Fast Disentanglement-based Blind Quantum Source Separation and Process Tomography Using a Feedforward Quantum-classical Adapting Structure

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***Abstract*—Our recent investigations of blind quantum source separation and process tomography methods for Heisenberg-coupled quantum bits (qubits) were focused on introducing a new separation principle, based on output disentanglement. We here extend them by proposing a more advanced implementation of their cost function and optimization algorithm. This leads us to move from a feedback to a feedforward adapting block, which avoids potential issues related to feedback in quantum circuits. The number of quantum source state preparations required to blindly adapt the separating system is thus strongly decreased (roughly from 10^7 to 10^4), yielding much faster adaptation.**

I. Prior work and problem statement

Within the information processing (IP) domain, various fields developed very rapidly during the last decades. One of these fields is Blind Source Separation (BSS), which led to various classes of methods, including Independent Component Analysis (ICA) [1]. Until recently, all BSS investigations were performed in a "classical", i.e. non-quantum, framework. Another growing field within the overall IP domain is Quantum Information Processing (QIP) [8]. QIP is closely related to Quantum Physics (QP). It uses abstract representations of systems whose behavior is requested to obey the laws of QP. This already made it possible to develop new and powerful IP methods, which manipulate the states of so-called quantum bits, or qubits.

In 2007, we bridged the gap between classical (B)SS and QIP/QP in [2], by introducing a new field, Quantum Source Separation (QSS) and especially its blind version (BQSS). The QSS problem consists in restoring (the information contained in) individual source quantum states, eventually using only the mixtures (in SS terms [3]) of these states which result from their undesired coupling. The blind version of this problem corresponds to the case when the parameter values of the mixing operator are initially unknown and are first estimated by using only mixtures of source quantum states, i.e. without knowing these source states (see [3] for (B)QSS applications).

A complete BQSS investigation consists of the definition of the same items as in classical BSS, namely: (1) considered mixing model, (2) proposed separating system structure, (3) proposed separation principle (which is the counterpart of e.g. forcing output independence in classical ICA) preferably with an analysis of the resulting so-called "indeterminacies", (4) proposed separation criterion (see e.g. output mutual information minimization in classical ICA), (5) proposed separation algorithm (e.g. gradient-based minimization of cost function).

Using this approach, we initially developed a first class of BQSS methods, which use a separation principle that has some relationships with classical ICA, although these methods address quantum sources. These methods were first introduced in [2] and their extensions were e.g. detailed in [3] and [5].

More recently, in [4] and [6], we started to develop a new class of BQSS methods, which use a different separation principle, based on the disentanglement of output quantum states of the separating system. Our investigations reported in [4] and [6] cover all five items of the above-defined procedure for developing BQSS methods. However, they required major efforts for the first three of these items, so that we then only resctricted ourselves to a simple approach for the cost function and associated optimization algorithm. Therefore, after summarizing the concepts from [4] and [6] which are needed here (see Sections II and III), a first contribution in this paper consists of an analysis of the properties of the above algorithm (see Section IV). The limitations which thus appear then motivate us to develop a modified version of this type of approach, which is a major evolution since even the separating system structure is thus changed and a much faster adaptation method is introduced.

Besides, classical BSS is mainly based on the blind inversion of the mixing model. BSS methods therefore typically also perform a blind identification of the mixing model. In [7], we started to develop similar considerations for the quantum counterpart of the above system identification problem, i.e. so-called Blind Quantum Process Tomography (BQPT). The second main contribution of this new paper therefore consists of an analysis of the capabilities of the proposed BQSS method from a BQPT point of view (see Section V). Conclusions are eventually drawn from this overall investigation in Section VI.

II. Mixing model

As stated above, computations of the field of QIP use qubits instead of classical bits [8]. In [4], we first detailed the required

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concepts for a single qubit and then presented the type of coupling between two qubits that we consider and that defines the "mixing model", in (B)SS terms, of our investigation. We hereafter summarize the major aspects of that discussion, which are required in the current paper.

A qubit with index i considered at a given time t_0 has a quantum state. If this state is pure, it belongs to a two-dimensional space $\mathcal{E}_i$ and may be expressed as

$$|\psi_i(t_0)\rangle = \alpha_i|+\rangle + \beta_i|-\rangle \tag{1}$$

in the basis of $\mathcal{E}_i$ defined by the two orthonormal vectors that we hereafter denote $|+\rangle$ and $|-\rangle$, whereas α_i and β_i are two complex-valued coefficients constrained to meet the condition

$$|\alpha_i|^2 + |\beta_i|^2 = 1 \tag{2}$$

which expresses that the state $|\psi_i(t_0)\rangle$ is normalized.

