Small Cycle Slip Detection using Singular Spectrum Analysis

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***Abstract*—In Global navigation Satellite System (GNSS) based positioning, the use of carrier phase measurements is widening day by day due to their preciseness as compared to code delay measurements. Although, carrier phase measurements are precise but they suffer from anomalies such as cycle slips and receiver clock jumps in addition to other error sources such as satellite-user dynamics and atmospheric delays. On one end, the detection and exclusion of these anomalies is critical for accurate and reliable positioning. On the other end, it is very difficult to detect these anomalies especially in dynamic environment due to irregular user dynamics. We propose a novel algorithm for separating and localizing these anomalies from the satellite-user dynamics. The proposed approach is based on extracting the singular spectrum of windowed carrier phase measurements. An optimal choice of different parameters ensures that the extracted singular spectrum is affected only by anomalies such as cycle slips and is independent of satellite-user dynamics. Simulation results, supported by real GNSS data analysis, indicate improved accuracy and enhanced robustness against such anomalies with respect to traditional approach using optimal time differencing.**

***Index Terms*—GNSS, Singular spectrum, Anomaly detection, Cycle slips, Single frequency receiver.**

I. Introduction

Global coverage of satellite navigation systems and advances in computation power have made Global navigation Satellite System (GNSS) receivers indispensable for many applications such as aircraft landing/take off, autonomous control of vehicles, lane change detection and collision avoidance systems. Many of these applications require access to precise positioning information for safe and reliable functioning. GNSS receivers are the primary choice for positioning information across the globe. These receivers provide us two types of measurements: code delay and carrier phase measurements. Pseudorange estimates from code delay measurements are unambiguous but noisy while those from carrier phase measurements are precise but suffer from integer ambiguity problem. Due to their preciseness, the use of carrier phase measurements for precise positioning is indispensable. As an example, various Intelligent Transport System (ITS) applications require the lane level accuracy of vehicle's position which might only be possible using carrier phase measurements. However, carrier phase measurements may contain some anomalies such as cycle slips in addition to other error terms like receiver noise, multipath, tropospheric and ionospheric errors. For a road vehicle moving in urban environment, several error sources exist on the roadside which may lead to frequent carrier phase anomalies due to poor environmental conditions and/or the poor visibility of satellites. These anomalies not only degrade the quality of position estimates obtained from carrier phase based positioning but also involve hazardous problems for reliable positioning solution. So, the first step towards accurate and reliable GNSS carrier phase based positioning solution is the detection and exclusion of these anomalies from carrier phase measurements.

Cycle slips are the most common and frequent type of anomaly present on carrier phase measurements. In this paper, we also focus on detection of cycle slip anomalies. Anomalies due to receiver clock jumps are more systematic, since they are known at the receiver and hence can be catered for by the receiver every time the clock is resynchronized [1]. Traditionally, cycle slip anomalies have been detected by forming various measurement combinations that are free from the affects of the geometric range and satellite-user dynamics through differencing the carrier phase measurements. In this regard, dual frequency receivers provide a lot of flexibility in forming different combinations. Also, dual frequency receivers provide the best possible positioning accuracy [2]. However, dual frequency receivers are very expensive and take about 20-40 minutes for convergence [2]. So, they cannot be used for applications that require real-time operations such as various safety critical ITS applications. For a single frequency receiver, the problem of detecting and excluding carrier phase anomalies becomes more challenging. Moreover, user dynamics pose an extra challenge in this problem. Depending on the trajectory of the user, satellite-user dynamics may be irregular and may introduce changes in carrier phase measurement that are difficult to distinguish from cycle slips and other such anomalies. So, the traditional approach towards detection of cycle slips and other anomalies is to obtain a time series which is not only independent of satellite-user dynamics but also reflects the affect of cycle slip anomalies. Several approaches have been proposed in the literature to obtain different time series by differencing either different measurements or same measurements at different time epochs. [1] forms triple-difference observables of the carrier phase measurements on both L1 and L2 bands for detecting possible cycle slips. [3] also formulates various quantities based on multiple differences of phase measurements for cycle slip and ionospheric scintillation detection.

