } \nu = \frac{k}{T}, |k| > M \\ \text{arbitrary} & \text{otherwise} \end{cases} \quad (5)$$

Thus, the spectrum of the resulting modulated waveforms is:

$$S_i(\nu) = G(\nu)P_i(\nu) = \sum_{k=-M}^M c_k^{(p_i)} G(\frac{k}{T}) \delta(\nu - \frac{k}{T}) \quad (6)$$

In time domain, we get:

$$s_i(t) = \sum_{k=-M}^M s_{ki} e^{j2\pi\frac{k}{T}t} \quad (7)$$

where $s_{ki} = c_k^{(p_i)} G(\frac{k}{T})$.

Finally, the combination z_i of the $K = 2M + 1$ Fourier coefficients at each channel i is given by:

$$\begin{aligned} z_i &= \frac{1}{T} \int_0^T x(t) \sum_{k=-M}^M s_{ki} e^{j2\pi\frac{k}{T}t} dt \\ &= \sum_{k=-M}^M s_{ki} c_{-k}^{(x)} = \sum_{k=-M}^M s_{ki}^* c_k^{(x)} \end{aligned} \quad (8)$$

Denoting by z the vector of length P of the values z_i , c the vector of length K containing the coefficients $\{c_k\}_{k=-M,\dots,+M}$ and S the matrix $P \times K$ where the ik^{th} coefficients are the complex conjugate s_{ik}^* , we get c from z by:

$$c = S^\dagger z \quad (9)$$

where S^\dagger is the Moore-Penrose pseudo-inverse of matrix S .

III. PROPOSED APPROACH FOR REDUCING THE NUMBER OF CHANNELS AND THE SAMPLING FREQUENCY

When sampling at the rate of innovation, the number of required Fourier coefficients is $K = 2L + 1$ and the minimum number of channels becomes $P = K = 2L + 1$. The scheme is easy to implement when the value of L is small (for instance, $L = 3$ corresponds to 7 channels). The problem occurs when more samples are needed for better reconstruction due to noise, or when L is very high, as it is the case of multipaths

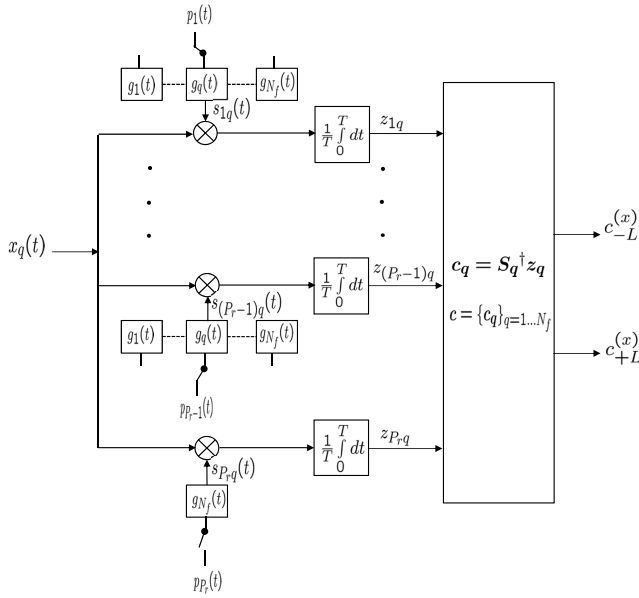


Fig. 2. MCMW sampling scheme with reduced number of channels.

channels. We would like to conserve the sampling frequency at its lowest possible to gain in energy consumption, cost and performance of ADCs. Therefore, we propose to reduce the required number of channels which allow to obtain $2L + 1$ Fourier coefficients, by increasing the number of pilot pulses $r(t)$.

On the other hand, for a fixed number of channels of the MCMW scheme, it is possible to reduce the sampling frequency when increasing the number of transmitted pulses $r(t)$. In this section, we define the new system model and detail the modified MCMW schemes which allow the number of channels or the sampling frequency to be reduced.