In the BQSS configuration studied in this paper, we first consider a system composed of two qubits, called "qubit 1" and "qubit 2" hereafter, at a given time t_0. This system has a quantum state. If this state is pure, it belongs to the four-dimensional space $\mathcal{E}$ defined as the tensor product (denoted $\otimes$) of the spaces $\mathcal{E}_1$ and $\mathcal{E}_2$ respectively associated with qubits 1 and 2, i.e. $\mathcal{E} = \mathcal{E}_1 \otimes \mathcal{E}_2$. We hereafter denote $\mathcal{B}_+$ the basis of $\mathcal{E}$ composed of the four orthonormal vectors $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$, where e.g. $|+-\rangle$ is an abbreviation for $|+\rangle \otimes |-\rangle$, with $|+\rangle$ corresponding to qubit 1 and $|-\rangle$ corresponding to qubit 2. Any pure state of this two-qubit system may then be expressed as

$$\begin{aligned} |\psi(t_0)\rangle &= c_1(t_0)|++\rangle + c_2(t_0)|+-\rangle \\ &\quad + c_3(t_0)|-+\rangle + c_4(t_0)|--\rangle \end{aligned} \tag{3}$$

and has unit norm. It may also be represented by the corresponding vector of complex-valued components in basis $\mathcal{B}_+$, which reads

$$C_+(t_0) = [c_1(t_0), c_2(t_0), c_3(t_0), c_4(t_0)]^T \tag{4}$$

where ^{T} stands for transpose. In particular, we study the case when the two qubits are independently initialized, with states defined by (1) respectively with $i = 1$ and $i = 2$. We then have

$$\begin{aligned} |\psi(t_0)\rangle &= |\psi_1(t_0)\rangle \otimes |\psi_2(t_0)\rangle & (5) \\ &= \alpha_1\alpha_2|++\rangle + \alpha_1\beta_2|+-\rangle \\ &\quad + \beta_1\alpha_2|-+\rangle + \beta_1\beta_2|--\rangle. & (6) \end{aligned}$$

Besides, we consider the case when the two qubits, which correspond to two spins 1/2, have undesired coupling after they have been initialized according to (5). The considered coupling is based on the Heisenberg model with a cylindrical-symmetry axis collinear to Oz, the direction common to the applied magnetic field and to our first chosen quantization axis. This coupling may be represented as

$$C_+(t) = MC_+(t_0) \tag{7}$$

where $C_+(t)$ is the counterpart of (4) at time t and defines the coupled (or "mixed", in BSS terms) state $|\psi(t)\rangle$ of the two-qubit system at that time. In basis $\mathcal{B}_+$, the evolution of the system's quantum state from t_0 to t is thus represented by the matrix M of (7). Our previous calculations show that, for the considered type of coupling

$$M = QDQ^{-1} = QDQ \tag{8}$$

with

$$Q = Q^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

and D equal to

$$\begin{bmatrix} e^{-\mathrm{i}\omega_{1,1}(t-t_0)} & 0 & 0 & 0 \\ 0 & e^{-\mathrm{i}\omega_{1,0}(t-t_0)} & 0 & 0 \\ 0 & 0 & e^{-\mathrm{i}\omega_{0,0}(t-t_0)} & 0 \\ 0 & 0 & 0 & e^{-\mathrm{i}\omega_{1,-1}(t-t_0)} \end{bmatrix} \tag{10}$$

with $\mathrm{i}^2 = -1$. The four real (angular) frequencies $\omega_{1,1}$ to $\omega_{1,-1}$ in (10) depend on the physical setup and their values are unknown in practice.

III. Previous BQSS method

In [4] and [6], to uncouple qubit states mixed according to the above model, we introduced a BQSS method that we are going to further analyze and modify in this paper. Therefore, we first summarize its main features hereafter.

