# Cognitive MIMO Radars: An Information Theoretic Constrained Code Design Method

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***Abstract*—A novel information theoretic code design approach for cognitive multiple-input multiple-output (MIMO) radars is proposed in the current paper. In the suggested solution, we consider an agile multi-antenna radar with spectral cognition and design optimized transmission codes considering practical limitations such as peak-to-average power ratio (PAR) and spectral coexistence in a spectrally crowded environment. As exact performance expressions for the detector is not analytically tractable in this case, an *information theoretic* approach is taken into account to design the transmission codes. Simulation results illustrate the efficiency of the proposed method.**

## I. Introduction

Compared to single-antenna radars, multiple-input multiple-output (MIMO) radars have shown a better detection performance, more accurate estimation of target parameters, better resolution, better interference rejection, and more flexibility in generating radiation patterns [1]. Accordingly, transmit waveform (code) design plays an important role in determining the performance of such radars.

Waveform design in MIMO radar systems depends on several parameters such as mobility of targets, the effect of signal-dependent clutter, and practical limitations. Moreover, depending on the desired system, different design criteria might be used, e.g., criteria related to detection, estimation, and classification [1]. The practical limitations may include energy, peak-to-average power ratio (PAR), and coexistence constraint. As an important example, in many applications, the sought code is supposed to be constant-amplitude/PAR-limited. However, this fact is usually not considered [2] or is partially cared about. In other words, in several works, the code is designed considering no PAR constraint and then, a PAR-constrained code is synthesized from the unconstrained code leading to a significant performance loss (see e.g. [2, 3]).

Spectral coexistence is another important consideration in radar code design specially in spectrally-crowded environments, when a radar is trying to transmit waveforms in frequency bands (fully or partially) assigned to primary licensed applications; e.g., coexistence of the radar and communication systems. In such conditions, radar systems need to be equipped with cognition tools such as radio environmental map (to get spectrum cognition about primary users) and dynamic environmental database (to predict the actual scattering scenario of themselves). This constraint has been addressed in single-antenna radars as in [4]. During the past decade, several works have considered the code design for detection performance improvement in multi-antenna radars. However, simplifying assumptions have been made especially on the target Doppler shift, clutter nature, and practical limitations (see [1, 3, 5]). Due to complex expressions of detection performance metrics in MIMO radars (if any [6]), information theoretic criteria have been employed as design metrics for such systems as they are usually associated with some bounds on the performance [7].

In this paper, we consider the problem of transmit code design for an agile MIMO radar system in the presence of clutter. The aim is to improve the detection performance of a moving target while dealing with practical/implementation limitations. Moreover, the system is assumed to be cognitive and designed to be adaptive in order to be able to coexist with other radio applications functioning in the same frequency band. To this end, we employ an information-theoretic approach and cast the problem of code design via maximization of the information-theoretic criterion, namely, *mutual information*. To account for the most important practical/implementation limitations in the radar signal design literature, PAR and spectral coexistence constraints are imposed to the design problem. To the best of our knowledge, no information-theoretic code design methodology is addressed for the MIMO radar that directly deals with the constrained design problem. The rest of this paper is organized as follows. In Section II, the system model is introduced. The new information theoretic approach is presented in Section III where we cast and tackle the constrained code design problem. Numerical results and discussions are provided in Section IV.

## II. System Model

We consider a colocated MIMO radar system with $N_T$ transmitter and $N_R$ receiver antennas. Let $\boldsymbol{a}_m = [a_m(1), \dots, a_m(N)]^T \in \mathbb{C}^N$ denote the fast-time transmit code from the $m$th transmit antenna. Assuming a target located at the direction $\theta_0$ and $L$ clutter patches located at $\theta_1, \cdots, \theta_L$, the matrix $\boldsymbol{X} \in \mathbb{C}^{N \times N_R}$ of the received signal from the target

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at the cell under test can be expressed as

$$\boldsymbol{X} = \underbrace{\alpha_0(\boldsymbol{A}\odot\boldsymbol{P})\boldsymbol{\Psi}(\theta_0)}_{\boldsymbol{S}} + \underbrace{\boldsymbol{A}\sum_{l=1}^{L}\alpha_l\boldsymbol{\Psi}(\theta_l)}_{\boldsymbol{C}} + \boldsymbol{V}, \tag{1}$$

