Optimal Quantization of TV White Space Regions for a Broadcast Based Geolocation Database

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***Abstract*—Currently, TV white space databases communicate the available channels over a reliable Internet connection to the secondary devices. For places where an Internet connection is not available, such as in developing countries, *we propose a broadcast based geolocation database*. This proposed geolocation database will broadcast the TV white space (or the primary services protection regions) on a rate-constrained channel.**

In this work, the feasibility of a broadcast based geolocation database transmission will be examined over rate constrained satellite channels. To address this problem, the quantization or digital representation of primary services protection regions is considered. Due to the quantization process, any point in the protection region must not be declared as white space region. Thus, this quantization problem is different than traditional quantizers which minimize the mean-squared error. A quantizer design algorithm is the main result of this work, which minimizes the TV white space area declared as protected region due to quantization. Performance results of our quantization algorithm on US and India UHF-band protection regions will be shown. The update-rate versus bandwidth tradeoff, while using satellite TV channels, for the proposed broadcast based geolocation database will also be explored in this work.

***Index Terms*—quantization (signal), cognitive radio, approximation error**

I. Introduction

The wireless spectrum is a limited and valuable resource. The demand for spectrum is increasing due to the increase in the number of wireless devices and this demand has led to research for efficient utilization techniques of the spectrum. The usage of TV white space by unlicensed secondary users is an example of efficient utilization of spectrum. The spectrum licensing agencies, Federal Communications Commission (FCC) in the United States and Office of Communication (Ofcom) in the United Kingdom, have permitted access of TV white space by an unlicensed secondary device [1], [2].

According to the existing regulations of FCC and Ofcom, TV white space can be accessed by a secondary or white space device (unlicensed user) via TV white space (geolocation) database access. A certified TV white space database service is queried before operation by the secondary device. This query includes the location of secondary device, and database is updated if a secondary is allocated a 'white' TV channel. The TV transmitter protection regions are calculated by the TV white space database service providers to avoid harmful interference to the primary devices of the licensed broadcasting services. The available TV white space changes with time and space, and it is mandatory for the secondary device to know the availability at the location and time of current operation. The access of TV white space database takes place over the Internet [1]. By design, TV white space database access is inaccessible for secondary devices in areas where there is unreliable or no Internet connection. The lack of internet connection is especially prevalent in many developing or under-developed countries where internet services are limited. In such areas, a *different scheme* for communication of the protection regions of TV transmitters are needed.

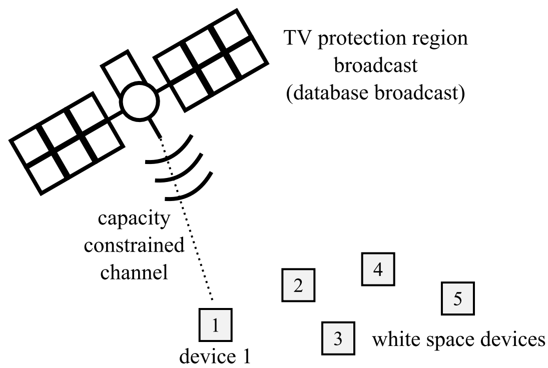


Fig. 1. A broadcast based geolocation database is illustrated. It is assumed that white space devices can receive the broadcast over a rate constrained channel. Thus, the database should quantize the protection regions, while ensuring the protection of primary from quantization error.

In this work, a *broadcast based* service for TV white space availability is proposed. A broadcast service, such as a satellite, can transmit the TV transmitters' protection regions, or simply *the protection regions*. This broadcast based approach is assumed to make use of a wireless or digital channel, and the transfer of information to a secondary device will be rate constrained (see Fig. 1). Accordingly, quantization of protection regions is of interest for a broadcast-style TV white space database. When the protection regions are quantized, some TV white space area will be 'lost' due to quantization error. This quantizer has to be designed to *minimize* the TV white space area lost due to quantization. In summary, protection region quantizer's *design and performance* are the key problems addressed in the current work. *To the best of our knowledge, this is the first work of its kind.*

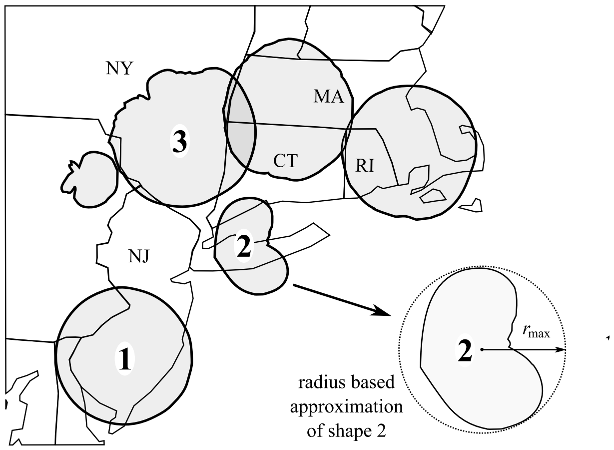


Fig. 2. This graph is obtained from the website of a certified TV white space service provider iconectiv in the United States [3]. The contours are protection regions for Channel 22 near New York.

