Robust Resource Allocation for Full-Duplex Cognitive Radio Systems

(Invited Paper)

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Abstract—In this paper, we investigate resource allocation algorithm design for full-duplex (FD) cognitive radio systems. The secondary network employs a FD base station for serving multiple half-duplex downlink and uplink users simultaneously. We study the resource allocation design for minimizing the maximum interference leakage to primary users while providing quality of service for secondary users. The imperfectness of the channel state information of the primary users is taken into account for robust resource allocation algorithm design. The algorithm design is formulated as a non-convex optimization problem and solved optimally by applying semidefinite programming (SDP) relaxation. Simulation results not only show the significant reduction in interference leakage compared to baseline schemes, but also confirm the robustness of the proposed algorithm.

I. INTRODUCTION

Bandwidth has become a scarce resource in wireless systems due to the tremendous demand for ubiquitous and high data rate communication. Recently, cognitive radio (CR) has emerged as a promising paradigm to improve spectrum efficiency. In particular, CR technology allows a secondary network to share the spectrum of a primary network without severely degrading the quality of service (QoS) of the primary network. The authors of [1] proposed an optimal beamforming and power control algorithm to guarantee communication security in multiuser CR networks. In [2], distributed beamforming and rate allocation for multiple secondary users were considered for maximization of the minimum data rate achieved by secondary users. However, the spectral resource is still underutilized in [1],[2]. Specifically, since the secondary network operates in the traditional half-duplex (HD) mode, orthogonal radio resources are used for uplink (UL) and downlink (DL) transmission which limits the spectral efficiency.

Full-duplex (FD) wireless communication has recently attracted significant research interest due to its potential to double the spectral efficiency by performing simultaneous DL and UL transmission using the same frequency [3]-[6]. Therefore, it is expected that the spectral efficiency of existing wireless communication systems can be further improved by employing an FD base station (BS) in CR networks. However, the simultaneous UL and DL transmission may lead to excessive interference leakage to the primary network and degrade the quality of communication. Therefore, different resource allocation designs for FD-CR networks were proposed to overcome this challenge. For example, the authors of [7] studied the rate region of a secondary single-antenna user served by a secondary FD BS while guaranteeing the primary user’s QoS. In [8], a suboptimal resource allocation algorithm was proposed for the maximization of the sum throughput of secondary

FD users. However, [7], [8] assumed that the channel state information (CSI) of the link between the secondary network and the primary network is perfectly known at the secondary FD BS which is a highly idealistic assumption. In fact, the perfect CSI of the primary users may not be available at the secondary FD BS since they do not directly interact with the secondary network. Besides, the objective of the resource allocation algorithms in [7], [8] was to improve the performance of the secondary network from the secondary network’s point of view. However, in FD-CR systems, interference leakage is more serious than in traditional HD-CR systems due to the simultaneous secondary DL and UL transmission. Therefore, in FD-CR systems, a careful design of the resource allocation is necessary.

Motivated by the aforementioned observations, we formulate an optimization problem to minimize the maximum interference leakage caused by the secondary FD network to the primary network while guaranteeing the QoS of all secondary users. The imperfectness of the CSI of the interference leakage channels is taken into account in the proposed problem formulation to facilitate a robust resource allocation.

II. SYSTEM MODEL

In this section, we present the considered FD-CR wireless communication system model.

A. Notation

We use boldface capital and lower case letters to denote matrices and vectors, respectively. $A^H$, Tr$(A)$, and Rank$(A)$ denote the Hermitian transpose, trace, and rank of matrix $A$, respectively; $A \succeq 0$ and $A > 0$ indicate that $A$ is a positive semidefinite and a positive definite matrix, respectively; $I_N$ is the $N \times N$ identity matrix; $C^{N \times M}$ denotes the set of all $N \times M$ matrices with complex entries; $\mathbb{H}^N$ denotes the set of all $N \times N$ Hermitian matrices; $|\cdot|$ and $\|\cdot\|$ denote the absolute value of a complex scalar and the Euclidean vector norm, respectively; $\mathcal{E}(\cdot)$ denotes statistical expectation; diag$(x_1,\cdots, x_K)$ denotes a diagonal matrix with diagonal elements $\{x_1,\cdots, x_K\}$ and diag$(X)$ returns a diagonal matrix having the main diagonal elements of $X$ on its main diagonal. $\Re(\cdot)$ extracts the real part of a complex-valued input; the circularly symmetric complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$ is denoted by $CN(\mu, \sigma^2)$; and $\sim$ stands for “distributed as”.