where $\boldsymbol{C}$ and $\boldsymbol{V}$ denote the clutter and signal-independent interference (receiver noise, jammer, interference of telecommunication systems, etc.) respectively. Herein, $\boldsymbol{\Psi}(\theta)$ is the steering matrix corresponding to the look angle $\theta$ given by $\boldsymbol{\Psi}(\theta) = \boldsymbol{s}_t(\theta)\boldsymbol{s}_r^T(\theta) \in \mathbb{C}^{N_T\times N_R}$ with $\boldsymbol{s}_r(\theta)$ and $\boldsymbol{s}_t(\theta)$ being the receive and transmit steering vectors, respectively. $\boldsymbol{A}\in\mathbb{C}^{N\times N_T}$ is the transmit code matrix with $\boldsymbol{A}_{n,m} = a_m(n)$. Also, the matrix $\boldsymbol{P} = [\boldsymbol{p}_d, ..., \boldsymbol{p}_d] \in \mathbb{C}^{N\times N_T}$ is the temporal steering matrix associated with target Doppler shift with $\boldsymbol{p}_d = [1, \exp(j2\pi f_d), ..., \exp(j2\pi f_d(N-1))]^T$, where $f_d$ is the non-negligible normalized Doppler shift associated with the moving target. Moreover, $\alpha_0$ and $\alpha_l$ denote the reflected coefficients associated with the radar cross section (RCS) as well as the propagation effects of the target and the $l$th interference source, respectively. Note that in model (1), for simplicity, we assume *sinc-shape* subpulses and neglect the effect of the clutter scatterers located at ranges other than that of the cell under test (see [4] for a similar assumption). However, the design methodology in this paper can be straightforwardly extended to account for the aforementioned scatterers or to be applied to a slow-time coding scenario [8].

The covariance matrix of the signal component $\boldsymbol{S}$ can be found using Kronecker properties as

$$\begin{aligned}\boldsymbol{R}_s =& \mathbb{E}\{\mathrm{vec}(\boldsymbol{S})\mathrm{vec}(\boldsymbol{S})^H\}\\ =&\mathbb{E}\Big\{\mathrm{vec}\Big(\alpha_0(\boldsymbol{A}\odot\boldsymbol{P})\boldsymbol{\Psi}(\theta_0)\Big)\mathrm{vec}\Big(\alpha_0(\boldsymbol{A}\odot\boldsymbol{P})\boldsymbol{\Psi}(\theta_0)\Big)^H\Big\}\\ =&(\boldsymbol{I}_{N_R}\otimes[\boldsymbol{A}\odot\boldsymbol{P}])\boldsymbol{T}\,(\boldsymbol{I}_{N_R}\otimes[\boldsymbol{A}\odot\boldsymbol{P}])^H,\end{aligned} \tag{2}$$

where $\boldsymbol{T} = \sigma_s^2\mathrm{vec}(\boldsymbol{\Psi}(\theta_0))\,\mathrm{vec}(\boldsymbol{\Psi}(\theta_0))^H \triangleq \boldsymbol{b}\boldsymbol{b}^H$ and $\sigma_s^2 = \mathbb{E}\{\alpha_0\alpha_0^*\}$. Following the same procedure, for the covariance matrix of the clutter component $\boldsymbol{C}$ we have

$$\boldsymbol{R}_c = \mathbb{E}\{\mathrm{vec}(\boldsymbol{C})\mathrm{vec}(\boldsymbol{C})^H\}\,(\boldsymbol{I}_{N_R}\otimes\boldsymbol{A})\,\boldsymbol{Q}\,(\boldsymbol{I}_{N_R}\otimes\boldsymbol{A})^H, \tag{3}$$

where $\boldsymbol{Q} = \sum_{l=1}^{L}\sigma_{c,l}^2\mathrm{vec}(\boldsymbol{\Psi}(\theta_l))\mathrm{vec}(\boldsymbol{\Psi}(\theta_l))^H$ and $\sigma_{c,l}^2 = \mathbb{E}\{\alpha_l\alpha_l^*\}$, and finally, $\boldsymbol{R}_{\boldsymbol{v}} = \mathbb{E}\{\mathrm{vec}(\boldsymbol{V})\mathrm{vec}(\boldsymbol{V})^H\}$.

## III. Information Theoretic Approach

The aim is to design the code matrix $\boldsymbol{A}$ to improve the detection performance of the Neyman-Pearson (NP) detector. In this section, we consider a cognitive MIMO radar system such that the second-order statistics of the target, clutter, and interference are known at the design stage. This can be fulfilled by using geological, meteorological, and previous scanned data in a cognitive setup [7].

