WORST-CASE JAMMING SIGNAL DESIGN AND AVOIDANCE FOR MIMO RADARS

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ABSTRACT

We optimize the jamming signal for disrupting the operation of a MIMO radar system in order to understand the threat jamming poses to such systems. The jamming signal optimization is formulated as a minimax problem minimizing the maximum SINR that the receivers can achieve, resulting in a semidefinite program for a Toeplitz jamming covariance matrix or a second-order cone program for a circulant approximation. In the simplest case of optimizing the average SINR of a single receiver, a waterfilling-type solution is obtained. Numerical studies suggest that distributed radar systems with waveform agility and mismatched filtering capabilities are resilient against jamming.

***Index Terms*—** MIMO radar, jamming, convex optimization, waterfilling, anti-jamming, interference mitigation, mismatched filtering

1. INTRODUCTION

Radars commonly need to operate in environments with hostile jamming. Fully adaptive radars can change their waveforms, receive filters, and beampatterns depending on jamming and target scenarios and radar channel conditions. In order to understand the potential threat of different jamming signals and to develop countermeasures to such signals, we consider optimizing a jamming signal to effectively disrupt the receivers. Knowing the optimal jamming signal design will help the MIMO radar receivers to protect themselves from the worst-case jamming scenarios.

In a simple case, the jamming signal has to be designed for a single known waveform and a particular receiver structure including the receive filter, which is typically the matched filter. However, the design is more demanding if multiple waveforms are launched from the radar transmitter simultaneously, as in the case of MIMO radars, and an unknown receive filter structure may be employed at the victim receiver.

Optimizing the jamming signal and designing the transmitted radar waveforms as a countermeasure to the jamming was considered in [1] from a game-theoretic perspective. In this paper, we use the minimax criterion in the jamming signal design. Given that the jammer does not know the exact filtering scheme the receiver is using, we take the approach of minimizing the maximum signal to jamming plus noise ratio that the receiver can achieve. A very important and broad class of receive filters, i.e. mismatched filters are considered at the MIMO radar receiver. Mismatched filters acknowledge the fact that there are no known radar waveforms that are orthogonal for all time delays and Doppler shifts. Mismatched filters are highly effective in minimizing the output interference power while maintaining desired autocorrelation sidelobe and cross-correlation levels [2].

The contributions of this paper are the following. We propose a jamming signal design such that the maximum SINR is minimized at the victim radar receivers. The goal is to find the covariance matrix of the optimal jamming signal, as the jamming signal is a random process. We show that the optimal covariance matrix of the jamming signal is found by solving a complex-valued, convex semidefinite program. The jamming signal is assumed to be quasi-stationary (stationary over the observation period) process in the wide sense so the covariance matrix has a Toeplitz structure. The Toeplitz structure of the covariance matrix and the idea of approximating large Toeplitz matrices by circulant matrices are exploited in the optimization. If the radar receiver characteristics are not known or cannot be estimated reliably, we propose to minimize the average SINR instead of finding a minimax solution. Performing the jamming signal optimization in the spectral domain, a waterfilling-type solution [3] is obtained in a such case. Only single target that the jammer is cooperating with is assumed, but the proposed design methods can be easily extended for multiple targets.

The implications of the jamming signal design on the jamming avoidance are studied in simulations. If a large number of codes are jammed simultaneously, the off-diagonal elements of jamming covariance matrix become small and jamming performance approaches that of white noise.

The rest of the paper is structured as follows. The optimal jamming signal design is formulated in Section 2 and numerical results are provided in Section 3. The concluding remarks are presented in Section 4.

2. JAMMING SIGNAL DESIGN

In this paper, we optimize a signal for jamming a radar system that is using mismatched filters to minimizing the output interference power. The goal of the jamming signal design is to minimize the SINR at the victim radar receiver. Knowing the optimal jamming signal design will help the MIMO radar receivers to protect themselves from the worst-case jamming scenarios.

We consider a radar system with K transmitters and N receivers. The waveform transmitted from the transmitter k is denoted by a $1 \times L$ vector $\mathbf{s}(m)$, where m is the propagation delay. We assume fast-time coding, so Doppler shift within each waveform is negligible. The received signal can then be expressed as

$$r_n(m) = \sum_{k=1}^{K} \sqrt{\sigma^2_{s,k,n}}\, s_k(m - \tau_{k,n}) + \boldsymbol{\nu}_n(m), \qquad (1)$$

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where the signal power for the of the waveform k is $\sigma^2_{s,k,n}$, $\tau_{k,n}$ is the propagation delay between transmitter k and receiver n, and $\boldsymbol{\nu}_n(m)$ is a noise and interference vector containing the jamming.