A. New system model

The transmitted signal $r'(t)$ contains N_f identical pulses $r(t)$ of duration T_r each, and periodized with a period T such that (2) is satisfied:

$$r'(t) = \sum_{q=1}^{N_f} r(t - (q-1)T) \quad (10)$$

The transmitted and received signals are both of duration $T_S = N_f T$, and we assume that the multipath channel is invariant during T_S . The received signal at the input of the sampling scheme is written as:

$$x'(t) = \sum_{q=1}^{N_f} x_q(t) + \lambda(t) \quad (11)$$

where $x_q(t) = \sum_{l=1}^L a_l r(t - (q-1)T - \tau_l)$. The spectrum ratio of $X'(\nu)$ and $R'(\nu)$ is identical to the one obtained in the case of transmitting one pulse (3). Thus, to find a_l and τ_l , it suffices to apply the spectrum analysis methods when having $2L$ coefficients every T_S and not every T . The rate ρ' is then reduced by a factor N_f , $\rho' = \frac{2L}{T_S} = \frac{\rho}{N_f}$.

| | $N_f = 1$ | $N_f = 10$ | $N_f = 50$ |
|--|-----------|------------|------------|
| Minimum number of channels | $P = 101$ | $P_r = 11$ | $P_r = 3$ |
| Number of integrators, ADCs and mixers | $P = 101$ | $P_r = 11$ | $P_r = 3$ |
| Number of filters | $K = 101$ | $K = 101$ | $K = 101$ |

TABLE I
EXAMPLE OF MCMW WITH REDUCED CHANNELS SCHEME
CHARACTERISTICS WHEN $L = 50$

B. Modified MCMW method for reducing the number of channels

Instead of acquiring $2L + 1$ coefficients from $x(t)$, the idea is to acquire a reduced number of coefficients from each $x_q(t)$ and that for a constant sampling frequency of $1/T$ at each channel. Consider for simplicity that $L = 6$ and $N_f = 3$. Instead of acquiring 13 coefficients $c_k^{(x)}$ from $x(t)$ ($k \in \{-6 \dots +6\}$), 4 coefficients are to be get from $x_1(t)$ ($k \in \{-6; -5; +5; +6\}$), 4 from $x_2(t)$ ($k \in \{-4; -3; +3; +4\}$) and 5 from $x_3(t)$ ($k \in \{-2; -1; 0; +1; +2\}$). The minimal number of channels is now reduced to:

$$P_r = 2 \frac{L}{N_f} + 1 \quad (12)$$

which corresponds to 5 channels in the example above. Note that L is a multiple of N_f . If N_f is equal to L , the number of channels of MCMW becomes equal to 3.

The modified scheme is given in Figure 2. At each channel i , N_f filters are put in series and a switch allows to choose the filter $g_q(t)$ corresponding to the signal $x_q(t)$. The last channel contains only one filter $g_{N_f}(t)$ and is only used when acquiring $x_{N_f}(t)$. Note that all filters are now bandpass filters except for $g_{N_f}(t)$ which remains a low-pass filter.

The T -periodic binary sequences $p_i(t)$ contain P pulses during every period T and not P_r pulses. One way is to consider one sequence $p_1(t)$ and to form the others by shifting it by $(i-1)T/P_r$ at each channel.

For each $x_q(t)$, $q = 1, \dots, N_f - 1$ the corresponding modulating waveform at channel i has a spectrum:

$$\begin{aligned} S_{iq}(\nu) &= G_q(\nu) P_i(\nu) \\ &= \sum_{k=-L+\frac{L}{N_f}(q-1)}^{-L+q(\frac{L}{N_f})-1} c_k^{(p_i)} G_q(\frac{k}{T}) \delta(\nu - \frac{k}{T}) \\ &\quad + \sum_{k=L-\frac{L}{N_f}(1-q)}^{L+\frac{L}{N_f}(1-q)} c_k^{(p_i)} G_q(\frac{k}{T}) \delta(\nu - \frac{k}{T}) \end{aligned} \quad (13)$$