Examples of the protection regions are illustrated in Fig. 2, which are obtained from the iconectiv website [3] for Channel 22 in the New York region. A circular approximation to the protection region number 2 is illustrated. The circular protection region model "marks" some portion of spatial TV white space as protected region, but it is the first approximation that can be used for quantizing protection regions. These circular protection regions have to be quantized or represented by a larger circle. This is to ensure that protection region is not labeled as unprotected (due to quantization). For this important regulatory reason, a traditionally well-studied mean-squared error optimal quantization method *cannot be employed* [4].

Key contributions: Optimal quantizer design algorithm will be discussed in this work to ensure primary's protection even after quantization! By optimal, it is meant that for a given quantizer precision, the TV white space area mislabeled (lost) as protection region is minimized. There is a fundamental limit of how much TV white space area is present. With larger transmission rate, the database can convey with increasingly accurate recovery of the TV white space region.

Related work: Geolocation database are well known in the literature [1], [2], [5]. Circular protection regions for TV white space regions are well known (see [6]). Primary service contours are available as databases for countries such as US and UK [1], [2]. To the best of our knowledge, a broadcast style geolocation database has not been studied in the literature. Quantization of real values where signal is always overestimated as well as envelope style approximations are also not known in the literature to the best of our knowledge.

Organization: Section II presents the optimal quantization of circular protection regions, while ensuring protection for the primary receivers. Section IV presents the bandwidth requirements needed to send the quantized protection regions using satellite TV standards. Finally, conclusions are presented in Section V.

Assumptions: The quantization of transmitters' center is not considered. Since TV transmitters are at fixed locations, this data can be exchanged first before repeated broadcast of TV white space takes place. It is assumed that the secondary devices will coexist by some media access control (MAC) mechanism, since the broadcast database facilitates only 'one-way' communication. The broadcast database will not register the secondary devices. Distributed coexistence techniques such as collision sense multiple access (CSMA) can be used by the secondary devices. It is assumed that the satellite used for broadcasting is geostationary and serves the entire country.

II. Optimal quantization of protection regions

This section deals with the quantization of (circular) protection region radius. The quantization has to be designed to minimize the white space area lost due to quantization across all transmitters. It is assumed that the centers of these protection regions are already available at the receiver, so that only radius of protection regions have to be quantized and communicated. In case if the protection region is not circular, the radius based approximation scheme depicted in Fig. 2 can be used to obtain a radius.

Let $\mathcal{R} := \{r_1, r_2, \ldots, r_n\}$ be the radius of protection regions. For simple exposition in this paper, it is assumed that this set $\mathcal{R}$ is fixed. These radius have to be quantized in the set $\mathcal{Q} = \{q_1, q_2, \ldots, q_m\}$, where $\log_2 m$ will be the number of bits being spent to communicate each circular region. These $\log_2 m$ bits will index various quantization levels in the set $\mathcal{Q}$. Without loss of generality, it is assumed that $q_1, q_2, \ldots, q_m$ and $r_1, r_2, \ldots, r_n$ are both in an increasing order. The quantized radius set $\mathcal{Q}$ is known to the broadcast-database transmitter as well as all the secondary white space devices. The protection region radius set $\mathcal{R}$, on the other hand, is only known to the transmitter.

To *ensure protection* for the primary, $Q(r_i) \geq r_i$ for all protection radius $r_i \in \mathcal{R}$. Unlike in traditional mean-squared or minimax optimal quantization, where $Q(r)$ is mapped to the closest quantization level [4], quantization level for protection region's radius is always the *nearest larger level*. Thus, the quantizer design for protection region radius is *different* than traditional mean-squared optimal quantizers.