B. Cognitive Radio System Model

The considered CR system comprises one secondary FD BS, $K$ secondary DL users, $J$ secondary UL users, one primary transmitter, and $R$ primary receivers. The secondary FD BS is equipped with $N_T > 1$ antennas for facilitating simultaneous DL transmission and UL reception in the secondary network in the same frequency band. The $K + J$ secondary users, the

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primary transmitter, and the secondary receivers are single-antenna HD devices that share the same spectrum, cf. Figure 1. The number of antennas at the secondary FD BS is assumed to be larger than the number of secondary UL users to facilitate reception of UL signals. Therefore, the received signal at secondary DL user \( r \) at the secondary FD BS is given by

\[
y_{DL}^r = h_{DL}^r x_k + \sum_{m \neq k} h_{DL}^r w_m + \sum_{j=1}^J \sqrt{P_j} f_{j,k} d_{j,DL} + n_{DL},
\]

(1)

\[
y_{UL} = \sum_{j=1}^J \sqrt{P_j} g_{j} d_{j,UL} + H_{SI} \sum_{k=1}^K x_k + n_{UL},
\]

(2)

\[
y_{PU}^r = \sum_{k=1}^K \sqrt{P_k} x_k + \sum_{j=1}^J \sqrt{P_j} e_{j,r} d_{j,UL} + n_{PU},
\]

(3)

respectively. The DL channel between the secondary FD BS and secondary DL user \( k \) is denoted by \( h_{DL}^k \in \mathbb{C}^{N_r \times 1} \) and \( f_{j,k} \in \mathbb{C} \) represents the channel between secondary UL user \( j \) and secondary DL user \( k \). Variables \( d_{j,UL} \), \( E \{ d_{j,UL}^2 \} = 1 \), and \( P_j \) are the data power and transmit power sent from secondary UL user \( j \) to the secondary FD BS, respectively. Vector \( g_{j} \in \mathbb{C}^{N_r \times 1} \) denotes the channel between secondary UL user \( j \) and the secondary FD BS. Matrix \( H_{SI} \in \mathbb{C}^{N_r \times N_r} \) denotes the self-interference (SI) channel of the secondary FD BS. The SI is caused by the signal leakage from DL transmission to UL reception in the secondary network. Vector \( l_r \in \mathbb{C}^{N_r \times 1} \) denotes the channel between the secondary FD BS and primary receiver \( r \). Scalar \( e_{j,r} \in \mathbb{C} \) denotes the channel between secondary UL user \( j \) and primary receiver \( r \). Variables \( h_{DL}^j, f_{j,k}, g_{j}, H_{SI}, l_r \), and \( e_{j,r} \) capture the joint effect of path loss and small scale fading. \( n_{UL} \sim \mathcal{CN}(0, \sigma^2_{UL}) \) and \( n_{DL} \sim \mathcal{CN}(0, \sigma^2_{DL}) \) represent the equivalent noise at the secondary FD BS and secondary DL user \( k \), which capture the joint effect of the received interference from the primary transmitter and thermal noise.

### III. Resource Allocation Problem Formulation

In this section, we formulate the resource allocation design as a non-convex optimization problem, after introducing the adopted performance metrics and the CSI assumed for resource allocation. For the sake of notational simplicity, we define the following variables: \( H_{x} = h_{DL} p_{DL}^r \), \( k \in \{1, \ldots, K\} \), \( G_{j} = g_{j} g_{j}^H \), \( j \in \{1, \ldots, J\} \), and \( V_{j} = v_{j} v_{j}^H \), \( j \in \{1, \ldots, J\} \).