A. Inverting block of separating system

The inverting block of the separating system is the part of this system which is to be used eventually (i.e. after this block has been adapted, as explained below) to derive the output quantum state $|\Phi\rangle$ of this system from its input quantum state, which is the above-defined coupled state $|\psi(t)\rangle$. That block appears in the upper right part of Fig. 1 and is used in both our previous BQSS method described here and in its extension proposed in Section IV. It uses quantum processing means only. The output quantum state of that block and therefore of our overall separating system is denoted as

$$|\Phi\rangle = c_1|++\rangle + c_2|+-\rangle + c_3|-+\rangle + c_4|--\rangle. \tag{11}$$

It may also be represented by the corresponding vector of components of $|\Phi\rangle$ in basis $\mathcal{B}_+$, denoted as

$$C = [c_1, c_2, c_3, c_4]^T. \tag{12}$$

We then have

$$C = UC_+(t) \tag{13}$$

where U defines the unitary quantum-processing operator applied by our separating system to its input $C_+(t)$. As justified below, we choose this operator U to belong to the class defined by

$$U = Q\tilde{D}Q \tag{14}$$

$$\text{with} \quad \tilde{D} = \begin{bmatrix} e^{\mathrm{i}\gamma_1} & 0 & 0 & 0 \\ 0 & e^{\mathrm{i}\gamma_2} & 0 & 0 \\ 0 & 0 & e^{\mathrm{i}\gamma_3} & 0 \\ 0 & 0 & 0 & e^{\mathrm{i}\gamma_4} \end{bmatrix} \tag{15}$$

where γ_1 to γ_4 are free real-valued parameters.

B. Adapting block of separating system

The above type of inverting block was selected because it can perfectly restore the quantum source state $|\psi(t_0)\rangle$ for adequate values of its free parameters γ_1 to γ_4: setting them so that $\tilde{D} = D^{-1}$ yields $U = M^{-1}$, which results in $C = C_+(t_0)$ and $|\Phi\rangle = |\psi(t_0)\rangle$. However, the condition $\tilde{D} = D^{-1}$ cannot be used as a *practical* procedure for directly assigning $\tilde{D}$, because D is unknown. Instead, a procedure for adapting the parameters γ_1 to γ_4 of $\tilde{D}$ by using only one of several values of the available mixed state $|\psi(t)\rangle$ is therefore required, which corresponds to a *blind* (quantum) source separation problem.

Briefly, the BQSS method that we developed to this end in [4] and [6] uses the output disentanglement separation principle that we introduced in these papers, which is based on the concept of quantum state entanglement. From this principle, we then derived a two-step adaptation procedure, where each step consists of the global minimization of a cost function expressed with respect to classical-form quantities, namely probabilities of discrete outcomes of spin component measurements performed at the output of the inverting block. The first cost function is defined as

$$F_z = \sum_{n=1}^{N_z} |f_z(n)|^p \tag{16}$$

with

$$f_z(n) = P_{1z}(n)P_{4z}(n) - P_{2z}(n)P_{3z}(n) \tag{17}$$

and e.g. $p = 1$ or 2. In these expressions, $P_{1z}(n)$ to $P_{4z}(n)$ are the above-mentioned probabilities, corresponding to the case when the considered spin components are measured along the above-defined axis Oz (we detail them in Section IV-A). Besides, the cost function in (16) involves $N_z \geq 2$ (non-redundant) source states, indexed by n. Each of these states is defined by (6) with corresponding parameter values $\alpha_1(n)$ to $\beta_2(n)$. The first step of the proposed procedure consists in performing a sweep on one of the parameters γ_2 and γ_3, while the other one, as well as γ_1 and γ_4, are constant. This procedure computes the corresponding (estimated) values of F_z and it eventually keeps the value of the tuned parameter γ_2 or γ_3 which minimizes F_z. It then freezes γ_2 and γ_3.

The second stage of the proposed procedure then performs a sweep on γ_1 or γ_4, in order to minimize a cost function which is similar to the above one, but which uses measurements of spins components along an axis Ox orthogonal to Oz.

IV. New BQSS method

A. Motivations

For any output two-spin quantum state defined by (11) in the basis $\mathcal{B}_+$ associated with the Oz axis, when measuring the components of these two spins along Oz, the four possible results are $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$ in normalized units. Moreover, their respective probabilities are

$$P_{1z} = |c_1|^2, P_{2z} = |c_2|^2, P_{3z} = |c_3|^2, P_{4z} = |c_4|^2. \tag{18}$$

The associated quantities which are available in practice and actually used in the cost function (16)-(17) are *estimates* of the above probabilities. These estimates are obtained by using our RWR procedure [4], [6]. For each source state (6) and set of separating system parameters γ_1 to γ_4, this procedure consists in first Repeatedly Writing (i.e. preparing) the considered source state and Reading (i.e. performing the above type of measurements for) the corresponding output of our separating system, and then computing the sample frequencies of all four possible measurement outcomes, which yields the above-mentioned four probability estimates.

