Combination of filtered-x adaptive filters for nonlinear listening-room compensation

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***Abstract*—Audio quality in sound reproduction systems can be severely degraded due to system nonlinearities and reverberation effects. In this context, linearization of loudspeakers has been deeply investigated but its combination with room equalization is not straightforward, mainly when the nonlinearities present memory. In this paper, a method relying on the convex combination of two linear filters using the filtered-x LMS (FXLMS) algorithm and based on the virtual path concept to preprocess audio signals is presented for nonlinear room compensation. It is shown that the combination of two linear adaptive filters behaves similarly to the filtered-x second-order adaptive Volterra (NFXLMS) filter. Moreover the new approach is computationally more efficient and avoids the generation of higher harmonics. Experimental results validate the performance of the new approach.**

I. Introduction

Sound reproduction systems inside a room can exhibit an undesirable behavior due to the room acoustics. Moreover, loudspeaker and amplifier systems can produce linear and nonlinear distortions, which deteriorate the sound quality. In order to remove these effects, an equalizer is used before driving the output signal through the loudspeakers. Thus, the combination of the equalizer filter and the nonlinear electroacoustic path $\mathcal{H}$ reproduces the desired audio signal. In this paper, the electroacoustic path that involves the loudspeaker-enclosure-microphone setup has been modeled as a Volterra filter. Specifically, a second-order Volterra filter of finite memory [1], as it is illustrated in Fig. 1, is considered. The output signal $z(n)$ of the nonlinear system $\mathcal{H}$ can be expressed as

$$z(n) = \mathcal{H}[y(n)] = \sum_{i_1=0}^{M_1-1} L(i_1)y(n-i_1) + \sum_{i_1=0}^{M_2-1}\sum_{i_2=0}^{M_2-1} N(i_1,i_2)y(n-i_1)y(n-i_2), \quad (1)$$

being $\mathcal{H}$ the nonlinear system modeled with 2 kernels and M_q, for $q = 1, 2$, the memory length of the qth-Volterra kernel. Moreover, $L(i_1)$ is the i_1-coefficient of the first kernel and $N(i_1, i_2)$ refers to the (i_1, i_2)-coefficient of the quadratic kernel, with a symmetric form [2].

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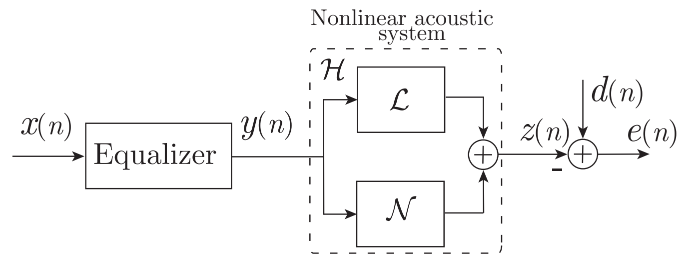


Fig. 1. Block diagram of a nonlinear room compensation system.

Although there are many papers addressing the loudspeaker linearization, only very few papers consider also the room compensation problem. Similarly to linear room equalization, adaptive nonlinear room compensation requires filtered-x ad hoc treatment to avoid instability, thus the input signal $x(n)$ is filtered through the electroacustic path. Therefore, the filtering of $x(n)$ involves the use of a *nonlinear filtered-x algorithm* as it was introduced in [3]. This structure has been previously used in active noise control (using the so-called virtual secondary path) with adaptive Volterra filters [4], [5], functional-link artificial neural networks (FLANN) [6], [7], or a general function expansion for both FLANN and Volterra filters [4]. In all those works, the nonlinear secondary paths were modeled as nonlinear memoryless systems. Recently, a second-order Volterra filter capable of compensating nonlinear distortion with memory in the context of room compensation has been proposed [8]. This adaptive filtered-x Volterra (NFXLMS) filter was based on the concept of virtual channel and therefore requires a single nonlinear filtering of $x(n)$. Present work provides a straighforward novel solution for nonlinear room compensation that exhibits a performace at least similar to that of the NFXLMS in terms of linear compensation. Motivated by [8] and inspired by the convex combination of filters [9], [10], two novel approaches are proposed. Since a linear filter is not able to compensate the nonlinearities and it can become unstable, we first propose an adaptive linear filter based on the estimated nonlinear channel as virtual path. The filter coefficients will be updated as in the conventional filtered-x LMS (FXLMS) [11] but with a nonlinear virtual channel (V-FXLMS) to ensure the filter convergence. In the second scheme we propose the convex combination of two