#### A. Performance Metrics

The receive signal-to-interference-plus-noise ratio (SINR) at secondary DL user \( k \) is given by

\[
\Gamma_{DL}^k = \frac{|h_{DL}^k w_k|^2}{\sum_{m \neq k} |h_{DL}^m w_m|^2 + \sum_{j=1}^J |f_{j,k} d_{j,DL}|^2 + \sigma^2_{n_k}}. \tag{4}
\]

On the other hand, the receive SINR of secondary UL user \( j \) at the secondary FD BS is given by

\[
\Gamma_{UL}^j = \frac{P_j |g_{j} v_j|^2}{\sum_{n \neq j} |P_n g_{n} v_n|^2 + |\sum_{k=1}^K H_{SI} x_k + n_{UL}|^2 + \sigma^2_{UL} |v_{j}|^2}, \tag{5}
\]

where \( v_{j} \in \mathbb{C}^{N_r \times 1} \) is the receive beamforming vector for decoding the information received from secondary UL user \( j \). Besides, we define \( I_{DL}^k = \text{Tr}(P_{DL}^r \text{diag}(\sum_{k=1}^K H_{SI} w_{k} w_{k}^H H_{SI}^H)) \), where \( 0 < \rho \ll 1 \) is a constant modelling the noise level of the SI cancellation at the secondary FD BS [9, Eq. (4)]. In this paper, we adopt zero-forcing receive beamforming (ZF-BF) [10] as it approaches the performance of optimal minimum mean square error beamforming (MMSE-BF) when the noise term is not dominating [10] or the number of antennas is sufficiently large [11]. Besides, ZF-BF facilitates the design of a computationally efficient resource allocation algorithm.

#### B. Channel State Information

In this paper, we assume that the CSI of all secondary users is perfectly known at the secondary BS because of frequent channel estimation. However, for the secondary network-to-primary network channels, the perfect CSI assumption may not hold since the primary receivers do not interact directly with the secondary network. Hence, the CSI of the link between the secondary BD BS and primary receiver \( r \in \{1, \ldots, R\} \), i.e., \( I_r \), and the link between the secondary UL user \( j \in \{1, \ldots, J\} \) and primary receiver \( r \), i.e., \( e_{j,r} \), are modeled as

\[
I_r = I_r + \Delta I_r, \quad \Omega_{DL}^r \Delta I_r \triangleq \{ l_r \in \mathbb{C}^{N_r \times 1} : |\Delta l_r| \leq \epsilon_{DLL} \}, \tag{6}
\]

\[
e_{j,r} = e_{j,r} + \Delta e_{j,r}, \quad \Omega_{UL}^j \Delta e_{j,r} \triangleq \{ e_{j,r} \in \mathbb{C} : |\Delta e_{j,r}| \leq \epsilon_{ULJ} \}, \tag{7}
\]

respectively, where \( \epsilon_{j,r} \) and \( \hat{l}_r \) are the CSI estimates, and \( \epsilon_{DLJ} \) is the unknown CSI estimation error. The continuous sets \( \Omega_{UL}^j \) and \( \Omega_{DL}^r \) contain all possible channel uncertainties, and \( \epsilon_{ULJ} \) and \( \epsilon_{DLJ} \) denote the bounded magnitude of \( \Omega_{UL}^j \) and \( \Omega_{DL}^r \), respectively.
C. Optimization Problem Formulation

The system objective is to minimize the maximum interference leakage from the secondary network to the primary receivers. The optimal power allocation and beamformer design are obtained by solving the following optimization problem:

\[
\begin{align*}
\text{minimize} & \quad \mathbf{w}_{k,j} \mathbf{w}_{k,j}^H \\
\text{subject to} & \quad \mathbf{w}_{k,j} \in \Omega_{DL_j}, \forall k, \forall j, \quad \sum_{k=1}^{K} ||\mathbf{w}_{k,j}||^2 \leq 2P_j |e_{j,r}|^2, \\
& \quad \Gamma_{\text{req}, j} \geq \Gamma_{\text{UL}, j}, \forall j, \forall v_k, \forall r, \\
& \quad \Delta_{\text{L}} \geq \Delta_{\text{UL}}, \forall k, \forall j, \forall r, \\
& \quad \mathbf{w}_{k,j} \succeq 0, \forall k, \forall j.
\end{align*}
\]

Now, we introduce a lemma which allows us to transform constraint C5a into an LMI.