The adaptation of γ_2 (or γ_3) is thus cumbersome, because it requires many Write/Read (WR) steps, i.e. typically 2×10^7 if one uses 10^4 such steps to accurately estimate each set of probabilities $P_{1z}(n)$ to $P_{4z}(n)$ and this is repeated for 10^3 values of γ_2 when tuning it and using $N_z = 2$ source states in the cost function (16). We hereafter introduce a method for reducing the complexity of the adaptation procedure.

B. Proposed method

As in all the BQSS methods that we proposed so far, we here consider adaptation methods which convert quantum data into classical-form data at a certain stage of the processing chain, using our RWR procedure. For our methods based on the disentanglement separation principle, this should be done for at least 2 quantum states. Such methods therefore typically require at least 2×10^4 WR steps, as shown by the above discussion. The much higher complexity reported in Section IV-A for our previous BQSS method results from the fact that, in that method, these measurements concern states at the *output* of the inverting block, which therefore e.g. depend on the value of the tuned parameter γ_2, so that these measurements must be repeated for each of the typically 10^3 values of γ_2 used in the sweep performed for this parameter, thus leading to a factor 10^3 in the complexity of the adaptation procedure. This suggests us to try and avoid this complexity increase by performing measurements only once, for states which do not depend on γ_2, i.e. (i) for the state $|\psi(t)\rangle$ at the *input* of the inverting block and/or (ii) for the state obtained by only transferring another instance[1] of $|\psi(t)\rangle$ through the first sub-block Q of the inverting block: as shown by (13)-(14) and Fig. 1, the overall inverting block may be seen as the cascade of three sub-blocks (quantum gates) whose behaviors are respectively defined by the matrices Q, $\tilde{D}$ and Q. Only the states available before sub-block $\tilde{D}$ do not depend on γ_2.

Let us first consider the two-spin quantum state $|\psi(t)\rangle$, defined by (3) with all coefficients $c_j(t_0)$ replaced by $c_j(t)$. Quantum Physics properties already used in (18) at the output of the inverting block here tell us that, when measuring the components along the Oz axis, of both spins, at the input of the inverting block, the four possible results are $(+\frac{1}{2}, +\frac{1}{2})$, $(+\frac{1}{2}, -\frac{1}{2})$, $(-\frac{1}{2}, +\frac{1}{2})$ and $(-\frac{1}{2}, -\frac{1}{2})$ in normalized units, and

[1] Providing a *quantum* state $|\psi(t)\rangle$ to several sub-systems requires a separate instance of that state (obtained with a separate preparation) for each sub-system [4], due to the no-cloning quantum theorem [8].

that the probabilities of these values are respectively the squared moduli of $c_1(t)$ to $c_4(t)$, i.e.

$$p_{1z} = |c_1(t)|^2, \qquad p_{2z} = |c_2(t)|^2, \tag{19}$$

$$p_{3z} = |c_3(t)|^2, \qquad p_{4z} = |c_4(t)|^2. \tag{20}$$

Similarly, let us now consider a couple of measurements performed along axis Oz for the two-qubit quantum state available at the output of the first sub-block Q of the inverting block. This again yields the above four possible results, but now with probabilities equal to the squared moduli of the elements of the vector which represents that state, which is equal to $QC_+(t)$, as shown by (13)-(14). Using (9), this yields

$$P_{1z}(Q) = |c_1(t)|^2, \quad P_{2z}(Q) = \frac{1}{2}|c_2(t) + c_3(t)|^2, \tag{21}$$

$$P_{4z}(Q) = |c_4(t)|^2, \quad P_{3z}(Q) = \frac{1}{2}|c_2(t) - c_3(t)|^2. \tag{22}$$

We therefore keep the cost function defined by (16)-(18) and we investigate whether it can be expressed only with respect to the probabilities defined in (19)-(20) and/or (21)-(22) and with respect to γ_1 to γ_4. It is not guaranteed that this is possible, because our previous BQSS method looses the phase information about complex-valued state coefficients at the output of the inverting block only, as shown by (18), whereas the new method that we are trying to build at this stage would loose phase information already at the input of that block and/or after its first Q sub-block, as shown by (19)-(22), and these two approaches are not guaranteed to be equivalent.