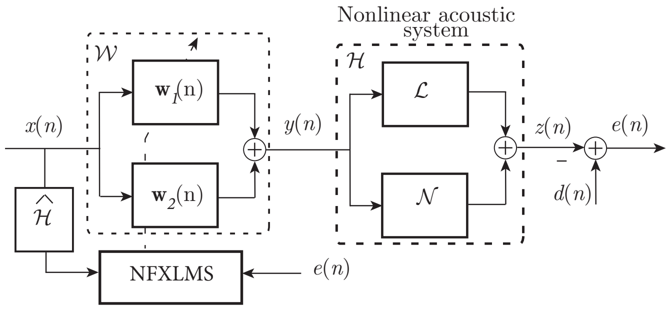


Fig. 2. Block diagram of the adaptive equalization system based on a filtered-x adaptive nonlinear algorithm (NFXLMS).

adaptive linear filters, which leads to the CV-FXLMS. The first component filter is the V-FXLMS and the second one is the FXLMS filter. These filters are combined in such a manner that the advantages of both component filters are kept: the fast convergence from the FXLMS at the transient state and the low steady-state error from the V-FXLMS.

The rest of the paper is organized as follows. In Section II we briefly described the NFXLMS filter. The two novel approaches are introduced in Section III: the V-FXLMS and the CV-FXLMS. The simulation results are provided in Section IV. Finally, a brief summary is presented in Section V.

II. Adaptive filtered-x Volterra filter

Recently, the authors proposed the NFXLMS filter to implement the adaptive compensation prefilter [8]. As illustrated in Fig. 2, the signal $y(n)$ is generated by a second-order Volterra filter $\mathcal{W}$ expressed as

$$y(n) = \mathcal{W}[x(n)] = \sum_{i_1=0}^{N_1-1} w_1(i_1;n)x(n-i_1) + \sum_{i_1=0}^{N_2-1}\sum_{i_2=0}^{N_2-1} w_2(i_1,i_2;n)x(n-i_1)x(n-i_2), \quad (2)$$

where N_p is the memory length of the pth-Volterra kernel ($p = 1, 2$) and $w_p(i_1, \ldots, i_p; n)$ is the specific coefficient at time n.

The error signal $e(n)$ is computed as the difference between the measured signal $z(n)$ and the desired signal $d(n)$, which corresponds to the input signal with a proper time delay

$$e(n) = d(n) - z(n) = x(n-\tau) - z(n). \quad (3)$$

The optimal filter coefficients are chosen to minimize the mean square error of the total system, $E\{e^2(n)\}$. Thus, the filter coefficients can be updated by applying a stochastic gradient algorithm [3], [11] by using the instantaneous estimate of the gradient of $E\{e^2(n)\}$ with respect to the filter coefficients and it can be formulated as

$$w_1(i_1;n) = w_1(i_1;n-1) + \mu_1 e(n)\frac{\partial e(n)}{\partial w_1(i_1;n)}, \quad (4)$$

$$w_2(i_1,i_2;n) = w_2(i_1,i_2;n-1) + \mu_2 e(n)\frac{\partial e(n)}{\partial w_2(i_1,i_2;n)}, \quad (5)$$

where μ_1 and μ_2 are the step size parameters.

From Fig. 2, we can derive

$$\frac{\partial e(n)}{\partial w_1(i_1;n)} = \sum_{m=0}^{M-1} \frac{\partial z(n)}{\partial y(n-m)} \cdot \frac{\partial y(n-m)}{\partial w_1(i_1;n)}, \quad (6)$$

$$\frac{\partial e(n)}{\partial w_2(i_1,i_2;n)} = \sum_{m=0}^{M-1} \frac{\partial z(n)}{\partial y(n-m)} \cdot \frac{\partial y(n-m)}{\partial w_2(i_1,i_2;n)}, \quad (7)$$

where M is the memory of $\mathcal{H}$, that is $M = max(M_1, M_2)$.