**Lemma 1 (S-Procedure [13]):** Let a function \( f_m(x), m \in \{1,2\}, x \in \mathbb{C}^{N \times 1} \), be defined as

\[
\begin{align*}
f_m(x) = x^H A_m x + 2 \Re \{ b_m^H x \} + c_m,
\end{align*}
\]

where \( A_m \in \mathbb{H}^N, b_m \in \mathbb{C}^{N \times 1}, \) and \( c_m \in \mathbb{R}^{1 \times 1} \). Then, the implication \( f_m(x) \leq 0 \iff f_m(x) \leq 0 \) holds if and only if there exists a variable \( \delta \geq 0 \) such that

\[
\begin{align*}
\begin{bmatrix} A_1 & b_1 \\ b_1^T & c_1 \end{bmatrix} - \begin{bmatrix} A_2 & b_2 \\ b_2^T & c_2 \end{bmatrix} \succeq 0,
\end{align*}
\]

provided that there exists a point \( x \) such that \( f_m(x) < 0 \).

By applying (6), constraint C5a can be equivalently expressed as

\[
\begin{align*}
\text{C5a:} \quad 0 \geq \Delta_{\text{L}}^H \sum_{k=1}^{K} \mathbf{w}_{k} \Delta_{\text{L}} + 2 \Re \{ \sum_{k=1}^{K} \mathbf{w}_{k} \Delta_{\text{L}} \} \succeq \mathbf{0}, \forall \theta, \forall \tau.
\end{align*}
\]

IV. SOLUTION OF THE OPTIMIZATION PROBLEM

To solve the non-convex problem in (8), efficiently, we follow a similar approach as in [12]. First, we reformulate the problem in an equivalent form and then transform the non-convex constraints into equivalent linear matrix inequality (LMI) constraints. Finally, the problem is solved by semidefinite programming (SDP) relaxation.

To facilitate the SDP relaxation, we define \( \mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H \) and rewrite the problem in the equivalent form:

\[
\begin{align*}
\text{minimize} & \quad \tau \\
\text{subject to} & \quad \Gamma_{\text{req}, j} \geq \Gamma_{\text{UL}, j}, \forall j, \\
& \quad \mathbf{W}_k \succeq 0, \forall k, \forall j, \\
& \quad \mathbf{W}_k \in \mathbb{H}^{N \times R}, \forall \theta, \forall \tau.
\end{align*}
\]

where \( \mathbf{W}_k \succeq 0, \mathbf{W}_k \in \mathbb{H}^{N \times R} \), and Rank(\( \mathbf{W}_k \)) \leq 1 in (9) is imposed to guarantee that \( \mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H \) holds after optimization. Furthermore, we use \( \Gamma_{\text{DL}} = \sum_{n=1}^{N} \Gamma_{\text{DL}, n} \) and \( \Gamma_{\text{UL}} = \sum_{n=1}^{N} \Gamma_{\text{UL}, n} \). The next theorem is a relaxation of (8). Constraint C5 involoves an infinite number of inequality constraints, as the estimation error variables \( \Delta_{\text{DL}} \) and \( \Delta_{\text{UL}} \) are involved. Here, we introduce a scalar slack variable \( \delta \) to handle the coupled estimation error variables in constraint C5. In particular, constraint C5 can be equivalently represented by

\[
\begin{align*}
\text{C5a:} \quad \sum_{k=1}^{K} ||\mathbf{w}_{k,j}||^2 \leq \delta, \forall \mathbf{V}_j \in \Omega_{DL_j}, \forall r, \\
\text{C5b:} \quad \delta \leq \tau - \sum_{j=1}^{J} P_j |e_{j,r}|^2, \forall e_{j,r} \in \Omega_{UL_j}, \forall j, r, \forall v_k, \forall r.
\end{align*}
\]

Next, we relax the non-convex constraint C7. Rank(\( \mathbf{W}_k \)) \leq 1 by removing it from the problem formulation such that the considered problem becomes a convex SDP:

\[
\begin{align*}
\text{minimize} & \quad \tau \\
\text{subject to} & \quad \Gamma_{\text{req}, j} \geq \Gamma_{\text{UL}, j}, \forall j, \\
& \quad \mathbf{W}_k \succeq 0, \forall k, \forall j, \\
& \quad \mathbf{W}_k \in \mathbb{H}^{N \times R}, \forall \theta, \forall \tau.
\end{align*}
\]

(16) The relaxed convex problem in (16) can be solved efficiently by standard convex program solvers such as CVX [14]. Besides, if the solution obtained for a relaxed SDP problem is a rank-one matrix, i.e., Rank(\( \mathbf{W}_k \)) = 1 for \( \mathbf{W}_k \neq \mathbf{0} \), \forall k, then it is also the optimal solution of the original problem. Next, we reveal the tightness of the SDP relaxation in the following theorem.

