Iterative Least Squares Algorithm for Inverse Problem in Microwave Medical Imaging

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Abstract— **The inverse problem in MicroWave Imaging (MWI) is an ill-posed one which can be solved with the aid of the sparsity prior of the solution. In this paper, an Iterative Least Squares Algorithm (ILSA) has been proposed as an inverse solver in MWI which seeks for the sparse vector satisfying the problem constraints. Minimizing a least squares cost function, we derive a relatively simple iterative algorithm which enforces the sparsity gradually with the aid of a reweighting operator. The simulation results confirm the superiority of the suggested method compared to the state-of-the-art schemes in the quality of the recovered breast tumors in the microwave images.**

Index Terms—**Microwave tomography, sparse signal processing, sparsity, inverse scattering, Microwave imaging technique.**

I. Introduction

Microwave imaging is a powerful tool for breast tomography which has gained a great deal of attention during the recent years [1]. The reason for the popularity of this scheme is that MWI is faster, less expensive, and more convenient for the patients as they do not have to endure the long time and steady stay in the MRI capsule [2]. The mechanism of MWI relies on the change of dielectric parameters from the healthy tissue to the malignant one. In order to derive the dielectric parameter values which would represent the tumors, the Distorted Born Iterative Method (DBIM) is used. The inverse problem which appears in the DBIM scheme is ill-conditioned which requires to be solved with the aid of some prior information. The conventional schemes were mostly based on classic optimization techniques such as Conjugate Gradient (CG) [5], [6] and Gauss Newton (GN) [2], [7]. With the introduction of Compressed Sensing (CS) [8]–[10], a new approach has been emerged to address the ill-posedness of the under-determined equations. The property of the signal which is being used in CS paradigm to solve the under-determined set of equations is its sparsity. Sparsity refers to the case where most of the signal entries are zero in some domain. The recent techniques exploits the sparsity assumption to solve the inverse problem. In [11], the elasticnet algorithm has been suggested. In [12]–[14], the thresholding techniques have been presented to enforce the sparsity of the signal. In this paper, a sparsity-based iterative weighted scheme is proposed to solve the MWI inverse problem at each iteration of the DBIM technique. We define a weighted L_2 norm cost function to encourage the sparsity at the same time of stabalizing the problem. The simulation results confirm the superiority of the proposed ILSA method over its counterparts in the quality of the recovered tumors.

The rest of the paper is organized as follows: In section II, a summery of the DBIM method is described. The suggested ILSA method is illustrated in section III. In section IV, the simulation results are reported. Section V concludes the paper.

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II. The DBIM method

In electromagnetical inverse scattering problems, the goal is to determine the dielectric profile of the tissues from the measured scattered fields. The Distorted Born Iterative Method (DBIM) inverse sattering scheme consists of two alternating stages of forward and inverse problems. The forward problem refers to the case where the dielectric profile is assumed to be known and the scattered fields are estimated; hence, the forward problem is well-posed which can be solved using the popular FDTD scheme. In inverse problem, the dielectric profile is estimated from the estimated fields. This is in general a non-linear problem, approximated as linear as shown in (1).

$$\mathbf{y} \simeq \mathbf{A}\mathbf{x} \tag{1}$$

where $\mathbf{y}$ represents the measurement vector (the measured scattered fields), the matrix $\mathbf{A}$ is the measurement matrix (the coefficient matrix), the vector $\mathbf{x}$ is the solution vector (contrast vector of parameters). Since the number of equations is less than the number of variables, the inverse problem is ill-conditioned which requires some prior information such as sparsity of the solution. The focus of this paper is to present a sparsity-based scheme to solve the inverse problem of MWI. The proposed method is illustrated in the next section.