We assume the step size parameters are small enough to allow slow variations of the filter coefficients and from (2) leads to

$$\frac{\partial y(n-m)}{\partial w_1(i_1;n)} \approx x(n-m-i_1), \quad (8)$$

$$\frac{\partial y(n-m)}{\partial w_2(i_1,i_2;n)} \approx x(n-m-i_1)x(n-m-i_2). \quad (9)$$

To implement the NFXLMS we use the virtual channel model as it was introduced in [3], [4]. To this end, we define a time-varying filter of M-length whose coefficients depend on the input signal $y(n)$ that from [8] can be expressed as the derivative of the nonlinear system defined in (1) with respect to the delayed inputs

$$\widehat{H}(m;n) = \frac{\partial z(n)}{\partial y(n-m)} = L(m) + 2\sum_{i=0}^{M_2-1} N(m,i)y(n-i), \quad (10)$$

where $0 \le m < \max(M_1, M_2)$. In other cases, the coefficients are 0.

Finally, by substituting (6)-(10) into (4) and (5), we obtain the update equations of the NFXLMS filter

$$w_1(i_1;n) = w_1(i_1;n-1) + \mu_1 e(n) \sum_{m=0}^{M-1} \widehat{H}(m;n)x(n-m-i_1), \quad (11)$$

as well as

$$w_2(i_1,i_2;n) = w_2(i_1,i_2;n-1) + \mu_2 e(n) \sum_{m=0}^{M-1} \widehat{H}(m;n)x(n-m-i_1)x(n-m-i_2) \quad (12)$$

with the virtual channel $\widehat{H}(m;n)$ defined in (10).

III. Novel adaptive schemes for nonlinear room compensation

This section introduces two adaptive filters for nonlinear room compensation. The first scheme we consider is based on a linear adaptive filter with a virtual channel (V-FXLMS). The second scheme, the CV-FXLMS is based on the convex combination of two adaptive linear filters, one is the FXLMS and the other is the V-FXLMS.

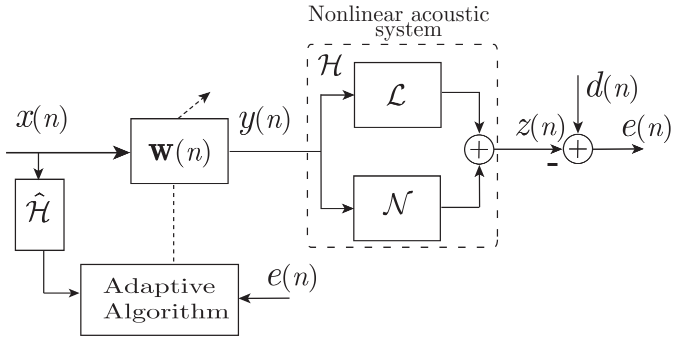


Fig. 3. Linear adaptive equalization using the V-FXLMS.

TABLE I
SUMMARY OF THE COMPUTATIONAL COMPLEXITY OF THE ADAPTIVE FILTERS IN TERMS OF MULTIPLICATIONS PER ITERATION. TYPICAL CASE: $N_1 = 1024$, $N_2 = 64$, $M_1 = 512$ AND $M_2 = 32$.

Algorithm	Multiplications	Typical case
FXLMS	$M_1N_1 + 5N_1 = C_F$	529.408
NFXLMS	$C_V + 2MN_2^2 + 5N_2^2 + N_2$	2.670.898
V-FXLMS	$M_2^2 + M_2 + MN_1 + 5N_1 = C_V$	530.464
CV-FXLMS	$C_V + C_F + M_1N_1 + 5N_1 + 3M$	1.061.929

A. Adaptive linear filter based on the virtual channel

The block diagram of the nonlinear room compensation system using the V-FXLMS adaptive filter is described in Fig. 3. The output signal of the adaptive filter $\mathcal{W}$ is given by

$$y(n) = \mathcal{W}[x(n)] = \sum_{i=0}^{N-1} w(i;n)x(n-i), \tag{13}$$

where N is the number of coefficients of the adaptive filter.

Similarly to (11), the update of filter coefficients by using the LMS algorithm is given by

$$w(i;n) = w(i;n-1) + \mu e(n) \sum_{m=0}^{M-1} \widehat{H}(m;n)x(n-m-i) \tag{14}$$

where the virtual path is defined as in (10).

Although this scheme is unable to completely remove the nonlinear distortions, it avoids the generation of higher harmonics as is the case for the NFXLMS. Moreover, as Table I illustrates, it needs lower computational requeriments lower than the NFXLMS.

B. Convex Combination of two linear filters for nonlinear room compensation

We also cosider the convex combination of two adaptive filters, one using the conventional filtered-x scheme and the other based on the virtual channel. The block diagram of the proposed approach is depicted in Fig. 4.