**Theorem 1:** Assume the considered problem is feasible, for \( \Gamma_{\text{req}, j} > 0 \), we can always obtain or construct an optimal rank-one matrix \( \mathbf{W}_k \).

**Proof:** Please refer to the Appendix.

V. RESULTS

In this section, we investigate the performance of the proposed resource allocation scheme through simulations. The most important simulation parameters are specified in Table I. There are \( K = 3 \) secondary DL users, \( J = 5 \) secondary UL users, and \( R = 2 \) primary receivers in the system. We assume that the primary transmitter is 100 meters away from the secondary FD BS. The secondary users and primary receivers are randomly and uniformly distributed between the reference distance of 5 meters and the maximum service distance of 50
TABLE I
SYSTEM PARAMETERS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier center frequency</td>
<td>1.9 GHz</td>
</tr>
<tr>
<td>System bandwidth</td>
<td>200 kHz</td>
</tr>
<tr>
<td>Path loss exponent</td>
<td>3.6 dB</td>
</tr>
<tr>
<td>SI cancellation</td>
<td>-90 dB</td>
</tr>
<tr>
<td>Secondary DL user equivalent noise power, $\sigma^2_{\text{UL,DL}}$</td>
<td>-90 dBm</td>
</tr>
<tr>
<td>Secondary FD BS equivalent noise power, $\sigma^2_{\text{SI,DL}}$</td>
<td>-90 dBm</td>
</tr>
<tr>
<td>Secondary FD BS antenna gain</td>
<td>Max. transmit power at the secondary FD BS, $P^\text{DMAX}_{\text{DL}}$</td>
</tr>
<tr>
<td>Max. transmit power at the secondary FD BS, $P^\text{DL}$</td>
<td>10 dB</td>
</tr>
<tr>
<td>Max. transmit power at the primary transmitter</td>
<td>-30 dBm</td>
</tr>
</tbody>
</table>

Fig. 2. Average maximum interference leakage (dBm) versus the minimum required DL SINR (dB), $\Gamma^\text{DL}_{\text{req}}$, for different resource allocation schemes.

Fig. 3. Average maximum interference leakage (dBm) versus the maximum normalized channel estimation error, $\kappa^2_{\text{est}}$, for $N_T = 9$.

The optimal MMSE receiver at the secondary HD BS [10]. The required SINRs for the secondary DL and UL users served by the secondary HD BS are $\Gamma^\text{DL}_{\text{req}} = (1 + \Gamma^\text{UL}_{\text{req}}) \frac{1}{2}$ and $\Gamma^\text{UL}_{\text{req}} = (1 + \Gamma^\text{UL}_{\text{req}}) \frac{1}{2}$, respectively. Besides, the power consumption of DL and UL transmission for the secondary HD network is divided by two as DL and UL transmission do not overlap. Then, we optimize $w_k$ and $P_j$ to minimize the maximum interference leakage to the primary users for the optimal MMSE receiver at the secondary HD BS [10]. It can be observed from Figure 2 that the average maximum interference leakage of the baseline schemes is higher than that of the proposed FD-CR system. In particular, the average maximum interference leakage increases with $\Gamma^\text{UL}_{\text{req}}$ for baseline scheme 1 due to the fixed beamforming design. Besides, the average maximum interference leakage of baseline scheme 2 is insensitive to $\Gamma^\text{UL}_{\text{req}}$ since the $w_k$ and $P_j$ are optimized for the considered system setting.

In Figure 3, we study the average maximum interference leakage versus the maximum normalized channel estimation error, $\kappa^2_{\text{est}}$, for a minimum required secondary DL SINR of $\Gamma^\text{DL}_{\text{req}} = 10$ dB and a minimum required secondary UL SINR of $\Gamma^\text{UL}_{\text{req}} = 5$ dB. As can be observed, the average maximum interference leakage increases with increasing $\kappa^2_{\text{est}}$. In fact, with increasing imperfection of the CSI, it is more difficult for the secondary FD BS to perform accurate DL beam-steering. In particular, more DoF are utilized to reduce interference leakage as the channel uncertainty increases which leads to a higher maximum interference leakage. Besides, as more DoF are consumed for interference leakage reduction, there are fewer DoF available to suppress the SI which degrades the UL reception in the secondary network. Thus, the secondary UL users are forced to transmit with a higher power to satisfy the UL QoS requirements which in turn results in a larger interference leakage to the primary network. Furthermore, we note that the baseline schemes cause significantly higher interference leakages compared to the proposed scheme due to their inefficient resource allocation.