The relevance of the proposed approach must therefore be analyzed, by deriving the expression of each term $f_z(n)$ of the cost function with respect to γ_1 to γ_4, to determine if the other quantities involved in this expression are restricted to the above probabilities. To this end, one first expresses the output state coefficients c_j by combining (12)-(15), (9) and the counterpart of (4) at time t. One then derives the squared moduli of these coefficients c_j and inserts them in (18) and (17). Tedious manipulations eventually yield

$$\begin{aligned} f_z(n) \;=\; & |c_1(t)|^2|c_4(t)|^2 - \frac{1}{16}|c_2(t) + c_3(t)|^4 \\ & -\frac{1}{16}|c_2(t) - c_3(t)|^4 \\ & +\frac{1}{8}[(|c_2(t)|^2 - |c_3(t)|^2)^2 - 4[\Im(c_2(t)c_3(t)^*)]^2] \\ & \quad \times \cos(2(\gamma_3 - \gamma_2)) \\ & -\frac{1}{2}(|c_2(t)|^2 - |c_3(t)|^2)\Im(c_2(t)c_3(t)^*) \\ & \quad \times \sin(2(\gamma_3 - \gamma_2)) \end{aligned} \tag{23}$$

where $\Im(.)$ stands for imaginary part, * represents complex conjugate and all coefficients $c_j(t)$ correspond to the state $|\psi(t)\rangle$ obtained for the source state with index n, but this index is here omitted for the sake of simplicity.

Estimating the probabilities (19)-(22) by means of the above-defined types of measurements therefore does not completely make it possible to derive an estimate of the cost function term in (23), for given γ_1 to γ_4: these measurements do not provide the required estimate of $\Im(c_2(t)c_3(t)^*)$. We solve this problem by extending this approach as follows. It may easily be shown that

$$\Im(c_2(t)c_3(t)^*) = \frac{1}{2}\left(|c_2(t) + \mathrm{i}c_3(t)|^2 - |c_2(t)|^2 - |c_3(t)|^2\right). \tag{24}$$

We therefore introduce a quantum gate which is the partly-imaginary counterpart of the gate defined above by (9), i.e. a gate whose operation is defined by the matrix

$$Q_{\mathrm{i}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \mathrm{i}\frac{1}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\mathrm{i}\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{25}$$

This matrix is unitary and therefore implementable in the standard QIP framework (see p. 213 and pp. 188-203 of [8]). In the same way as for gate Q, feeding gate Q_{i} with another instance of the state $|\psi(t)\rangle$ and applying our RWR procedure to a couple of measurements performed along axis Oz at the output of that gate Q_{i} yields estimates of the probabilities

$$P_{1z}(Q_{\mathrm{i}}) = |c_1(t)|^2,\, P_{2z}(Q_{\mathrm{i}}) = \frac{1}{2}|c_2(t) + \mathrm{i}c_3(t)|^2, \tag{26}$$

$$P_{4z}(Q_{\mathrm{i}}) = |c_4(t)|^2,\, P_{3z}(Q_{\mathrm{i}}) = \frac{1}{2}|c_2(t) - \mathrm{i}c_3(t)|^2. \tag{27}$$

Using the (estimated) probabilities of (26)-(27) in addition to those of (19)-(22) therefore completely makes it possible to derive an estimate of the cost function term in (23), for given γ_1 to γ_4. This opens the way to various algorithms for adapting γ_2 (or γ_3). The simplest algorithm consists in performing a sweep on the value of γ_2 and keeping the value which minimizes F_z. Each value of F_z is here derived from (23) and (16) by using classical processing means, as shown in the lower part of Fig. 1.

The adaptation of γ_1 (or γ_4) should then be addressed by transposing all the approach that we introduced in this paper to the second cost function mentioned in Section III-B. This investigation is skipped here, due to space limitations.

The overall separating system structure thus obtained is shown in Fig. 1. Both its inverting and adapting blocks have a feedforward structure, i.e. with a data flow from the mixed state $|\psi(t)\rangle$ to quantities derived from it. This is a major difference as compared with the feedback structure used in the adapting block of our previous method of [4] and [6], which required special care since at least part of the considered data have a *quantum* form (see [8] p. 23 concerning feedback in quantum circuits). Once γ_1 to γ_4 have been obtained in classical form in the adapting block, they are "downloaded" into the quantum sub-block $\tilde{D}$ of the inverting block, i.e. they are e.g. used to generate voltages which control the parameters of that sub-block $\tilde{D}$.