The output signal $y(n)$ is obtained as the weighted sum of the single outputs $y_1(n)$ and $y_2(n)$,

$$y(n) = \lambda(n)y_1(n) + \Big[1-\lambda(n)\Big]y_2(n) \tag{15}$$

$$= \lambda(n) \sum_{k=0}^{K_1-1} w_1(k;n)x(n-k) + \big[1-\lambda(n)\big] \sum_{k=0}^{K_2-1} w_2(k;n)x(n-k), \tag{16}$$

being K_j the number of coefficients of each component adaptive filter, for $j = 1, 2$. The mixing parameter $\lambda(n) \in [0, 1]$ is defined by using a sigmoid activation function ($\mathrm{sgm}[a(n)] = \{1 + exp[-a(n)]\}^{-1}$) as

$$\lambda(n) = \frac{\mathrm{sgm}[a(n)] - \mathrm{sgm}[-a^+]}{\mathrm{sgm}[a^+] - \mathrm{sgm}[-a^+]}, \tag{17}$$

where $a(n)$ has been restricted to the interval $[-a^+, a^+]$ [9] and it is updated by the following LMS rule,

$$a(n) = a(n-1) \tag{18}$$

$$+\frac{\overline{\mu}_a}{p(n)} e(n)\Big[y_{1\mathrm{f}}(n) - y_{2\mathrm{f}}(n)\Big]\mathrm{sgm}[a(n)]\Big\{1-\mathrm{sgm}[a(n)]\Big\}, \tag{19}$$

being $\overline{\mu}_a$ the step size for the mixing parameter update,

$$\overline{\mu}_a = \frac{\mu_a}{\mathrm{sgm}[a^+] - \mathrm{sgm}[-a^+]}, \tag{20}$$

and $y_{j\mathrm{f}}(n)$ corresponds to the filter output signal $y_j(n)$ filtered through the estimated first kernel $\hat{\mathcal{L}}$ for $j = 1$ and through the virtual channel $\widehat{H}$ for $j = 2$. Moreover $p(n)$ is an estimate of its power obtained from

$$p(n) = \beta p(n-1) + (1-\beta)\Big[y_{1\mathrm{f}}(n) - y_{2\mathrm{f}}(n)\Big]^2, \tag{21}$$

being β a constant between 0 and 1 that we have set close to one ($\beta = 0.9$) according to [12]. The error signals of the component adaptive filters are given by

$$e_j(n) = d(n) - y_{j\mathrm{f}}(n), \quad j = 1, 2. \tag{22}$$

The update rule of the component filters can be written as

$$w_1(k;n) = w_1(k;n-1) + \mu_1 e_1(n)x_{1\mathrm{f}}(n-k), \tag{23}$$

and

$$w_2(k;n) = w(k;n-1) + \mu_2 e_2(n) \sum_{m=0}^{M-1} \widehat{H}(m;n)x(n-m-k) \tag{24}$$

where $x_{1\mathrm{f}}(n-k)$ is the input signal $x(n)$ filter through $\hat{\mathcal{L}}$, and where μ_1 and μ_2 are the step size parameters of each component filter. The computational complexity of this approach is slightly higher than the V-FXLMS but lower than the NFXLMS (see Table I).