### A. Problem Formulation

The detection problem associated with the model in (1) leads to the following binary hypothesis test

$$\begin{cases}\mathrm{H}_0: \boldsymbol{y}\sim\mathcal{CN}(\boldsymbol{0},\boldsymbol{I})\\ \mathrm{H}_1: \boldsymbol{y}\sim\mathcal{CN}(\boldsymbol{0},\boldsymbol{I}+\boldsymbol{F})\end{cases} \tag{4}$$

where $\boldsymbol{x} = \mathrm{vec}(\boldsymbol{X})$, $\boldsymbol{y} \triangleq \boldsymbol{D}^{-1/2}\boldsymbol{x}$ with $\boldsymbol{D} = \boldsymbol{R}_c + \boldsymbol{R}_{\boldsymbol{v}}$, and $\boldsymbol{F} = \boldsymbol{D}^{-\frac{1}{2}}\boldsymbol{R}_s\boldsymbol{D}^{-\frac{1}{2}}$. Note that the performance metrics of the NP detector associated with above problem are too complicated for using in code design (see e.g., [6]). In such cases, other design criteria referred to as *information-theoretic* criteria can be employed for transmission code design [5, 7]. Herein, we employ mutual information metric as the information theoretic criterion for code design. It has been shown that maximizing the mutual information ($\mathcal{M}$) between the received signal and the target response leads to a better detection performance in the case of Gaussian distributions (see e.g. [7] and references therein–see also Remark 2 below). Using the results of [9], the $\mathcal{M}$ metric associated with (4) becomes

$$\mathcal{M} = \log\det(\boldsymbol{I}+\boldsymbol{F}). \tag{5}$$

Thus, the code design problem considering criterion $\mathcal{M}$ can be cast as

$$\begin{aligned}\max_{\boldsymbol{A},\boldsymbol{F},\boldsymbol{D},\boldsymbol{R}_s}\quad & \log\det(\boldsymbol{I}+\boldsymbol{F})\\ \text{subject to}\quad & \boldsymbol{F} = \boldsymbol{D}^{-\frac{1}{2}}\boldsymbol{R}_s\boldsymbol{D}^{-\frac{1}{2}},\ \boldsymbol{A}\in\mathcal{C}.\end{aligned} \tag{6}$$

Here, $\mathcal{C}$ represents a typical constraint set on the code matrix.

### B. The Proposed Method

The problem in (6) is non-convex and belongs to a class of NP-hard problems in general. Note also that $\log\det(\boldsymbol{I}+\boldsymbol{F})$ is a concave function of $\boldsymbol{F}\in\mathcal{S}_+^{NN_R}$ where $\mathcal{S}_+^M$ denotes the positive semidefinite cone in $\mathbb{C}^{M\times M}$. We deal with the problem in (6) in two steps: first, the iterative technique majorization-minimization (MaMi) [10] is used to obtain a quadratically constrained quadratic program (QCQP) for each iteration of the MaMi procedure. Then, the QCQP is handled with PAR and coexistence constraints.

*1) Devising the QCQP:* We present the results of applying MaMi to the design problem in the following Theorem.

*Theorem 1:* A solution $\boldsymbol{A} = \boldsymbol{A}_\star$ to the problem in (6) can be obtained iteratively by solving the following QCQP in the $i$th iteration

$$\min_{\widetilde{\boldsymbol{a}}\in\widetilde{\mathcal{C}}}\quad \widetilde{\boldsymbol{a}}^H\boldsymbol{H}^{(i)}\widetilde{\boldsymbol{a}} + 2\Re\{(\boldsymbol{g}^{(i)})^H\widetilde{\boldsymbol{a}}\}, \tag{7}$$

where $\widetilde{\boldsymbol{a}} = \mathrm{vec}(\boldsymbol{A})$ and $\widetilde{\mathcal{C}}$ denotes the constraint set (associated with $\mathcal{C}$) imposed on $\widetilde{\boldsymbol{a}}$. The matrices $\{\boldsymbol{H}^{(i)}\}\succeq\boldsymbol{0}$ and the vectors $\{\boldsymbol{g}^{(i)}\}$ are given in the sequel. ■

*Proof.* We begin by noting that the covariance of the signal component $\boldsymbol{R}_s$ is rank-1; namely, $\boldsymbol{R}_s = \boldsymbol{k}\boldsymbol{k}^H$ with $\boldsymbol{k} = [\boldsymbol{I}_{N_R}\otimes(\boldsymbol{A}\odot\boldsymbol{P})]\,\boldsymbol{b}$ and $\boldsymbol{b} = \sigma_s\mathrm{vec}(\boldsymbol{\Psi}(\theta_0))$. Let $J = \boldsymbol{k}^H(\boldsymbol{R}_c+\boldsymbol{R}_{\boldsymbol{v}})^{-1}\boldsymbol{k}$ and observe that the objective function in (6) can be rewritten w.r.t. the positive scalar $J$. More precisely, using the determinant property we obtain

$$\begin{aligned}\log\det(\boldsymbol{I}+\boldsymbol{F}) &= \log\det\left(\boldsymbol{I}_{NN_R}+\boldsymbol{D}^{-\frac{1}{2}}\boldsymbol{R}_s\boldsymbol{D}^{-\frac{1}{2}}\right)\\ &= \log\det\left(\boldsymbol{I}_{NN_R}+\boldsymbol{D}^{-\frac{1}{2}}\boldsymbol{k}\boldsymbol{k}^H\boldsymbol{D}^{-\frac{1}{2}}\right)\\ &= \log\left(1+\boldsymbol{k}^H(\boldsymbol{R}_c+\boldsymbol{R}_{\boldsymbol{v}})^{-1}\boldsymbol{k}\right) = \log(1+J).\end{aligned} \tag{8}$$