It was shown in [2] that the receiver can mitigate non-white interference very effectively using mismatched filters. The interference can be attenuated with only a small loss in the sidelobe and cross-correlation levels, so it is sensible for the receivers to use the mismatched filters as a countermeasure against jamming.

It is assumed that the jammer attempting to interfere with the operation of the receivers knows which waveforms the transmitters are using. A jammer could transmit a copy of the used waveform to interfere with the operation of the receivers, but upon detection of the jamming at the receiver, a waveform-agile transmitter could switch the used waveform. The receiver would then only need to place additional constraints for the mismatched filter in order to attenuate the previous waveform that the jammer is still using. The same principle holds for any deterministic signal. For this reason, the jammer should use a random signal in order to be effective against countermeasures.

Typically, the objective of the jammer is to reduce the signal to interference plus noise ratio (SINR) at the receiver so that target detection and tracking are disrupted. Denote the mismatched filter coefficient vector with a $1 \times L$ vector $\mathbf{w}_{k,n}$. The SINR for the waveform k received at the receiver n is given by

$$\text{SINR}_{k,n} = \frac{\sigma^2_{s,k,n} |\mathbf{w}^H_{k,n}\mathbf{s}_k(0)|^2}{\mathbf{w}^H_{k,n}\mathbf{R}_{\boldsymbol{\nu},n}\mathbf{w}_{k,n}}, \tag{2}$$

where $\mathbf{R}_{\boldsymbol{\nu},n}$ is the covariance matrix of noise plus interference, including the jamming.

The problem with the minimization of the SINR given in (2) is that the jammer has no knowledge of the filters $\mathbf{w}_{k,n}$ that the receivers are using. Therefore, we take the approach of minimizing the maximum SINR that the receivers can achieve.

The filter design minimizing the interference and noise power at the filter output can be written as a optimization problem [4]

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{\boldsymbol{\nu},n}\mathbf{w} \tag{3a}$$

$$\text{s.t } \mathbf{w}^H \mathbf{s}_k(0) = 1, \tag{3b}$$

The solution to this problem,

$$\mathbf{w}_{k,n} = \frac{\mathbf{R}^{-1}_{\boldsymbol{\nu},n}\mathbf{s}_k(0)}{\mathbf{s}^H_k(0)\mathbf{R}^{-1}_{\boldsymbol{\nu},n}\mathbf{s}_k(0)}, \tag{4}$$

is the well-known MVDR beamformer [5, 6]. Substituting (4) into (2) yields

$$\text{SINR}_{k,n} = \sigma^2_{s,k,n}\mathbf{s}^H_k(0)\mathbf{R}_{\boldsymbol{\nu}}^{-1}\mathbf{s}_k(0), \tag{5}$$

which can be then used in the jamming signal design.

To be precise, we want to find the covariance matrix of the optimal jamming signal, as the jamming signal is considered to be a random process; a deterministic jamming signal would be significantly easier to mitigate. The jamming signal is assumed to be quasi-stationary (stationary over the observation period) process in the wide sense so the covariance matrix has a Toeplitz structure. Thus, optimizing the jamming signal can be formulated as finding a positive-semidefinite (PSD) Toeplitz matrix $\mathbf{R}_j$ that minimizes the maximum SINR in (5) over all transmitters k and receivers n.

We can assume without loss of generality that the jammer has unit power. The noise and interference covariance matrix at the receiver can then be written as $\sigma^2_{j,n}\mathbf{R}_j + \mathbf{R}_{\boldsymbol{\nu},n}$, where $\sigma^2_{j,n}$ is the jamming signal power at the receiver n, and $\mathbf{R}_{\boldsymbol{\nu},n}$ contains now the noise and interference components other than jamming. The objective function for the optimization is thus

$$\max_{k,n} \sigma^2_{s,k,n}\mathbf{s}^H_k(0)(\sigma^2_{j,n}\mathbf{R}_j + \mathbf{R}_{\boldsymbol{\nu},n})^{-1}\mathbf{s}_k(0). \tag{6}$$

The problem of designing the jamming covariance matrix can be expressed in the epigraph form as

$$\min t \tag{7a}$$

$$\text{s.t } \sigma^2_{s,k,n}\mathbf{s}^H_k(0)(\sigma^2_{j,n}\mathbf{R}_j + \mathbf{R}_{\boldsymbol{\nu},n})^{-1}\mathbf{s}_k(0) \le t, \forall k, n \tag{7b}$$

where t is now the maximum SINR over all k and n. In addition, $\mathbf{R}_j$ has to be constrained to be a positive-semidefinite Toeplitz matrix with diagonal elements equal to one due to the power constraint for the jammer.