The actual area of a circular protection region with radius r is πr^2. After quantization, the circular protection region will have a radius of $\pi Q(r)^2$. Since $Q(r) \geq r$ by design requirements, so a part of TV white space region will be *lost* or mislabeled as protection region. This motivates the following cost function

$$C(\mathcal{R}, \mathcal{Q}) = \pi \sum_{r \in \mathcal{R}} \left\{ Q^2(r) - r^2 \right\} \quad (1)$$

which signifies the white space area lost due to quantization. This is explained with an example having four protection radius and two quantization levels in Fig. 3. The radius r_1, r_2 get mapped to q_1. Even though q_1 is nearer to r_3, still r_3 gets mapped to q_2. For any given number of quantization levels m, an *optimal* quantization map $Q : \mathcal{R} \to \mathcal{Q}$ has to be designed to minimize the lost TV white space area, subject to a primary protection condition—the actual (unquantized) protection region should be a *subset* of the quantized protection region.

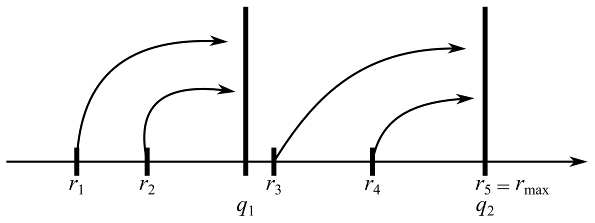


Fig. 3. Radius r_1, r_2 get translated to the quantization level q_1 and r_3, r_4 get translated to the quantization level q_4 as shown in the image. This is for the purpose of protecting the primary transmitting device from any harmful interference.

The cost function in (1) can be rewritten as

$$C(\mathcal{R},\mathcal{Q}) = \pi\left\{\sum_{i=1}^{n} Q^2(r_i) - r_i^2\right\} \tag{2}$$

where $Q(r_i)$ maps r_i to the next largest quantization level in the set $\mathcal{Q} = \{q_1, q_2, \ldots, q_m\}$. The levels in $\mathcal{Q}$ have to be chosen to minimize the lost TV white space area to quantization, or $C(\mathcal{R},\mathcal{Q})$ in (2). The usual technique for minimization will be to evaluate

$$\frac{\partial C(\mathcal{R},\mathcal{Q})}{\partial q_j} = 0 \text{ and } \frac{\partial^2 C(\mathcal{R},\mathcal{Q})}{\partial q_j^2} > 0 \tag{3}$$

for all $q_j \in \mathcal{Q}$. Let $r_{\max} = r_n$ be the radius of the largest protection region in $\mathcal{R}$. Then $q_m = r_{\max}$. However, the derivative does not exist since the set $\mathcal{R}$ is discrete and therefore a change in $q_j \geq r_i$ to $q_j < r_i$ causes a discontinuous change in $C(\mathcal{R},\mathcal{Q})$.[1] For this reason, the following empirical (and optimal) approach is developed by us.

An iterative algorithm will be obtained to find the set $\mathcal{Q}$ which minimizes $C(r,\mathcal{Q})$ or $C(\mathcal{R},\mathcal{Q})$. This iterative algorithm is explained next for minimizing $C(\mathcal{R},\mathcal{Q})$, i.e., assuming that $\mathcal{R}$ is given. At first, note that the largest quantization level must be equal to the largest protection radius, i.e.,

$$q_m = r_n \tag{4}$$

since the largest values of $Q(r_i)$ need not exceed r_n (the maximum protection radius) in (2). Apart from q_m, each q_j will be chosen from the discrete-set $\mathcal{R}$ only. This can be understood using Fig. 3. If q_1 is chosen between r_2 and r_3, then the cost contribution of q_1 is $q_1^2 - r_1^2 + q_1^2 - r_2^2$, which gets minimized when $q_1 = r_2$.

At a high-level, one notes that q_1 has $(n-m)$ choices in $\mathcal{R}$, subsequently q_2 has $(n-m-1)$ choices in $\mathcal{R}$, and so on. So the total number of choices for the entire set $\mathcal{Q}$ is $(n-m)(n-m-1)\ldots(1) = (n-m)!$, which is huge! A fast algorithm will be developed next to solve the selection of $\mathcal{Q}$.

[1]One can assume a continuous distribution whose statistical realizations govern the actual radius of transmitters and setup an average cost function which is differentiable. However, in such a setup, in what we observed, the Hessian and its properties are analytically difficult. The details are omitted due to space constraints.