*Remark 1:* In light of the above observation, we note that the objective function in (6) can be replaced with the scalar

$J$. Also, one may consider $J$ as a type of SINR at the output of the detector and observe that for the employed model of MIMO radars, code design via maximizing SINR is equivalent to that of maximizing the information theoretic criterion. Note also that SINR maximization has been addressed in the literature for MIMO radar code design but to the best of our knowledge, they are all based on a synthesis stage/suboptimal procedure to deal with constraints such as PAR.

*Remark 2:* It can be shown that the design problem associated with other information theoretic criteria, e.g., J and KL divergences as well as Bhattacharyya distance can be dealt with using Theorem 1. Indeed, the corresponding objective functions become monotonically increasing functions of the scalar $J$.

*Lemma 1:* [1] For the function $f_{\mathcal{M}}(\boldsymbol{F}) = \log\det(\boldsymbol{I}+\boldsymbol{F}) : \mathcal{S}_{+}^{NN_R} \to \mathbb{R}_{+}$, we have

$$\log\det(\boldsymbol{I}+\boldsymbol{F}) = \log(\boldsymbol{u}^H\boldsymbol{B}^{-1}\boldsymbol{u}), \tag{9}$$

where $\boldsymbol{u} = [1, \boldsymbol{0}^T_{NN_R\times 1}]^T$ and

$$\boldsymbol{B} = \begin{bmatrix} 1 & \boldsymbol{k}^H \\ \boldsymbol{k} & \boldsymbol{R}_s+\boldsymbol{R}_c+\boldsymbol{R}_{\boldsymbol{v}} \end{bmatrix} \in \mathcal{S}_{+}^{NN_R+1}. \tag{10}$$

Moreover, for any full-column rank matrix $\boldsymbol{U}$, $\log\det(\boldsymbol{U}^H\boldsymbol{B}^{-1}\boldsymbol{U})$ is convex over $\boldsymbol{B}\succ\boldsymbol{0}$. ■

Using Lemma 1, the solution to the problem in (6) can be found by considering the equivalent optimization

$$\begin{aligned} \max_{\boldsymbol{A},\boldsymbol{k},\boldsymbol{B}} \quad & \log(\boldsymbol{u}^H\boldsymbol{B}^{-1}\boldsymbol{u}) \\ \text{subject to} \quad & \boldsymbol{k} = [\boldsymbol{I}_{N_R}\otimes(\boldsymbol{A}\odot\boldsymbol{P})]\,\boldsymbol{b}, \quad \boldsymbol{A}\in\mathcal{C} \end{aligned} \tag{11}$$

According to Lemma 1, the objective function in (11) is convex w.r.t. $\boldsymbol{B}^{-1}$. More concretely, here, the matrix $\boldsymbol{U}$ is given by the vector $\boldsymbol{u}$ and determinant of the scalar $\boldsymbol{u}^H\boldsymbol{B}^{-1}\boldsymbol{u}$ is $\boldsymbol{u}^H\boldsymbol{B}^{-1}\boldsymbol{u}$. Therefore, this term can be minorized using its tangent plane at a given $\widetilde{\boldsymbol{B}}$ as

$$\log\left(\boldsymbol{u}^H\boldsymbol{B}^{-1}\boldsymbol{u}\right) \geq \log\left(\boldsymbol{u}^H\widetilde{\boldsymbol{B}}^{-1}\boldsymbol{u}\right) + \operatorname{tr}\left\{\widetilde{\boldsymbol{\Upsilon}}\big(\boldsymbol{B}-\widetilde{\boldsymbol{B}}\big)\right\},$$

where $\widetilde{\boldsymbol{\Upsilon}} = -\widetilde{\boldsymbol{B}}^{-1}\boldsymbol{u}\big(\boldsymbol{u}^H\widetilde{\boldsymbol{B}}^{-1}\boldsymbol{u}\big)^{-1}\boldsymbol{u}^H\widetilde{\boldsymbol{B}}^{-1} \succeq \boldsymbol{0}$. Moreover, the first term on the right hand side of above equation is constant for a given $\widetilde{\boldsymbol{B}}$. Thus, by defining $\boldsymbol{\Upsilon} = -\widetilde{\boldsymbol{\Upsilon}}$, the optimization problem for the $i$th iteration of MaMi can be handled via solving