Using the Schur complement [7], we can rewrite the jamming signal design problem as a complex-valued semidefinite program (SDP)

$$\min t \tag{8a}$$

$$\text{s.t } \begin{bmatrix} \sigma^2_{j,n}\mathbf{R}_j + \mathbf{R}_{\boldsymbol{\nu},n} & \sigma_{s,k,n}\mathbf{s}_k(0) \\ \sigma_{s,k,n}\mathbf{s}^H_k(0) & t \end{bmatrix} \succeq 0, \quad \forall k, n \tag{8b}$$

$$(\mathbf{R}_j)_{ii} = 1, \quad i = 1, \ldots, L \tag{8c}$$

$$\mathbf{R}_j \in \mathcal{T}_+ \tag{8d}$$

where $\mathcal{T}_+$ is the set of positive-semidefinite Toeplitz matrices. Problem (8) is a convex problem, so a global optimum can be found efficiently in polynomial time.

In practice, the jammer would not know $\sigma_{j,n}$, $\sigma_{s,k,n}$, or $\mathbf{R}_{\boldsymbol{\nu},n}$, but would have to use estimates based on the knowledge of the radar system and its setup. For example, if the receiver locations are known, $\sigma_{j,n}$ and $\sigma_{s,k,n}$ can be estimated using signal power measurements and knowledge of the target RCS profile. (Since the jammer cooperating with the target, knowledge of the target RCS can be assumed.) Then only $\mathbf{R}_{\boldsymbol{\nu},n}$ would be unknown. If intelligence of the receiver hardware is available, the jammer might, for example, have knowledge of typical noise power at the receiver. In the absence of better information, the jammer may have to assume $\sigma_{j,n} = \sigma_{s,k,n} = 1$ and white noise with a low power.

The number of $(L + 1) \times (L + 1)$ positive-semidefinite matrix variables needed in the optimization problem (8) is KN. This might lead to a computational complexity that is too high for a practical implementation. One way to reduce the complexity of the optimization is assuming that $\sigma_{j,n}$, $\sigma_{s,k,n}$, and $\mathbf{R}_{\boldsymbol{\nu},n}$ are equal for all receivers, which would reduce the complexity by a factor of N. This would be the case when the receiver locations are unknown. However, the computational complexity might still be prohibitively high, particularly if L is large.

Another way to reduce the computational complexity significantly is to assume that the covariance matrices are circulant. If the dimension of the waveform vector is large, the covariance matrix $\mathbf{R}_{\boldsymbol{\nu}}$ is a large Toeplitz matrix that can be approximated well with a circulant matrix. Furthermore, if the vector $\mathbf{s}_k(0)$ is zero-padded, it has the finite-term structure defined in [8]. In order to prove the convergence of the result with a circulant $\mathbf{R}_j$ to the Toeplitz case based on the results of [8], however, we would first need to solve the SDP in (8) to obtain the Toeplitz $\mathbf{R}_j$ for the particular setup.

Assuming the circulant covariance matrices and using the fact that any circulant matrix can be diagonalized using a discrete Fourier transform (DFT) matrix, the SINR at the receiver can be written as

$$\begin{aligned}\mathrm{SINR}_{k,n} &= \sigma_{s,k,n}^2 \mathbf{s}_k^H(0)(\sigma_{j,n}^2 \mathbf{R}_j + \mathbf{R}_{\boldsymbol{\nu},n})^{-1}\mathbf{s}_k(0) \\ &= \sigma_{s,k,n}^2 \mathbf{s}_k^H(0)(\sigma_{j,n}^2 \mathbf{F}\boldsymbol{\Lambda}\mathbf{F}^H + \mathbf{F}\boldsymbol{\Xi}_n\mathbf{F}^H)^{-1}\mathbf{s}_k(0) \\ &= \sigma_{s,k,n}^2 \mathbf{s}_k^H(0)\mathbf{F}(\sigma_{j,n}^2 \boldsymbol{\Lambda} + \boldsymbol{\Xi}_n)^{-1}\mathbf{F}^H\mathbf{s}_k(0) \\ &= \sigma_{s,k,n}^2 \sum_{i=1}^{L} \frac{|(\tilde{\mathbf{s}}_k)_i|^2}{\sigma_{j,n}^2\lambda_i + \xi_{n,i}},\end{aligned} \tag{9}$$