III. Iterative Least Squares Algorithm (ILSA)

In this section, our proposed Iterative Least Squares Algorithm (ILSA) is presented. In order to solve the under-determined inverse problem, the following minimization is used:

$$\min \|\mathbf{x}\|_1 + \lambda_2 \|\mathbf{x}\|_2^2 + \lambda_1 \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 \tag{2}$$

where the first L_1 norm term is to encourage the sparsity of the solution vector, and the second L_2 term is to increase the robustness of the problem against instabilities occurring due to the non-linear approximation. The third term is the measurement fidelity term. We intend to replace the non-differentiable L_1 norm term with the differentiable weighted L_2 norm term as:

$$\min \sum_{j=1}^{N} \mathbf{x}_j^2(\omega_j + 2\lambda_2) + \lambda_1 \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 \quad (3)$$

where the weighting coefficients ω_i's are set as:

$$\omega_i = \frac{1}{\left(\mathbf{x}_i{}^2 + \epsilon^2\right)^{1/2}} \quad (4)$$

By considering a very small value for ϵ, it is trivial to see that weighted L_2 norm term can be a good approximation of the L_1 norm.

The minimization task is simply conducted by taking the derivative of the cost function as:

$$\mathbf{x} = \lambda_1 \left(diag(\omega) + 2\lambda_2\mathbf{I} + \lambda_1\mathbf{A}^H\mathbf{A}\right)^{-1} \mathbf{A}^H\mathbf{y} \quad (5)$$

Using the reweighting and minimization steps alternatively, we obtain the ILSA method illustrated in the following algorithm. $iter_{max}$ is the maximum number of iterations.