$$\begin{aligned} \min_{\boldsymbol{A},\boldsymbol{k},\boldsymbol{B}_{\mathcal{M}}} \quad & \operatorname{tr}(\boldsymbol{\Upsilon}^{(i)}\boldsymbol{B}) \\ \text{subject to} \quad & \boldsymbol{k} = [\boldsymbol{I}_{N_R}\otimes(\boldsymbol{A}\odot\boldsymbol{P})]\,\boldsymbol{b}, \quad \boldsymbol{A}\in\mathcal{C} \end{aligned} \tag{12}$$

where $\boldsymbol{\Upsilon}^{(i)}$ denotes $\boldsymbol{\Upsilon}$ in the $i$th iteration. Let $\boldsymbol{\Upsilon} = \begin{bmatrix} v_{11} & \boldsymbol{v}_{12} \\ \boldsymbol{v}_{21} & \boldsymbol{\Upsilon}_{22} \end{bmatrix}$ with the same partitioning as that of $\boldsymbol{B}$ in (10). Therefore, by neglecting the constant terms and using hermitian property of $\boldsymbol{\Upsilon}$ and $\boldsymbol{R}_s$, the following equivalent problem is obtained for the above optimization

$$\begin{aligned} \min_{\boldsymbol{A},\boldsymbol{R}_s,\boldsymbol{R}_c,\boldsymbol{k}} \quad & 2\Re\big\{\boldsymbol{k}^H\boldsymbol{v}_{21}^{(i)}\big\} + \operatorname{tr}\left\{\boldsymbol{\Upsilon}_{22}^{(i)}(\boldsymbol{R}_{\boldsymbol{s}}+\boldsymbol{R}_{\boldsymbol{c}})\right\} \\ \text{subject to} \quad & \boldsymbol{R}_{\boldsymbol{c}} = (\boldsymbol{I}_{N_R}\otimes\boldsymbol{A})\,\boldsymbol{Q}\,(\boldsymbol{I}_{N_R}\otimes\boldsymbol{A})^H \\ & \boldsymbol{k} = [\boldsymbol{I}_{N_R}\otimes(\boldsymbol{A}\odot\boldsymbol{P})]\,\boldsymbol{b} \\ & \boldsymbol{R}_{\boldsymbol{s}} = \boldsymbol{k}\boldsymbol{k}^H, \quad \boldsymbol{A}\in\mathcal{C}. \end{aligned} \tag{13}$$

Now, letting $\boldsymbol{G} = \boldsymbol{b}\boldsymbol{v}_{21}^H$, $\boldsymbol{g} = \sum_{k=1}^{N_R}\operatorname{vec}\big(\boldsymbol{G}_{kk}{}^H\big)$, $\widetilde{\boldsymbol{g}} = (\boldsymbol{g}^H\odot\widetilde{\boldsymbol{p}})^H$, $\widetilde{\boldsymbol{p}} = \operatorname{vec}(\boldsymbol{P})$, and $\widetilde{\boldsymbol{a}} = \operatorname{vec}(\boldsymbol{A})$, where $\boldsymbol{G}_{kk}\in\mathbb{C}^{N\times N_T}$ is a submatrix of $\boldsymbol{G}$ containing rows $(k-1)N_T+1$ to $kN_T$ and columns $(k-1)N+1$ to $kN$, it is straightforward to verify

$$\Re\left\{\boldsymbol{k}^H\boldsymbol{v}_{21}\right\} = \Re\{\widetilde{\boldsymbol{g}}^H\widetilde{\boldsymbol{a}}\}, \tag{14}$$

$$\operatorname{tr}\left\{\boldsymbol{\Upsilon}_{22}(\boldsymbol{R}_s+\boldsymbol{R}_c)\right\} = \widetilde{\boldsymbol{a}}^H\boldsymbol{H}\,\widetilde{\boldsymbol{a}}, \tag{15}$$

where

$$\boldsymbol{H} = \sum_{l=1}^{N_R}\sum_{k=1}^{N_R}\left[(\boldsymbol{T}_{kl}^H\otimes\boldsymbol{\Upsilon}_{22,lk})\odot(\widetilde{\boldsymbol{p}}\widetilde{\boldsymbol{p}}^H)^T + \boldsymbol{Q}_{kl}^H\otimes\boldsymbol{\Upsilon}_{22,lk}\right],$$

in which $\boldsymbol{T}_{kl}$ and $\boldsymbol{Q}_{kl}$ are submatrices of $\boldsymbol{T}$ and $\boldsymbol{Q}$, respectively, containing rows $(k-1)N_T+1$ to $kN_T$ and columns $(l-1)N_T+1$ to $lN_T$, and $\boldsymbol{\Upsilon}_{22,lk}$ is a submatrix of $\boldsymbol{\Upsilon}_{22}$ containing rows $(l-1)N+1$ to $lN$ and columns $(k-1)N+1$ to $kN$. Replacing (14) and (15) in (13) concludes the proof of Theorem 1. □

*2) Solving the Derived QCQP:* We solve the derived QCQP in Section III-B1 with the following two constraints.