For $x_{N_f}(t)$, the coefficient at K_0 is taken in addition:

$$S_{iN_f}(\nu) = \sum_{k=-L+\frac{L}{N_f}(N_f-1)}^{L+\frac{L}{N_f}(1-N_f)} c_k^{(p_i)} G_{N_f}(\frac{k}{T}) \delta(\nu - \frac{k}{T}) \quad (14)$$

Thus we can write the sample z_{iq} , $q = 1, \dots, N_f - 1$ at each channel as follows:

$$z_{iq} = \sum_{k=-L+\frac{L}{N_f}(q-1)}^{-L+q(\frac{L}{N_f})-1} s_{k iq}^* c_k^{(x_q)} + \sum_{k=L-q(\frac{L}{N_f})+1}^{L+\frac{L}{N_f}(1-q)} s_{k iq}^* c_k^{(x_q)} \quad (15)$$

For $x_{N_f}(t)$, $z_{iN_f}^*$ is written as:

$$z_{iN_f}^* = \sum_{k=-L+\frac{L}{N_f}(N_f-1)}^{L+\frac{L}{N_f}(1-N_f)} s_{k iN_f}^* c_k^{(x_{N_f})} \quad (16)$$

where $s_{k iq} = c_k^{(p_i)} G_q(\frac{k}{T})$.

Similarly to (9), denoting by \mathbf{z}_q the vector of the samples z_{iq} , \mathbf{c}_q the vector of the Fourier coefficients $c_k^{(x_q)}$ and \mathbf{S}_q the matrix where the ik^{th} coefficients are the complex conjugate s_{ik}^* , we get \mathbf{c}_q from \mathbf{z}_q by:

$$\mathbf{c}_q = \mathbf{S}_q^\dagger \mathbf{z}_q \quad (17)$$

Once all $x_q(t)$ pass through the sampling scheme, vector \mathbf{c} of the overall $2L + 1$ Fourier coefficients $c_k^{(x)}$ is formed by concatenating vectors $\{\mathbf{c}_q\}_{q=1, \dots, N_f}$. Consider that $L = 50$ for an UWB channel estimation example. Table I shows the number of resources needed for different N_f values. We observe that for $N_f > 1$, less ADCs, integrators and mixers are required allowing complexity and cost reduction, therefore the implementation of the scheme becomes more practical.

C. Modified MCMW method for reducing the sampling frequency

Now the number of channels is kept equal to $P \geq K$, and the idea is to get $K = 2M + 1$ Fourier coefficients from $x'(t)$ in order to reduce the sampling frequency at each channel to $\frac{1}{T_S}$. Therefore, the signal at the input of the MCMW scheme is considered to be $x'(t)$ and the values of the spectrum $R'(\nu)$ are known at the receiver. The modifications in the MCMW method aim to obtain K Fourier coefficients $c_k^{(x')} = \frac{1}{T_S} \int_0^{T_S} x'(t) e^{-j2\pi \frac{k}{T_S} t} dt$ uniformly spaced by $1/T = N_f/T_S$. The scheme is slightly modified than the original one illustrated in Figure 1.

The binary sequences $p_i(t)$ are periodized with a period T_S with exactly P pulses during each period. At each channel i , the sequence is obtained by a circular shifting of $(i-1)T_S/P$ of $p_1(t)$. Therefore, we get in the frequency domain:

$$P_i(\nu) = \sum_{k \in \mathbb{Z}} c_k^{(p_i)} \delta(\nu - \frac{k}{T_S}) \quad (18)$$

The low-pass filter of impulse response $g(t)$ allows to keep only K Fourier coefficients of $p_i(t)$ spaced by N_f/T_S . The