The main technique in our algorithm is that if q_j has to be selected while other elements in $\mathcal{Q}$ are fixed, then q_j depends only on q_{j-1} and q_{j+1}. That is, if the odd elements in the set $\mathcal{Q}$ are fixed, then the even elements can be found separately. Similarly, if the even elements in the set $\mathcal{Q}$ are fixed, then the odd elements can be found separately. This results in a separable optimization algorithm as detailed next. The cost function in (2) can be rewritten as

$$\begin{aligned}
&\frac{1}{\pi}C(\mathcal{R},\mathcal{Q}) \\
&\quad= \sum_{r\in\mathcal{R}} Q^2(r) - \sum_{r\in\mathcal{R}} r^2 \\
&\quad= \sum_{r\in(q_{j-1},q_j]} Q^2(r) + \sum_{r\in(q_j,q_{j+1}]} Q^2(r) + E_j - \sum_{r\in\mathcal{R}} r^2 \\
&\quad= \sum_{r\in(q_{j-1},q_j]} q_j^2 + \sum_{r\in(q_j,q_{j+1}]} q_{j+1}^2 + E_j - \sum_{r\in\mathcal{R}} r^2 \qquad (5)
\end{aligned}$$

where the term E_j is positive, depends on $q_1, \ldots, q_{j-1}, q_{j+1}, \ldots, q_m$, and is independent of q_j. In (5), it is also noted that $r \in (q_{j-1}, q_j]$ will get quantized to q_j and $r \in (q_j, q_{j+1}]$ will get quantized to q_{j+1}. Let the number of protection region radius $r \in (q_{j-1}, q_{j+1}]$ be n_j and number of radius $r \in (q_{j-1}, q_j]$ be k_j. Then (5) can be rewritten as

$$\frac{1}{\pi}C(\mathcal{R},\mathcal{Q}) = k_j q_j^2 + n_j q_{j+1}^2 + E_j - \sum_{r\in\mathcal{R}} r^2 \tag{6}$$

The last term is independent of $\mathcal{Q}$, while the second last term E_j is independent of q_j and can be ignored during optimization. Since q_{j-1} and q_{j+1} are fixed, so is n_j. Therefore, the only choice variables are q_j, which subsequently determines k_j as well. The minimization of $k_j q_j^2 + n_j q_{j+1}^2$ and subsequently the expression in (6) (for fixed q_{j-1} and q_{j+1}) can be performed by an exhaustive search over various values of q_j in between q_{j-1} and q_{j+1}. In summary, for given fixed values of q_{j-1} and q_{j+1}, the value of q_j that (locally) minimizes $C(\mathcal{R},\mathcal{Q})$ can be found out by an exhaustive search.[2]

This motivates the following *Even-Odd* algorithm for the minimization of cost function in (2), subject to the condition that quantized protection radius is always larger than the actual protection radius:

1) A random initialization for the quantization levels in the set $\mathcal{Q}$ is assumed. It must be noted that the quantization levels belong in the set $\mathcal{R}$.
2) The largest quantization level q_m is fixed to the largest protection radius r_n.
3) After a random initialization, the even quantization levels are fixed and the odd quantization levels are exhaustively searched according to the process outlined in (6). It is restated that the exhaustive search for each quantization level is separate. This results in optimal

[2]It is noted that exhaustive search can be computationally suboptimal, and more efficient methods such as bisection can be used for reduced complexity of optimization. This is left as a future work.

values for odd quantization levels with respect to the cost function in (2).

4) The (locally) optimized values of the odd quantization levels, as obtained in the previous step, are fixed. The even quantization levels are now exhaustively searched according to the process outlined in (6). This results in optimal values for the even quantization levels with respect to the (2).
5) The steps 3 and 4 above and used in an iterative manner, since till the quantization levels do not change. The resultant quantization levels minimize the desired cost function stated in (2).

The cost in (2) reduces at each step of the above algorithm. Since the cost is bounded below by zero, a decreasing sequence of cost will converge. The data used in simulation experiments is outlined next.

III. Quantization on available data-sets

For circular protection regions, two sets of data were available to us. The first data-set contains the protected service contours' bounding radius calculated using protection and pollution viewpoints [6]. This set of radius is available for all the channels between 2 to 51 (49 channels) of United States, where transmission by a white space device is permitted by the FCC. There were 8012 protection regions, and hence radius, in total. The second data-set contains the protected service contours' bounding radius obtained for India [7]. There are no white space regulations in India as of now. The protection radius are available only for 15 channels in the UHF Band-III (470-590MHz). There were 374 protection regions, and hence radius, in total for India.