*a) PAR Constraint:* For the considered MIMO system, we impose the PAR constraint on the emitted signals as

$$\begin{aligned} \min_{\widetilde{\boldsymbol{a}}} \quad & \widetilde{\boldsymbol{a}}^H\boldsymbol{H}\widetilde{\boldsymbol{a}} + 2\,\Re\{\widetilde{\boldsymbol{g}}^H\widetilde{\boldsymbol{a}}\} \\ \text{subject to} \quad & \max_{n=1,\cdots,NN_T}|\widetilde{\boldsymbol{a}}(n)|^2 \leq \gamma,\ \|\widetilde{\boldsymbol{a}}\|_2^2 = NN_T \end{aligned} \tag{16}$$

with $\gamma$ being the PAR threshold. The QCQP in (16) is NP-hard in general [11]. However, (local) solutions to the problem can be obtained via iterative solving of the following *nearest-vector* optimization (under some mild conditions [10])

$$\begin{aligned} \min_{\widetilde{\boldsymbol{a}}^{(k+1)}} \quad & \|\widetilde{\boldsymbol{a}}^{(k+1)} - \boldsymbol{a}^{(k)}\|_2^2 \\ \text{subject to} \quad & \max_{n}|\widetilde{\boldsymbol{a}}^{(k+1)}(n)|^2 \leq \gamma,\ \|\widetilde{\boldsymbol{a}}^{(k+1)}\|_2^2 = NN_T \end{aligned} \tag{17}$$

where $\boldsymbol{a}^{(k)}$ represents the vector containing the first $NN_T$ entries of $\widetilde{\boldsymbol{H}}\hat{\boldsymbol{a}}^{(k)}$ with $\widetilde{\boldsymbol{H}} = \mu\boldsymbol{I}_{N+1} - \boldsymbol{\mathcal{K}}$ for any $\mu > \lambda_{\max}(\boldsymbol{\mathcal{K}})$, $\boldsymbol{\mathcal{K}} = \begin{bmatrix} \boldsymbol{H} & \widetilde{\boldsymbol{g}} \\ \widetilde{\boldsymbol{g}}^H & 0 \end{bmatrix}$, and $\hat{\boldsymbol{a}} = [\widetilde{\boldsymbol{a}}^T\ 1]^T$. The results of [12] can be used for solving (17) (see [5]).

*b) Spectral Coexistence Constraint:* In some applications, radars need to function in coexistence with other legacy radio users, i.e., using the same frequency band in an underlay paradigm. That is, the energy emitted by the radar has to remain under a certain threshold such that the intra-channel interference caused to the legacy user meets a given bound. This is an important feature of cognitive MIMO radar systems when operating in spectrally-crowded environments.

[1]Due to limitation of space, here we omit proof of the lemma.

This requirement for the single-antenna case can be managed by imposing the following constraint (see [4] for a similar argument)

$$\boldsymbol{a}^H \boldsymbol{R} \boldsymbol{a} \leq \tilde{\mathcal{E}}, \quad (18)$$

where $\boldsymbol{a}$ is the code, $\tilde{\mathcal{E}}$ is a given energy level, and

$$\{\boldsymbol{R}\}_{m,n} = \begin{cases} f_2 - f_1, & \text{if } m = n \\ \frac{e^{j2\pi f_2 (m-n)} - e^{j2\pi f_1 (m-n)}}{j2\pi (m-n)}, & \text{otherwise} \end{cases} \quad (19)$$

with $[f_1, f_2]$ being the frequency band granted to the primary licensed application and to be protected. The constraint in (18) can be generalized to the multi-antenna case as

$$\sum_{k=1}^{N_T} \boldsymbol{a}_k^H \boldsymbol{R} \boldsymbol{a}_k = \boldsymbol{a}_k^H (\boldsymbol{I}_{N_T} \otimes \boldsymbol{R}) \boldsymbol{a}_k \leq \mathcal{E}, \quad (20)$$

with $\mathcal{E}$ being the maximum emitted energy allowed in $[f_1, f_2]$.

*Remark 3:* Note that the transmitted energy by the multiple-antenna radar over $[f_1, f_2]$ is related to (20) and hence can be controlled by it. However, the energy depends on the correlation among the signals transmitted by different antennas as well as energy of the basic pulse of the radar system. Moreover, it is straightforward to extend the above formulation to the case where there are more than a single frequency band.