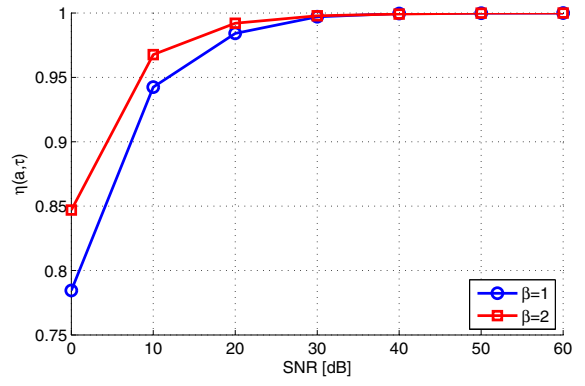


Fig. 3. Performance of MUSIC algorithm for channel estimation in terms of correlation between the real and estimated composite channel impulse responses as a function of the SNR and the number of measurements K .

transfer function $G(\nu)$ of the proposed filter is then expressed as :

$$G(\nu) = \begin{cases} \neq 0 & \text{si } \nu \in \{-\frac{N_f M}{T_S}, \dots, \frac{N_f M}{T_S}\} \\ = 0 & \text{si } \nu = \frac{N_f k}{T_S}, |k| > M \\ \text{arbitrary} & \text{otherwise} \end{cases} \quad (19)$$

The modulating waveforms are the result of the binary sequences filtered by $g(t)$:

$$s_i(t) = \sum_{k=-M}^M s_{N_f k i} e^{j2\pi \frac{N_f k}{T_S} t} \quad (20)$$

with $s_{N_f k i} = c_{N_f k}^{(p_i)} G(\frac{N_f k}{T_S})$.

Consequently, the resulting Fourier coefficients combination on each channel becomes:

$$z_i = \sum_{k=-M}^M s_{N_f k i}^* c_{N_f k}^{(x')} \quad (21)$$

Consider a typical value of $T = 40$ ns for UWB channel estimation. When taking $N_f = 10$ for example, the sampling frequency at each channel becomes equal to $1/T_S = 2.5$ MHz. Therefore, affordable ADCs with high resolutions can be used.

IV. NUMERICAL RESULTS

A. UWB channel estimation

We study the system performance in the presence of additive white gaussian noise for the UWB channel estimation with the superresolution MUSIC algorithm. The transmitted pulse $r(t)$ of duration $T_r = 1$ ns, is the first derivative of the Gaussian pulse. The UWB channel $h(t)$ is a realization of the channel model CM1 IEEE.802.15.3a. It is truncated to $L = 15$ paths and its resolution is equal to the pulse duration (1 ns). We limit our study to the case of ideal filters and perfect synchronization. To evaluate the performance of the proposed schemes, we consider 10000 realizations of the channel and we calculate the correlation coefficient η between the real

and estimated discrete composite channel impulse response $g(t) = h(t) * r(t)$ given by:

$$\eta = \frac{\langle g, \hat{g} \rangle}{\|g\| \cdot \|\hat{g}\|} \quad (22)$$

Figure 3 depicts the correlation coefficient as a function of the signal to noise ratio (SNR) for different number of measurements $K = 2\beta L + 1$ which correspond to 31 and 61 measurements for $\beta = 1$ and 2, respectively. These K measurements are obtained with the proposed methods presented in Section III-B and Section III-C for $N_f = 3$. For the method in Section III-B, we obtain $P_r = 11$ and 21 channels for $\beta = 1$ and 2, respectively instead of $P = 31$ and 61 for the original MCMW method. For the method in Section III-C, this corresponds to a reduced frequency by a factor $N_f = 3$ with respect to the original MCMW method at each channel.

For $\beta = 1$, the correlation coefficient is equal to 0.78 for SNR= 0 dB and $\cong 1$ for SNRs greater than 30 dB and it is the same for both methods.

B. UWB detection

We study now the performance of a Rake receiver in terms of bit error rate (BER) as a function of the SNR, when estimating the channel at the finite rate of innovation ($\beta = 1$). For each symbol, 3 Binary Phase Shift Keying (BPSK) modulated pulses are transmitted. The channels considered are CM1 IEEE 802.15.3a realizations truncated to 30 paths. Their estimations are realized with the algorithm MUSIC for SNRs equal to 20 and 10 dB. We consider $S = 8$ strongest paths in order to observe the performance of a selective Rake (S-Rake) for both configurations. The performance is compared with a perfect channel estimation scenario for the same S-Rake and an A-Rake (All Rake).