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Similarly, [4] detects anomalies in the phase measurements after subtracting the satellite and user related terms, estimated through a model, from the differenced phase measurements. [5] uses independent Doppler measurements, free from the effect of cycle slips, for maintaining the integrity of carrier phase measurements. Recently, [6] proposed an algorithm for selection of an optimum differencing order of the carrier phase measurements. It also detects cycle slips by thresholding the optimum differenced phase observables. The drawback of all these methods is that they rely on observables that suffer from enhanced noise due to differencing and hence generate a lot of false positives in case the threshold is set too low or they miss cycle slips if the threshold is set too high. Most of these methods rely on computationally intensive hypothesis testing and validation methods after generating cycle slip candidates.

In this paper, we propose a novel method to detect and exclude cycle slips from carrier phase measurements based on singular spectrum of windowed carrier phase measurements. The proposed technique makes use of the fact that any anomaly present on the carrier phase measurements is independent from the satellite-user dynamics and other slowly varying error terms and hence can be separated in singular spectrum domain. By carefully decomposing the incoming phase measurements into overlapping windows, in general, we first map each window to a *trajectory* matrix. Singular Value Decomposition (SVD) of this matrix then decomposes it into a number of interpretable components with different subspaces. An optimal choice of parameters for singular spectrum ensures that the satellite-user dynamics are captured in the first principal component and the random errors in the remaining principal components. Any anomaly present will be reflected in the second and onwards principal components and will result in an increase in their singular values. Conventional approaches of singular spectrum analysis (SSA) are usually applicable to stationary data [7]. Carrier phase measurements are an ambiguous measure of the satellite-user range and thus inherently non-stationary. We deal with the non-stationarity nature of the data by segmenting it into overlapping windows and applying Singular Spectrum Analysis (SSA) to each window individually. We compare the detection performance of the proposed method with optimum time-difference (OTD) based method using simulation results which indicate improved detection performance. Moreover, we also verify and validate the proposed method and its efficacy in the presence of satellite-user dynamics by collecting real Global positioning System (GPS) data by a moving vehicle in an urban area.

Section II describes the GNSS carrier phase measurement model. Section III describes the proposed algorithm based on SSA. Section IV presents the algorithm with application to cycle slips on both simulated and real GPS data while Section V concludes the paper.

II. GNSS Carrier Phase Signal Model

The carrier phase measurement, $\rho_\phi(n)$, at discrete time epoch n as measured by the receiver from a single satellite can be written as

$$\rho_\phi(n) = r(n) - \lambda A + T(n) - I(n) + M_\phi(n) + c\Delta t_{sr}(n) + \epsilon_\phi(n) \tag{1}$$

where c is the speed of light, $\Delta t_{sr}(n)$ is the clock misalignment error between user and satellite clocks, $r(n)$ is the user-satellite geometric range, λ is the wavelength of incoming radio-frequency carrier wave which is equal to 0.19 m for GPS C/A signal, $A = 0, \pm1, \pm2, \cdots$ is an unknown integer ambiguity associated with carrier phase measurement at the time of satellite lock-on, $I(n)$ is the ionospheric error term, $T(n)$ is the tropospheric error, $M_\phi(n)$ is the multipath error on carrier and $\epsilon_\phi(n)$ is receiver noise on carrier phase observables. It is well-known that the levels of receiver noise and multipath on carrier phase measurements is almost negligible. The single frequency receiver operating in non-differential mode can mitigate the atmospheric errors up to an extent using some mathematical model and method as described in [8]. We can model $\epsilon_\phi(n)$ as zero mean Gaussian random process with variance σ_ϕ^2 *i.e.* $\epsilon_\phi(n) \sim \mathcal{N}(0, \sigma_\phi^2)$.