where $\mathbf{F}$ is the $L \times L$ DFT matrix, $\boldsymbol{\Lambda}$ is the diagonal matrix of eigenvalues λ_i of $\mathbf{R}_j$, $\boldsymbol{\Xi}$ is the diagonal matrix of eigenvalues $\xi_{n,i}$ of $\mathbf{R}_{\boldsymbol{\nu},n}$, and $\tilde{\mathbf{s}}$ is the DFT of $\mathbf{s}_k(0)$. The optimization is thus done in the spectral domain.

Using (9), the optimization of the jamming covariance matrix can be written as

$$\min\ t \tag{10a}$$

$$\text{s.t.} \sum_{i=1}^{L} \frac{\sigma_{s,k,n}^2 |(\tilde{\mathbf{s}}_k)_i|^2}{\sigma_{j,n}^2\lambda_i + \xi_{n,i}} \le t \quad \forall k, n \tag{10b}$$

$$\sum_{i=1}^{L} \lambda_i = L \tag{10c}$$

$$\lambda_i \ge 0, \tag{10d}$$

where (10c) results from the main diagonal of $\mathbf{R}_j$ consisting of ones and (10d) from the positive-semidefiniteness requirement. This problem can be formulated as a second-order cone program, which is convex. The computational complexity is considerably smaller than for the SDP problem (8). Naturally, jamming is likely to be less efficient using the circulant covariance matrix compared to the optimal Toeplitz matrix due to the smaller number of degrees of freedom.

If many of the receiver characteristics cannot be estimated, it might be more sensible to minimize the average SINR instead of the maximum. This also leads to further simplification of the optimization problem. Assuming circulant $\mathbf{R}_j$, white noise with a variance σ_ν^2, as well as equal signal, jamming, and noise powers at all the receivers (or alternatively, only a single receiver), the average SINR can be written as

$$\frac{1}{K}\sum_{k=1}^{K}\sum_{i=1}^{L} \frac{\sigma_{s,k}^2 |(\tilde{\mathbf{s}}_k)_i|^2}{\sigma_j^2\lambda_i + \sigma_\nu^2} = \sum_{i=1}^{L} \frac{d_i^2}{u_i^2 + \rho}, \tag{11}$$

where $u_i^2 = \lambda_i$, $\rho = \sigma_\nu^2/\sigma_j^2$ is the inverse of the jamming to noise ratio, and

$$d_i = \frac{1}{K}\sum_{k=1}^{K} \frac{\sigma_{s,k}^2}{\sigma_j^2} |(\tilde{\mathbf{s}}_k)_i|^2. \tag{12}$$

The minimization of the average SINR can thus be written as

$$\min \sum_{i=1}^{L} \frac{d_i^2}{\rho + u_i^2} \qquad \text{s.t.} \qquad \sum_{i=1}^{L} u_i^2 = L, \tag{13}$$

where $u_i^2 = \lambda_i$ will guarantee a positive-semidefinite $\mathbf{R}_{\boldsymbol{\nu}}$.

The Lagrangian of (13) is

$$\mathcal{L} = \sum_{i=1}^{L} \frac{d_i^2}{\rho + u_i^2} + \mu\Big(\sum_{i=1}^{L} u_i^2 - L\Big). \tag{14}$$

Setting the derivative of $\mathcal{L}$ with respect to u_k equal to zero yields

$$-2u_k \frac{d_k^2}{(\rho + u_k^2)^2} + 2\mu u_k = 0 \tag{15}$$

so we obtain five solutions $u_k = 0, \pm\big(\pm\mu^{-1/2}d_k - \rho\big)^{1/2}$. Since u_k^2 must be nonnegative and it appears in the denominator, a minimum is obtained with

$$u_k = \begin{cases} \pm\big(\mu^{-1/2}d_k - \rho\big)^{1/2}, & \mu^{-1/2}d_k > \rho \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Since the actual eigenvalue is u_k squared, the negative solution can be ignored. Therefore, a general waterfilling solution [3] with single water level, $\lambda_k = \max\big(0, \mu^{-1/2}d_k - \rho\big)$, is obtained.