These datasets are used to analyse the recovered TV white space area and test our optimal quantization Even-Odd algorithm. As India has extensive rural areas with negligible broadband services, Internet connection is extremely unreliable. It is once again emphasized that a broadcast based TV white space geolocation database will be quite useful for such scenarios.

If b bits are used to index each radius in the set $\mathcal{R}$, then $m = 2^b$. It is recalled that the radius set $\mathcal{R}$ is only known to the database, points in $\mathcal{R}$ will be mapped into m quantization levels, and $\mathcal{Q}$ is agreed upon between the broadcasting geolocation database and the secondary devices. For comparison of our Even-Odd algorithm discussed in Section II, a uniform scalar quantizer is used. For uniform scalar quantizer, the quantization levels are $\mathcal{Q} = \{r_{\min} + \Delta, r_{\min} + 2\Delta, \ldots, r_{\max}\}$ where $\Delta = (r_{\max} - r_{\min})/m$. All the values of radius (r) greater than $i-1^{\text{th}}$ level and less than or equal to i^{th} level are translated to the i^{th} level in order to protect the primary.

The results obtained by applying our algorithm on the data-set from United States are illustrated in Fig. 4(a). Increasing the number of bits decreases the area of white space region lost, and with 5 bits (per protection radius) or 32 quantization levels recover most of the white space area. The results obtained by applying our algorithm on the data-set from India are illustrated in Fig. 4(b). With 4 bits (per protection radius) or 16 quantization levels recover most of the white space area.

The evolution of quantization levels in our Even-Odd algorithm is illustrated in Fig. 5. The initial quantization levels are obtained by using a scalar quantizer for $b = 3$. The quantization levels obtained using the algorithm, help in recovering more white space area as compared to uniform quantization for every bit that is sent as illustrated in Fig. 4(a) and Fig. 4(b).

IV. Bandwidth and broadcast based geolocation database

In this section, we will explore the tradeoffs between TV white space area recovered, the update-rate, and the bandwidth needed by a broadcast based geolocation database. As explained in Section II and Section III, each TV transmitter will have a protection radius, which can be quantized. The broadcast based geolocation database can use a satellite communication, such as satellite broadcast TV, to periodically notify the secondary users about the protection radius. It is assumed that there are a fixed number of n TV transmitters in the frequency band of interest. Their locations are fixed and accessible to the secondary users based on an id-number. Let the n transmitters have identity numbers $(\text{id}_1, \ldots, \text{id}_n)$. In the most compressed form, these id-numbers will need $\log_2 n$ number of bits per transmitter for identification. Let $\mathcal{N}$ denote the database. Then the database needs to communicate

$$\mathcal{N} := \{(id_1, r_1), \ldots, (id_n, r_n)\}$$

to the secondary receivers using a broadcasting satellite.

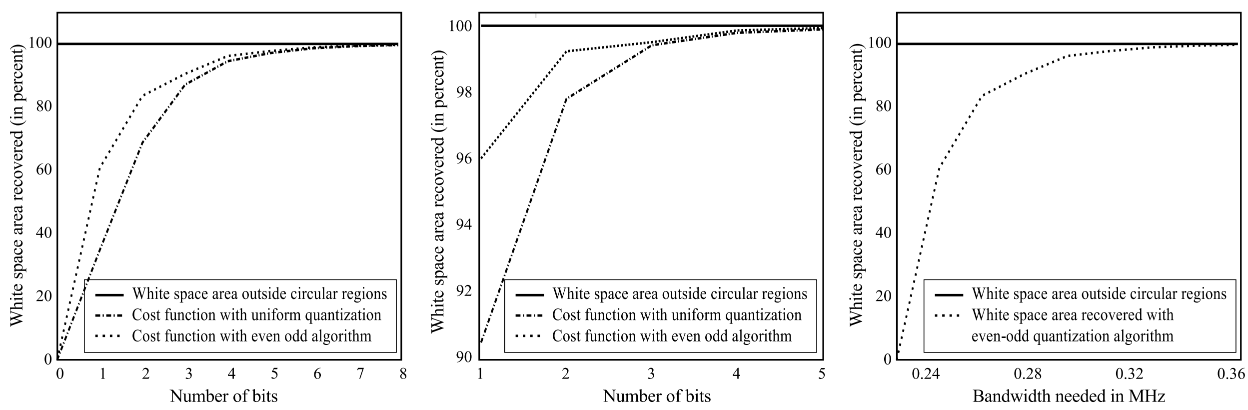
For United States, there are 8012 primary transmitter across channels 2 to 51. This data-set has been obtained from the work done by Harrison, Mishra, and Sahai [6]. To index each of the 8012 transmitters with a unique id we need at least 13-bits. If b-bits are used for sending the protection region of each transmitter, then it would result in a data volume of