Figure 4 shows the results. As it is expected, S-Rake performance with a channel estimation at 20 dB is not far from that of S-Rake with perfect channel estimation. However, performance degrades when estimating the channel at 10 dB. Furthermore, it is interesting to see that the performance of S-Rake with 8 channels, is close to that of A-Rake with 30 channels. This is due to the fact that most of the channel energy is contained in the strongest paths.

V. CONCLUSION

In this paper, we have introduced a new FRI based sampling scheme for UWB channel estimation. The proposed approach allows reducing its complexity in terms of number of processing channels, and thus results in realistic implementation constraints. Moreover, our approach leads to low sampling rates on each processing channel, so that lower cost ADCs may be used for a given resolution. We have shown that even for low SNRs, good performance may be obtained for UWB channel estimation using superresolution algorithms, such as MUSIC. The same performance level is also highlighted in terms of BER, in the framework of coherent Rake reception based on UWB channel estimation result.

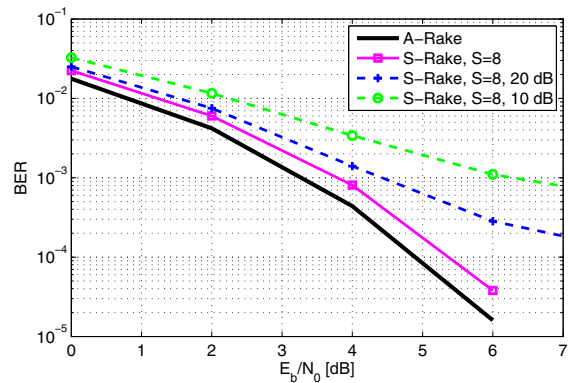


Fig. 4. Performance in terms of BER as a function of the SNR, with a channel estimation at 20 and 10 dB with MUSIC.

REFERENCES

- [1] L. Yang and G. Giannakis, "Ultra-wideband communications: an idea whose time has come," *Signal Processing Magazine, IEEE*, vol. 21, no. 6, pp. 26–54, 2004.
- [2] I. Oppermann, M. Hamalainen, and J. Iinatti, *UWB Theory and Applications*. Finland: John Wiley & Sons, 2004.
- [3] M. Di Benedetto, T. Kaiser, A. Molisch, I. Oppermann, C. Politano, and D. Porcino, *UWB Communication Systems: A Comprehensive Overview*. Finland: EURASIP Book Series on Signal Processing and Communications, Hindawi Publishing Corporation, 2006.
- [4] M. Vetterli, P. Marziliano, and T. Blu, "Sampling signals with finite rate of innovation," *Signal Processing, IEEE Transactions on*, vol. 50, no. 6, pp. 1417 – 1428, 2002.
- [5] I. Maravic and M. Vetterli, "Sampling and reconstruction of signals with finite rate of innovation in the presence of noise," *Signal Processing, IEEE Transactions on*, vol. 53, no. 8, pp. 2788 – 2805, 2005.
- [6] K. Gedalyahu, R. Tur, and Y. C. Eldar, "Multichannel sampling of pulse streams at the rate of innovation," *Signal Processing, IEEE Transactions on*, vol. 59, no. 4, pp. 1491–1504, 2011.
- [7] R. Tur, Y. Eldar, and Z. Friedman, "Innovation rate sampling of pulse streams with application to ultrasound imaging," *Signal Processing, IEEE Transactions on*, vol. 59, no. 4, pp. 1827–1842, 2011.
- [8] A. Molisch, J. Foerster, and M. Pendergrass, "Channel models for ultrawideband personal area networks," *Wireless Communications, IEEE*, vol. 10, no. 6, pp. 14–21, 2003.