According to the above discussions, the optimization problem in the $i$th iteration of MaMi with coexistence constraint can be cast as

$$\begin{aligned} \min_{\widetilde{\boldsymbol{a}}} \quad & \widetilde{\boldsymbol{a}}^H \boldsymbol{H} \widetilde{\boldsymbol{a}} + 2\,\Re\{\widetilde{\boldsymbol{g}}^H \widetilde{\boldsymbol{a}}\} \\ \text{subject to} \quad & \widetilde{\boldsymbol{a}}^H (\boldsymbol{I}_{N_T} \otimes \boldsymbol{R}) \widetilde{\boldsymbol{a}} \leq \mathcal{E}, \ \|\widetilde{\boldsymbol{a}}\|_2^2 = e_s, \end{aligned} \quad (21)$$

where $e_s$ is the total transmitted energy. This QCQP has a hidden convexity and we present its semidefinite programming (SDP) form in the following. We first let

$$\boldsymbol{Y} = \begin{bmatrix} \widetilde{\boldsymbol{a}}\widetilde{\boldsymbol{a}}^H & \widetilde{\boldsymbol{a}} t^* \\ \widetilde{\boldsymbol{a}}^H t & |t|^2 \end{bmatrix} \in \mathbb{C}^{N+1}, \quad (22)$$

where $t$ is an auxiliary variable. Thus, (21) is equivalent to

$$\begin{aligned} \min_{\boldsymbol{Y}} \quad & \operatorname{tr}\left\{ \begin{pmatrix} \boldsymbol{H} & \widetilde{\boldsymbol{g}} \\ \widetilde{\boldsymbol{g}}^H & 0 \end{pmatrix} \boldsymbol{Y} \right\} \\ \text{subject to} \quad & \operatorname{tr}\left\{ \begin{pmatrix} \boldsymbol{R} & 0 \\ 0 & 0 \end{pmatrix} \boldsymbol{Y} \right\} \leq \mathcal{E} \\ & \operatorname{tr}\left\{ \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \boldsymbol{Y} \right\} = 1 \\ & \operatorname{tr}\left\{ \begin{pmatrix} \boldsymbol{I} & 0 \\ 0 & 0 \end{pmatrix} \boldsymbol{Y} \right\} = e_s. \end{aligned} \quad (23)$$

A rank-1 solution $\boldsymbol{Y}_\star$ can be obtained for this SDP according to [13] as

$$\boldsymbol{Y}_\star = \begin{bmatrix} \hat{\widetilde{\boldsymbol{a}}} \\ t \end{bmatrix} \begin{bmatrix} \hat{\widetilde{\boldsymbol{a}}} \\ t \end{bmatrix}^H. \quad (24)$$

Therefore, the solution to (21) is obtained by $\widetilde{\boldsymbol{a}}_\star = \hat{\widetilde{\boldsymbol{a}}}/t$. Algorithm 1 summarises the proposed constrained code design method.

**Algorithm 1** The Proposed Method for the Constrained Code Design Method Using the Mutual Information Metric

1: Initialize $\widetilde{\boldsymbol{a}}$ with a random vector in $\mathbb{C}^{NN_T}$ and set the iteration number $i$ to 0.
2: Solve the QCQP of Theorem 1 to obtain $\widetilde{\boldsymbol{a}}^{(i+1)}$; set $i \leftarrow i+1$.
3: Update the parameters of the objective function in the QCQP of step 2.
4: Repeat steps 2 and 3 until a pre-defined stop criterion is satisfied.

## IV. Numerical Examples and Discussions

In this section, we provide numerical simulations to evaluate the performance of the proposed code design algorithm using information theoretic approach for MIMO radar systems. In the simulation setup, a MIMO radar with $N_T = 5$ transmit antennas, $N_R = 3$ receive antennas, and a code length of $N = 11$ is considered. As for the constraints, the transmit energy $e_s$ is supposed to be equal to 1. Moreover, the phase code design is addressed meaning that the PAR constraint with $\gamma = 1$ is imposed. A colored noise with the exponential correlation shape is considered for which the $(m, n)$th entry of the covariance matrix is $\sigma_n^2 \rho^{|m-n|}$ with $\sigma_n^2 = 1, \rho = 0.5$. As for the coexistence constraint, we set $\mathcal{E} = 10^{-7}$ and $[f_1, f_2] = [0.2, 0.3]$. In order to solve the convex optimization problems, the `cvx` toolbox [14] is employed.

We let the target at the angle $\theta_0 = 25$ degrees with a normalized Doppler shift of $f_D = 0.1$; also, the number of interfering clutter patches is supposed to be $L = 7$ around $\theta_0$, viz. $\{\theta_l\} = \{22, 23, 24, 25, 26, 27, 28\}$. A homogenous clutter environment is dealt with herein implying that $\sigma_{c,1} = \sigma_{c,2} \cdots = \sigma_{c,L}$. For this case, signal to noise ratio and clutter to noise ratio are defined as $\text{SNR} = \frac{\sigma_s^2}{\sigma_n^2} e_s$ and $\text{CNR} = \frac{\sigma_{c,l}^2}{\sigma_n^2} e_s$. Fig. 1 shows the value of the mutual information metric vs. the number of iterations for SNR = 0 dB and CNR = 0 dB under different constraints. The value of the metric increases by increasing the number of iterations until the improvement in the successive iterations reaches the predefined threshold ($10^{-3}$). In the case of PAR and coexistence constraints, smaller values are observed for the metric compared to the case of energy constraint which is due to the smaller feasibility region. A minor performance degradation can be seen for the phase-code design which highlights the effectiveness of the design methodology; this can be justified considering the fact that the proposed method directly deals with the constraint instead of employing a suboptimal synthesis procedure (see e.g., [3]).