It is necessary to find the correct water level, which corresponds to identifying those λ_i that are equal to zero. Sorting the coefficients d_i in ascending order, the eigenvalues λ_i will also be in ascending order. There are then L possible choices for the water level. Selecting the k smallest eigenvalues to be zero, the constraint equation of (13) yields

$$\sum_{i=k+1}^{L} (\mu^{-1/2}d_i - \rho) = L \Rightarrow \mu^{-1/2} = \frac{L + (L-k)\rho}{\sum_{i=k+1}^{L} d_i}. \tag{17}$$

If then $\mu^{-1/2}d_{k+1} < \rho$, k was chosen to be too small. Otherwise, k can be increased. Therefore, it is possible to use binary search to find the value of k that gives the correct water level. The complexity of sorting d_i's is $O(L\log_2 L)$, the complexity of the binary search is $O(log_2 L)$, and finally, checking the validity of the water level is an operation with a complexity of $O(L)$. The total complexity of the algorithm is thus $O(L\log_2 L)$.

3. NUMERICAL EXAMPLES

Numerical examples of a jamming signal optimization and countermeasures are provided in this section.

The jamming signal was optimized to minimize the signal to jamming plus noise ratio (SJNR) of a MIMO radar system employing the Oppermann codes. The Oppermann codes are given by [9]

$$s_k(m) = (-1)^{km} \exp\left(\frac{\mathrm{j}\pi(k^a m^b + m^c)}{N}\right), \tag{18}$$

where j is the imaginary unit, N is the sequence length, k is the sequence number, m is the symbol index, and a, b, as well as c are design parameters.

The transmitted waveforms in this example were the codes with the parameter values $N = 61$, $k = 1, \ldots, 10$, $a = 2$, $b = 3$, and $c = 3$. The JNR was assumed to be 20 dB at the receiver and unknown to the jammer, whereas SNR was equal to 5 dB. The jamming signal was optimized to target one, two, or four first sequences in the Oppermann code set.

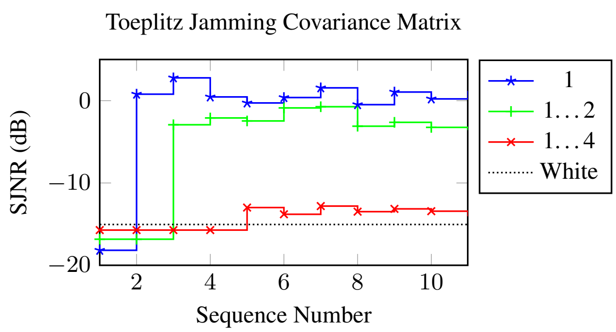
Table 1 shows the matched filter and the MVDR SJNR at the filter output for various jamming signal designs in dB, when the first code sequence was targeted with the jamming. The MVDR can easily attenuate the jamming designed against the matched filter. When the jamming is designed to minimize the SJNR of the MVDR, the MVDR is naturally ineffective. However, the difference to jamming employing white noise is only a few dB. Having more degrees of freedom, the Toeplitz covariance matrix is typically few dB better that the circulant one.

SJNR of the MVDR filter output for the different codes is shown in Fig. 1 for both a Toeplitz and a circulant covariance matrix. As a Toeplitz matrix has more degrees of freedom than a circulant matrix, lower SJNR is achieved with the Toeplitz solution. It can be seen that targeting specific sequence leads to low SJNR for that sequence,

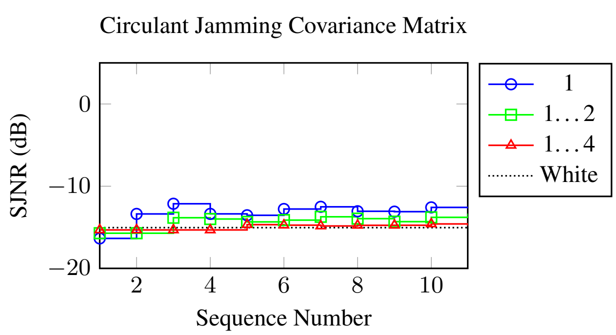
Table 1. Filter Output SJNR

	Circ. vs. Matched	Toepl. vs. Matched	Circ. vs. MVDR	Toepl. vs. MVDR	White Noise
Matched	-21.0	-21.7	-17.0	-18.6	-15.0
MVDR	4.7	4.7	-16.2	-18.2	-15.0

Comparison of matched filter and MVDR output SJNR with various jamming signal designs in dB. The MVDR can easily mitigate the jamming designed against matched filter, but not the worst case jamming signal.