Algorithm 1 Iterative Least Squares Algorithm (ILSA)

```
input:
A measurement matrix A ∈ R^{M×N}.
A measurement vector y ∈ R^M.
The maximum number of iterations iter_max.
output:
A recovered estimate x̂ ∈ R^N of the original signal.
procedure ILSA(y, x)
    x^1 ← 0
    ε ← 10^{-5}
    ω^1 ← (1, 1, ··· , 1)
    for n=1:iter_max do
        x^{n+1} ← λ1 (diag(ω^{n+1}) + 2λ2 I + λ1 A^H A)^{-1} A^H y
        w_i^{n+1} = 1 / ((x_i^{n+1})^2 + ε_n^2)^{1/2}
    end for
    return   x̂ ← x^{iter_max}
end procedure
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IV. Simulation Results

In this section, the simulation results are presented. The dataset is adopted from the UW-Madisons repository data [17]. The FDTD forward solver has been applied in a uniform grid cell of size 2 mm. The proposed ILSA method is compared against two of the methods in the literature, L_2-IMATCS [12], [13] and L_2-ISATCS [14]. In the simulations, the time constant parameter τ is set to be 1.7 ps. For the background medium, we set $\epsilon_r = 2.6$. Moreover, the data has been obtained in 6 equally spaced frequencies ranging from 1.2-2.7 GHz.

The proposed ILSA technique has two parameters to be adjusted, λ_1 and λ_2. The convergence of the algorithm depends on the value of λ_1. Increasing λ_2 would boost the robustness of the algorithm, while decreasing the quality of the recovered tumors. Setting λ_2 to 0.001 provides this tradeoff. The maximum number of iterations is set to $iter_{max} = 3$. Figure 1 depicts the synthetic breast tumors used as database for the first simulation scenario.

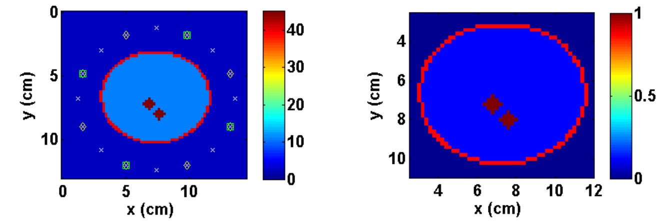

Fig. 1: (left) ϵ_r, (right) σ

The recovered tumors using the proposed method and the other two state-of-the-art methods for the SNR of 60 dB are shown in Figure 2. The parameter λ_1 is set to 0.08 for this simulation.

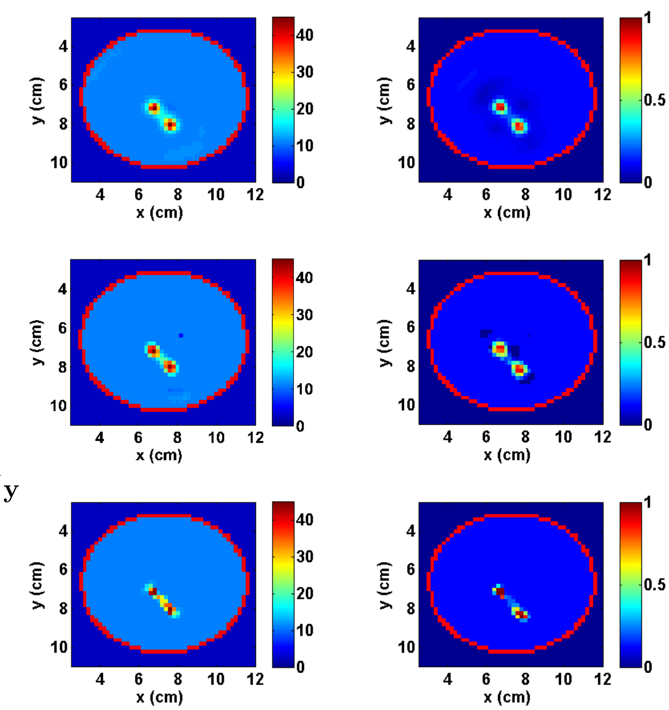

Fig. 2: The left column represents the reconstructed dielectric constant ϵ_r, the right column represents the reconstructed conductivity σ distributions for the small tumors in Fig. 1 for 16 antennas and SNR=60 dB. The first row is for ILSA, the second row is for L_2-ISATCS, and the third row is for L_2-IMATCS.

According to this figure, we observe that the proposed ILSA method offers better recovery compared to the L_2-IMATCS scheme, while its performance is slightly better than the L_2-ISATCS method. We should also study the behaviors of the three methods for the reconstruction of heterogeneous numerical breast phantoms. The dielectric constant and the conductivity maps of the heterogeneous breast model which is used as the dataset for the next simulation scenario are depicted in Fig. 3.

For the heterogeneous breast models used in this scenario, we have used the low-frequency reconstruction at 1 GHz as the initial solution. The reconstructions of the three methods,

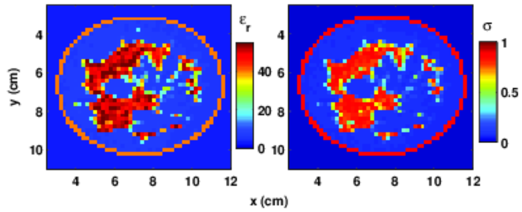

Fig. 3: Dielectric constant ϵ_r (left) and the conductivity σ (right) maps of the heterogeneous breast model calculated at 1 GHz.

the proposed ILSA, L_2-ISATCS, and L_2-IMATCS, for the low SNR value of 30 dB are depicted in Figure 4. In this scenario, the parameter λ_1 has been increased to 0.6 to provide the convergence of the algorithm.