VI. CONCLUSIONS
In this paper, we studied the robust resource allocation design for CR secondary networks employing an FD BS for serving
multiple secondary HD DL and UL users simultaneously. The algorithm design was formulated as a non-convex optimization problem with the objective to minimize the maximum interference leakage to the primary network while taking into account the QoS requirements of all secondary users. The imperfection of the CSI of the secondary-network-to-primary network channels was taken into account for robust resource allocation algorithm design. The proposed non-convex problem was solved optimally by SDP relaxation. Simulation results unveiled a significant reduction in interference leakage with the QoS requirements of all secondary users which are collected in $C^a$, $C^b$, $C^5$, and $C^8$. 

Since the problem in (17) has the same feasible set as problem (16), problem (17) is also feasible. Now, we claim that for a given $P^*$ and $\Xi^*$ in (17), the solution $W^*_k$ of (17) is a rank-one matrix. First, the problem in (17) is jointly convex with respect to the optimization variables and satisfies the Slater’s constraint qualification. Therefore, strong duality holds and solving the dual problem is equivalent to solving the primal problem [13]. For obtaining the dual problem, we first need the Lagrangian function of the primal problem in (16) which is given by

$$L = -\sum_{k=1}^{K} \text{Tr}(W_k H_k) + \sum_{j=1}^{J} \sum_{k=1}^{K} \text{Tr}(\rho V_j \text{diag}(W_k H_k^H H_{SI})) + (1+\mu) \sum_{k=1}^{K} \text{Tr}(W_k Y_k) - \sum_{r=1}^{R} \text{Tr}(R_{\text{SI}_r} W_k H_k) + \Delta,$$

where $Y_k^* D_{\text{SI}_r}^* \lambda_k^* \theta_j^* \mu^* \geq 0$, and $\mu^*$ are the optimal Lagrange multipliers for dual problem (19). $\nabla W_k L$ denotes the gradient of Lagrangian function $L$ with respect to matrix $W_k$. The KKT condition in (22) can be expressed as

$$(1+\mu)^*\mathbb{J}_{N} + \sum_{j=1}^{J} \theta_j^* \rho V_j \text{diag}(H_{SI}^H H_{SI}) + \sum_{r=1}^{R} B_r D_{\text{SI}_r}^* B_r^H,$$

where $\Pi^*_k = (1+\mu)\mathbb{J}_{N} + \sum_{j=1}^{J} \theta_j^* \rho V_j \text{diag}(H_{SI}^H H_{SI}) + \sum_{r=1}^{R} B_r D_{\text{SI}_r}^* B_r^H$. Premultiplying both sides of (24) by $W_k^*$, and utilizing (21), we have $W_k^* \Pi_k^* = \lambda_k^* W_k^* H_k$. By applying basic inequalities for the rank of matrices, the following relation holds:

$$\text{Rank}(W_k^*) \leq \text{Rank}(\Pi_k^*) \leq \text{Rank}(\lambda_k^* W_k^* H_k) \leq \text{Rank}(H_k^*),$$

where $(a)$ is due to $\Pi_k^* \succeq 0$, $(b)$ is due to the basic result $\text{Rank}(AB) \leq \text{min} \{ \text{Rank}(A), \text{Rank}(B) \}$, and $(c)$ is due to the fact that $\text{min}(a,b) \leq a$. Since $\text{Rank}(H_k^*) \leq 1$, by utilizing (25), the rank of $W_k^*$ is given by

$$\text{Rank}(W_k^*) \leq \text{Rank}(H_k^*) \leq 1.$$

We note that $W_k^* \neq 0$ for $\Gamma_{\text{req},k} > 0$. Thus, $\text{Rank}(W_k^*) = 1$. Therefore, an optimal rank-one matrix $W_k^*$ for (16) is constructed.

References