(a)



(b)

Fig. 1. Signal to jamming plus noise ratio of MVDR output for the receiver filters when the jamming covariance matrix is (a) Toeplitz and (b) circulant. The Toeplitz matrix decreases SJNR more due to more degrees of freedom compared to the circulant solution. The jamming performance approaches that of white noise as the number of sequences increases.

whereas the jamming does not affect the other sequence as much. As the number of targeted codes increases, the jamming performance approaches that of white noise.

The reason for this can be seen in Fig. 2, which shows Frobenius norm of the difference of the trace-normalized circulant jamming signal covariance matrix $\mathbf{R}_j$ and an $L \times L$ identity matrix, i.e. $\left\|\frac{1}{\mathrm{tr}(\mathbf{R}_j)}\mathbf{R}_j - \frac{1}{L}\mathbf{I}\right\|_F$. When $\mathbf{R}_j$ is a scaled identity matrix, the norm is equal to zero. As the number of sequences being jammed increases, the covariance matrix resembles more and more the covariance matrix of uncorrelated noise. This means that the jamming performance will also approach that of uncorrelated noise.

The provided examples demonstrated that mismatched filtering can be effectively used to suppress jamming. As a further countermeasure against jamming, the transmitters can frequently switch the waveforms that they transmit. This forces the jammer to target

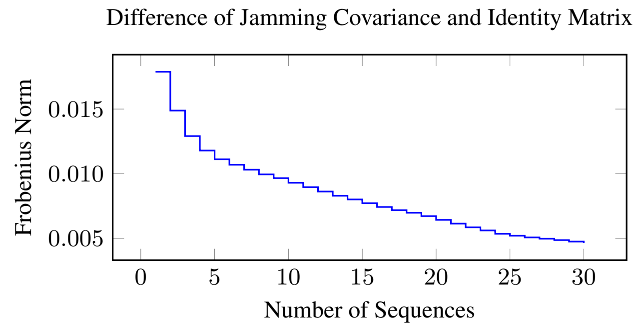


Fig. 2. Frobenius norm of the difference of the trace-normalized optimal jamming signal covariance matrix and an identity matrix. As the number of sequences being jammed increases, the covariance matrix resembles more and more a scaled identity matrix, so the jamming signal becomes temporally white.

multiple codes limiting the effectiveness of jamming. Using such a countermeasure requires a large set of suitable waveforms, however. Disrupting the operation of a distributed MIMO radar would likely require spoofing-type attacks in which target-like signals are created to mislead the radar instead of jamming with noise-like signals.

4. CONCLUSIONS

In this paper, the jamming signal of a radar jammer was optimized to disrupt the operation of a waveform-agile radar with possibly many transmitters and receivers. Since the receivers can use mismatched filters as a countermeasure against jamming, the minimax approach was taken so that jamming signal minimizes the largest signal to interference plus noise ratio the receivers can achieve.

In order to reduce the receivers' capability to suppress the jamming signal, a non-deterministic signal that is a random process should be used. Assuming quasi-stationarity, the covariance matrix of the signal has a Toeplitz structure. The optimization of Toeplitz covariance matrix to minimize the maximum receiver SINR was formulated as a convex semidefinite program that can be solved in polynomial time. It was shown that the computational complexity of the optimization can be reduced by assuming the covariance matrices to be circulant. This decreases the effectiveness of the jamming compared to the optimal Toeplitz structure. Furthermore, in case of a single receiver or equal receiver characteristics, it was shown that the circulant jamming signal covariance matrix minimizing the average SINR can be obtained with a fast, water-filling algorithm.

Distributed radar systems in which the transmitters and receivers are in different locations are resilient against jamming, as the jammer does not typically know the positions of the receivers and thus, cannot direct energy towards them. It was demonstrated that by the proposed jamming signal optimization, the jammer is able to improve the effectiveness of the jamming signal. However, if the receiver uses a mismatched filter with a performance close to that of MVDR, the effectiveness of the optimized jamming signal is significantly diminished. Furthermore, as the number of transmitted sequences increase, the jamming performance approaches the performance of white noise as the transmitted waveforms are typically designed have as low correlation as possible. As a countermeasure for jamming, a waveform-agile transmitter can switch the sequence being transmitted frequently to minimize the impact of jamming. As the receiver can estimate the direction of the jammer, that information could be used to suppress the jamming further.

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