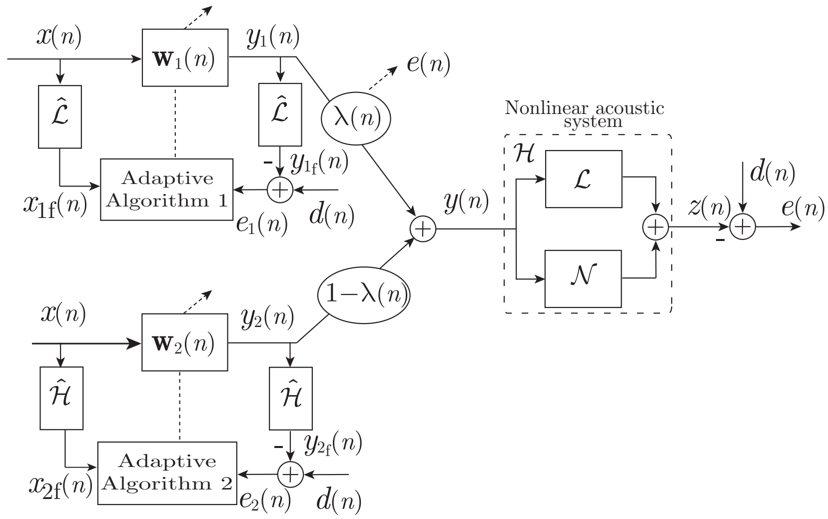
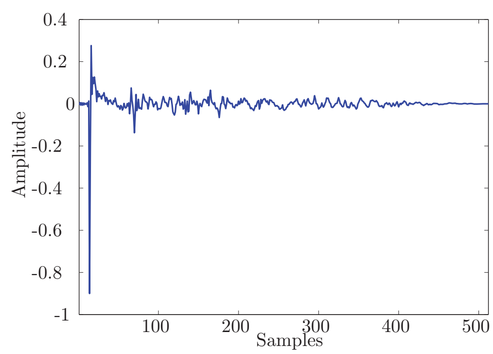
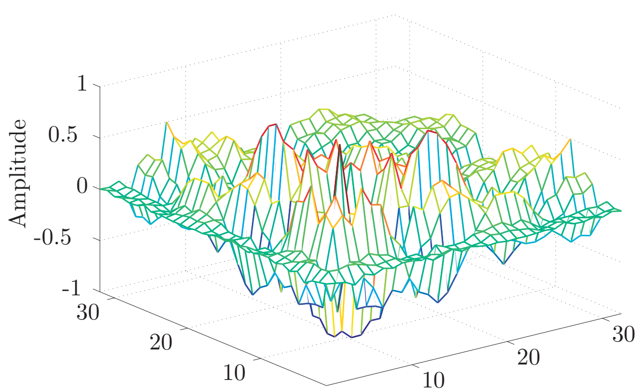


Fig. 4. Combination of filtered-x adaptive filters for listening-room compensation.



(a) Linear distortion



(b) Nonlinear distortion

Fig. 5. (a) Linear and (b) quadratic kernels of $\hat{\mathcal{H}}$.

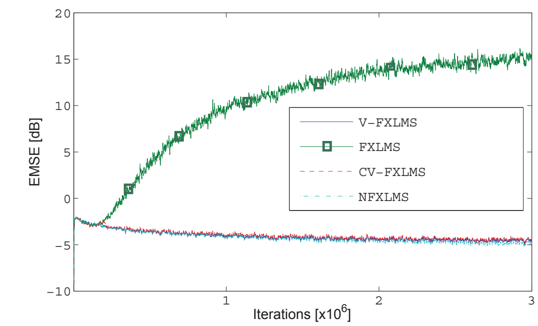
IV. Simulation Results

Several experiments are carried out in this section to illustrate the effectiveness of the proposed schemes in applications of listening-room compensation.

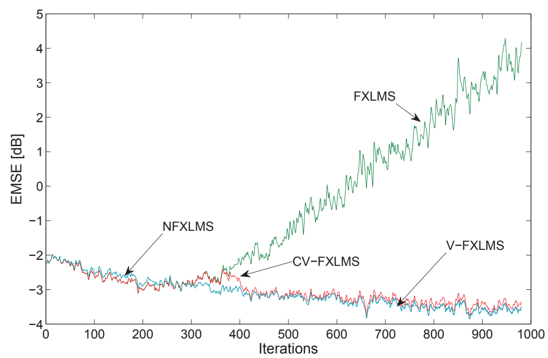
The nonlinear system $\mathcal{H}$ has been measured at a sampling frequency of 8 kHz using a second-order Volterra filter with $M_1 = 512$ and $M_2 = 32$. The linear and quadratic kernels are shown in Fig. 5. Notice that the amplitudes exhibited by the second-order kernel are similar to those of the linear kernel resulting in a high degree of nonlinearitites. As a result, a Linear-to-NonLinear Ratio (LNLR) [8] of 0 dB is obtained. The input signal $x(n)$ is a white Gaussian noise with zero mean and unit variance. Moreover, an uncorrelated noise signal $r(n)$, with zero mean and signal-to-noise ratio (SNR) of 40 dB, has been added to the microphone signal. The adaptive filters have been designed to have $N_1 = K_1 = K_2 = 1024$ and $N_2 = 32$ coefficients.