Next, we illustrate the detection performance associated with the designed code via the proposed method. To this end, the receiver operating characteristic (ROC) of the optimal detector associated with (4) is considered. Fig. 2 shows the ROC for the detector when the system employs the designed code based on the mutual information metric. The ROC is obtained numerically using the results of [6]. The figure also includes the performance of the systems employing random

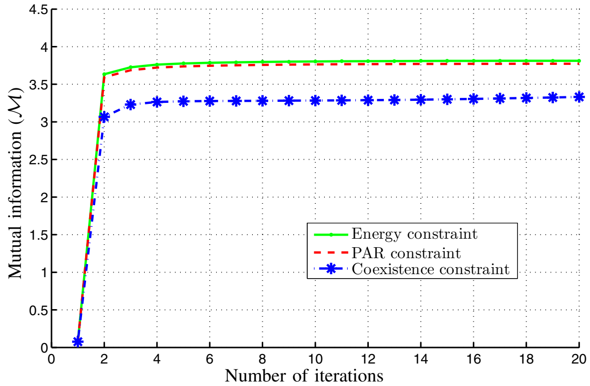


Fig. 1. The values of the mutual information versus the number of iterations for the proposed constrained code design method.

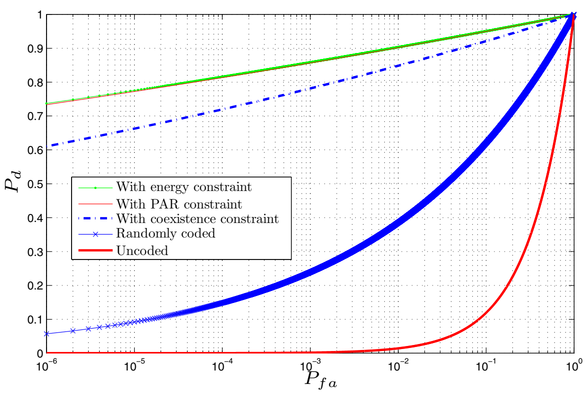


Fig. 2. ROC of the optimal detector for the designed code via mutual information criterion.

coding (with i.i.d. Gaussian elements for the code) as well as all-one codes (an uncoded system with a scaled version of $\boldsymbol{A} = \mathbf{1}$) at the transmitter side. A significant performance improvement can be observed for the system that employs the designed code compared to the randomly coded and uncoded systems. Similar to Fig. 1, the loss in the detection probability is negligible considering the phase-code design.

In order to evaluate the performance of the proposed method under the coexistence constraint, energy spectral density (ESD) of the resulting code matrix is illustrated in Fig. 3. Here, ESD is estimated as

$$\mathrm{ESD}(k) = \left| \sum_{m=1}^{N_T} \sum_{n=0}^{N-1} a_m(n) e^{-j2\pi kn/N} \right|^2, \qquad (25)$$

where $k$ denotes the discrete sampled frequency. As it can be seen from the figure, imposing the constraint in (20) reduces the spectrum by about 70 dB in the rejection band of $[f_1, f_2]$, whereas there is a significant energy emission in this frequency band when only PAR or energy constraint is considered.

We herein remark on the fact that in this paper the cognitive radar with a perfect a priori knowledge was assumed for the target and interference; however, in practice there might be

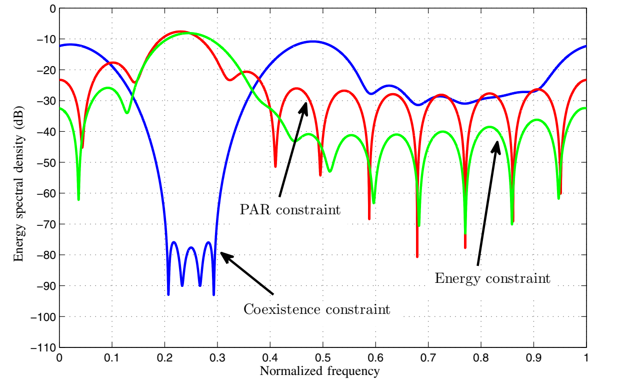


Fig. 3. Energy spectral density of the designed code using the proposed method under different constraints.

some inaccuracies in the statistics of the target/interfernce. Therefore, a robust design w.r.t. the mentioned inaccuracies can be a topic for future researches.

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