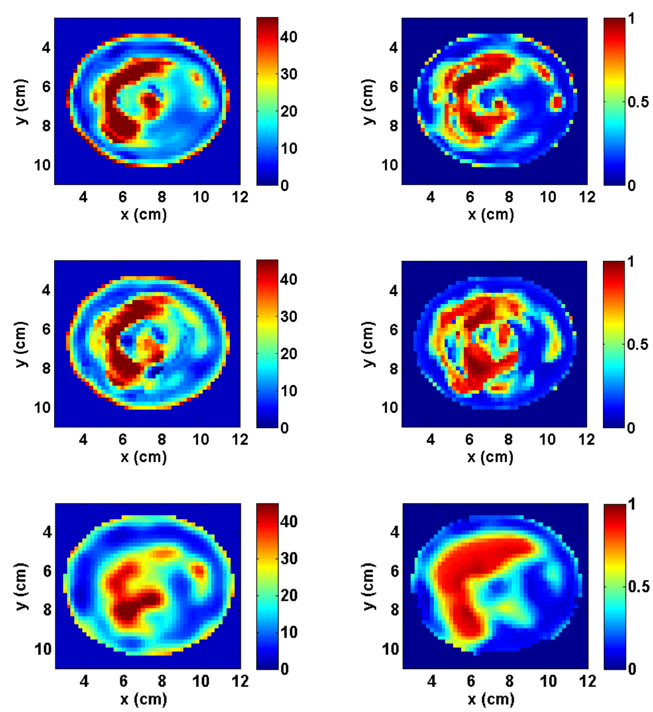

Fig. 4: The left column represents the reconstructed dielectric constant ϵ_r, the right column represents the reconstructed conductivity σ distributions for the heterogeneous tumors in Fig. 3 for 16 antennas and SNR=30 dB. The first row is for ILSA, the second row is for L_2-ISATCS, and the third row is for L_2-IMATCS.

The results indicate that the proposed scheme offers subjectively better recovery of the heterogeneous tumors compared to both of the other techniques. In order to have an objective measure to evaluate the efficiency of the methods, we use the Normalized Mean Square Error (NMSE) defined as:

$$NMSE = \frac{\|\mathbf{x}_{rec} - \mathbf{x}\|^2}{\|\mathbf{x}\|^2} \quad (6)$$

where $\mathbf{x}_{rec}$ indicates for the estimated parameters and $\mathbf{x}$ represents for the original parameters (σ or ϵ_r).

The NMSE of the three schemes in various scenarios are given in Table I.

TABLE I: Normalized Mean Square Error (NMSE) at 1GHz

scenario	parameter	L2-IMATCS	L2-ISATCS	ILSA
Fig. 1 (SNR=30 dB)	NMSE(ϵ_r) NMSE(σ)	0.0742 0.0892	0.0562 0.0974	0.0643 0.1403
Fig. 1 (SNR=60 dB)	NMSE(ϵ_r) NMSE(σ)	0.0758 0.1004	0.0553 0.0635	0.0568 0.0627
Fig. 3 (SNR=30 dB)	NMSE(ϵ_r) NMSE(σ)	0.4193 0.3665	0.4878 0.8370	0.4811 0.7675
Fig. 3 (SNR=60 dB)	NMSE(ϵ_r) NMSE(σ)	0.2650 1.3709	0.4816 0.8048	0.4645 0.6980

According to this table, the NMSE of the ILSA scheme is much lower than the other two methods in most of the cases. This difference is more evident in the case of the heterogeneous breast model. The overall conclusion on the recovery performances of the methods is that ILSA scheme is more efficient than the L_2-ISATCS which is also superior to the L_2-IMATCS. Using the reweighting strategy, the proposed ILSA scheme converges to the ultimate solution much faster than the other methods. The number of required iterations is 3 for the ILSA scheme while this number is around 12 for the L_2-ISATCS and 24 for the L_2-IMATCS. Of course, it should be noted that the complexity of each iteration in the proposed method is more than that of the L_2-ISATCS and L_2-IMATCS. As a concrete example, the simulation time per iteration for the ILSA method is approximately 1 second, while it is around 0.1 seconds for the L_2-ISATCS and 2.5 seconds for the L_2-IMATCS. Hence, comparing from the total computational comlpexity, the ILSA scheme behaves better than the L_2-IMATCS and worse than L_2-ISATCS.

V. Conclusion

In this paper, an iterative least squares technique is suggested for the inverse problem of MWI. The premise of the method is to encourage the sparsity of the solution with the application of the reweighting strategy. The suggested method offers promisable performance compared to its counterparts in the recovery accuracy. Despite the fact that the method requires very few number of iterations, say 3, for convergence, its computational complexity is more than L_2-ISATCS. However, compared to the l_2-IAMTCS scheme, the proposed method is faster.

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