For performance comparisons, we use the excess mean square error (EMSE) achieved by each adaptive filter versus the number of iterations defined as $EMSE = E\left\{[e(n) - r(n)]^2\right\}$ that has been estimated by averaging over 50 independent runs of the algorithm. Fig. 6 illustrates the EMSE evolution of the two novel approaches (V-FXLMS and the CV-FXLMS) compared with the FXLMS and the NFXLMS filters. As might be expected, the FXLMS shows an unstable tendency and does not converge. Moreover, it seems that the other three algorithms, the NFXLMS, the V-FXLMS and the CV-FXLMS, exhibit a similar behaviour. If we look at the first iterations in Fig. 6.(b), it can be observed how the FXLMS presents a convergence speed faster than the NFXLMS at the first iterations. Furthermore, the CV-FXLMS follows the FXLMS at the transient state and then changes to follow the second component filter, that is the V-FXLMS. This can be also noted in Fig. 7 that shows the mixing parameter evolution, where a $\lambda(n)$ value close to 1 means a FXLMS behavior whereas a value close to 0 means the filter follows the V-FXLMS.

To further evaluate the ability of the proposed filters to reduce the nonlinear distortions, we perform simulations assuming that the different filters use both the coefficients and the mixing parameter obtained at steady state. A sinusoidal wave of 500 Hz is generated as input signal. Fig. 8 presents



(a) EMSE evolution



(b) First 1000 iterations of the EMSE evolution

Fig. 6. Performance comparison of the FXLMS, V-FXLMS, NFXLMS and CV-FXLMS filters for a single frequency tone as evaluation signal.

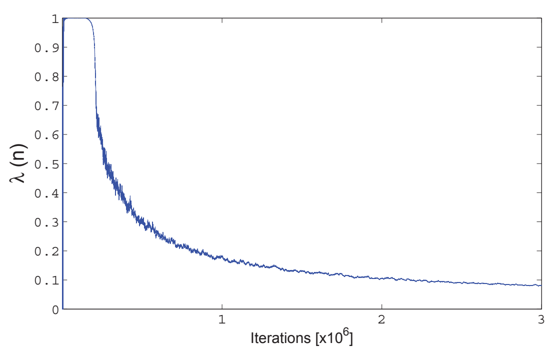


Fig. 7. Combination parameter for the CV-FXLMS filter

the spectral power of the signal pick up at the microphone. The nonlinear distortion of the system generates a first harmonic at 1000 Hz. The V-FXLMS and the CV-FXLMS exhibit a similar performance, slightly reducing the level of the first harmonic signal without generating other harmonics. This similar behavior is reasonable since the mixing parameter of the CV-FXLMS tends to zero at steady state, so the CV-FXLMS follows the V-FXLMS. However, although the nonlinear Volterra filter (NFXLMS) also reduces the first harmonic, it generates a second harmonic at 1500 Hz.

V. Conclusion

In this paper we discussed the equalization of room acoustic systems when the electroacustic system can produce linear

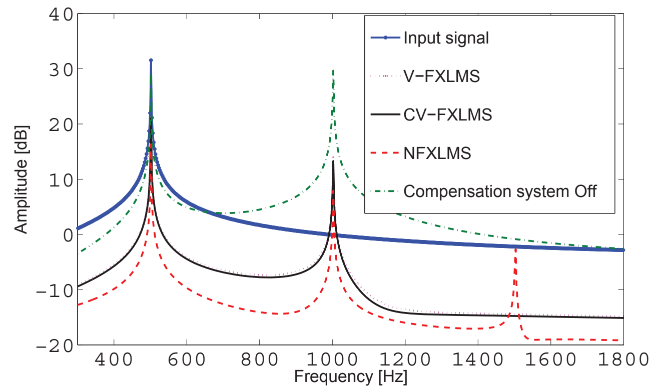


Fig. 8. Output signal obtained with the different adaptive filters for a single frequency input signal of 500 Hz

and nonlinear distortions. Two novel adaptive filters based on the virtual path concept have been introduced for linear room compensation in the presence of nonlinearities: the V-FXLMS and the CV-FXLMS. The V-FXLMS relies on the adaptation of a filtered-x linear filter but using the virtual path concept. The second approach, the CV-FXLMS, is based on the convex combination of two linear filters, the V-FXLMS and the conventional FXLMS. We show that both schemes, despite using linear filters, do not become unstable and can provide good performance in terms of linear distortions. Simulation results validate the proposed approaches.

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