$$\mathcal{V} := (13 + b) \times 8012. \tag{7}$$

For b bits, there would be 2^b quantization points and the Even-Odd algorithm in the radius set $\mathcal{R}$ can be used to minimize the lost TV white space area (see Section II). The set of quantization levels $\mathcal{Q}$ are assumed to known by both the broadcast based geolocation database and the unlicensed secondary device.

This data can be broadcasted using some existing standard Digital Video Broadcast standards [8], [9]. For the DVB-S2 standard [8], the spectral efficiency can range from 0.5 (bits/sec)/Hz to 4.5 (bits/sec)/Hz which depends on the FEC encoding technique and signal energy to noise power spectral density ratio. The bandwidth requirements with the pessimistic spectral efficiency number of 0.5 (bits/sec)/Hz with 1 update/sec requirement of the database at the secondary device is illustrated in Fig. 4(c).

Observe that a bandwidth of only 0.36MHz is needed to get updates about protection region every second for 8012 protection regions and to recover almost all of TV white



(a) (b) (c)

Fig. 4. (a) The recovered white space area for various values of bits/protection region is plotted for the data-set from United States [6]. The solid line represents the actual real valued white space area. For smaller values of bits/protection region, the Even-Odd algorithm based quantizer outperforms the uniform quantizer. The area is reported in percent, which is proportional to the actual area of United States. (b) The recovered white space area for various values of bits/protection region is plotted for the data-set from India [7]. (c) The graph shows the bandwidth requirement for sending the protection radius and identification number of each of the protected contours of United States for spectrum efficiency value of 0.5bps/Hz and updates/sec value of 1.

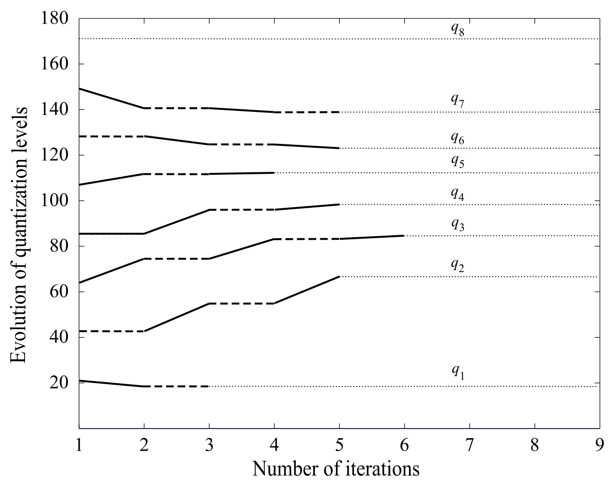


Fig. 5. The evolution of quantization levels in the Even-Odd algorithm is illustrated. The initial points are obtained from a uniform quantizer. The largest level q_8 is always equal to $r_{\max}$. In the odd steps (q_1, q_3, q_5, q_7) are calculated while (q_2, q_4, q_6) are held fixed. The roles are reversed in the odd steps. The movement is illustrated by solid line, the fixed behavior is illustrated by a dashed line, and convergence of the quantization level is shown with a dotted line.

space region after quantization. The *required bandwidth is pessimistic* since the spectral efficiency was assumed at a minimum value, the number of transmitters in UHF band within India is smaller than 8012, and the update rate of 1 per second is very high. The *required is slightly optimistic* because the various $\text{id}_1, \text{id}_2, \ldots, \text{id}_n$ and associated protection region radius will have to be considered with packetization overheads. The treatment of packetization is omitted from this work due to space constraints.

V. CONCLUSIONS

A geolocation database that broadcasts the TV white space or the primary services protection regions on rate-constrained digital channel was considered. The key issue addressed was quantization or digital representation of the protection regions. A fast algorithm for optimal quantizer design was developed. The algorithm minimizes the white space area identified as protection region, while ensuring that protection region is not labeled as white space region due to quantization. The approximation methods were tested using two experimental data-sets. These data-sets included circular protection regions across all TV channels in the United States, circular protection regions in the UHF band TV channels in India. The bandwidth requirement for implementing a broadcast based geolocation database using existing Digital Video Broadcast standards was also examined.

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