V. Blind quantum process tomography

As explained in Section I, we here analyze the capabilities of the method proposed in Section IV in terms of BQPT, i.e.

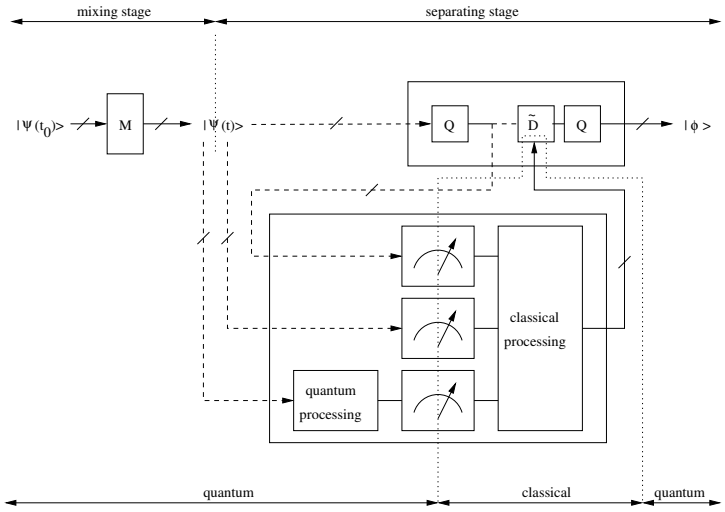

Fig. 1. Mixing stage and proposed separating system: inverting block (upper right part of figure) and adapting block (lower part) including quantum processing, quantum/classical conversion (measurements) and classical processing. Each quantum state is used only once (dashed lines): see Section IV-B (no-cloning).

its ability to blindly identify the mixing model (8)-(10). More precisely, in [7] we explained that its unknown parameters $\omega_{1,1}$ to $\omega_{1,-1}$ in (10) read

$$\omega_{1,1} = \frac{1}{\hbar}\left[GB - \frac{J_z}{2}\right], \quad \omega_{1,0} = \frac{1}{\hbar}\left[-J_{xy} + \frac{J_z}{2}\right], \quad (28)$$

$$\omega_{0,0} = \frac{1}{\hbar}\left[J_{xy} + \frac{J_z}{2}\right], \quad \omega_{1,-1} = \frac{1}{\hbar}\left[-GB - \frac{J_z}{2}\right] \quad (29)$$

where all quantities are physical parameters defined in [7] and only J_{xy} and J_z have unknown values, which should be blindly estimated.

Although the method of Section IV is quite different from the method of [6] summarized in Section III, both methods adapt γ_1 to γ_4 so as to reach the global minimum of the same cost function (16) and of the same second cost function outlined in Sections III and IV. Moreover, for the considered coupling model, this minimization may be shown to be equivalent to the disentanglement of the output states of the separating system, for the considered source states. The investigation reported in [7] then entails that, when applying the new method of Section IV, γ_1 to γ_4 are tuned to final values which are such that

$$J_{xy} = \frac{\hbar}{2(t-t_0)}(\gamma_3 - \gamma_2 - m\pi) \quad (30)$$

$$J_z = \frac{\hbar}{2(t-t_0)}(\gamma_2 + \gamma_3 - \gamma_1 - \gamma_4 + 2k\pi - m\pi) \quad (31)$$

where k and m are integers. This yields estimates of J_{xy} and J_z (up to the indeterminacies due to $2k\pi$ and $m\pi$).

VI. Conclusion

In this paper, we further extended the new class of BQSS methods that we recently introduced in [4] and [6], by proposing a more advanced implementation based on the considered separation principle. This led us to modify the structure of the separating system, moving from a feedback to a feedforward adapting block, which avoids potential issues related to feedback in quantum circuits. The overall approach thus obtained yields a major improvement for the adaptation of the separating system parameters: the number of quantum source state preparations required during adaptation is roughly reduced from 10^7 to 10^4. We also derived an associated BQPT method. Since the physical implementation of qubits is only an emerging topic today, our future works will especially consist in developing a software package including simulated Heisenberg-coupled qubits and BQSS and BQPT methods, to assess their numerical performance.

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Erratum: replace two terms $E\{r_i\}E\{q_i\}$ in (33) of [2] by $E\{r_i q_i\}$, since q_i depends on r_i.

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