III. Proposed Approach

Suppose, we have total N consecutive carrier phase measurements $\rho_\phi(n)$ *i.e.* $n = 0, 1, 2, \cdots N-1$ in (1). We define a window of size N_w. Divide these N carrier phase measurements into $T = N - N_w + 1$ temporary time series each of size N_w where each window is obtained by an advance of one sample from the previous time step. Let $\mathbf{X}_t = [\rho_\phi(t) \quad \rho_\phi(t+1) \quad \cdots \quad \rho_\phi(t+N_w-1)]$ denotes one such temporary time series extracted at instant t where $t = 0, 1, \cdots T-1$. By considering the individual $\mathbf{X}_t$, the proposed method can be described in the following three steps:

1) Windowing & Decomposition : Following $L \times K$ *trajectory* matrix H_t corresponding to series $\mathbf{X}_t$ can be formed as

$$H_t = \begin{bmatrix} \mathbf{X}_t(0) & \dots & \mathbf{X}_t(K-1) \\ \mathbf{X}_t(1) & \dots & \mathbf{X}_t(K) \\ \vdots & \vdots & \vdots \\ \mathbf{X}_t(L-1) & \dots & \mathbf{X}_t(N_w-1) \end{bmatrix} = [\mathbf{c}_0 \cdots \mathbf{c}_{K-1}] \tag{2}$$

where $K = N_w - L + 1$. Each column of H_t is a time-lagged vector $\mathbf{c}_k = [\mathbf{X}_t(k) \cdots \mathbf{X}_t(k+L-1)] \in \mathbb{R}^L$ for $k = 0, 1 \cdots K-1$. Note that the *anti-diagonal* elements of H_t are equal in value indicating its Hankel structure. The parameters defining the decomposition step are window length N_w and *subwindow* length L where $2 \leq L \leq N_w - 1$.

2) Singular Spectrum: Now, we perform SVD of the matrix H_t which yields L^* eigenvalues $(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{L^*} \geq 0)$ of matrix $H_t^T H_t$ taken in decreasing order of magnitude, where $L^* = min(L, K)$ and $\sqrt{\lambda_i}$ is the ith singular value of H_t. At each t, we define a variable Λ_t denoting the sum of singular values $(\lambda_2, \lambda_3, \cdots \lambda_{L^*})$ of H_t

$$\Lambda_t = \sum_{i=2}^{L^*} \lambda_i \tag{3}$$

The presence of anomalies such as cycle slips on carrier phase measurements are reflected in Λ_t. Note that the rationale behind (3) is that the largest singular value of H_t corresponds to all the slowly varying terms in (1) while remaining singular values correspond to sudden jumps and independent noise component.

3) Thresholding: In normal conditions, Λ_t maintains a constant value, very small in comparison to λ_1, i.e. $\Lambda_t \ll \lambda_1$, with very slight perturbations due to noise depending upon the choice of N_w and L. The occurrence of any anomaly is detected through a sharp rise in the value of Λ_t as compared to its running average. At this point, a threshold can be set based on the running average of Λ_t. We denote the running average of Λ_t by the symbol $\hat{\Lambda}_t$ which is given by

$$\hat{\Lambda}_t = \frac{1}{M} \sum_{j=t-M+1}^{t} \Lambda_j \text{ for } t \geq M \tag{4}$$

where M is the window size over which running average of Λ_j is computed. A threshold γ can subsequently be used on $\hat{\Lambda}_t$ to detect any anomaly such as cycle slip and resolve its position within window of length N_w. Note that the choice of γ is very critical for increasing the detection rate and minimizing the false alarms.

IV. Simulation and Results

In this section, we first compare the detection performance of the proposed algorithm with OTD method using simulation results. We also describe the effect of different parameters present in the proposed method. After thoroughly testing the proposed algorithms in simulations, we present the results using real GPS data to show the efficacy of the proposed method in the presence of irregular user dynamics.

A. Simulations

For simulations, International GNSS Service (IGS) Final Products data was used to generate the carrier phase measurements according to (1). The amount of noise was simulated on these measurements for Monte Carlo scenario. First, we study the proposed method in order to understand the effect of different parameters on its performance.

1) Effect of Parameters: There are two parameters, N_w and L, that govern the separability of various components of the carrier phase measurement and ensure that the satellite-user range along with other slowly varying error terms is confined to the first principal component. The choice of N_w has a clear dependency on the pseudorange dynamics $\rho(n)$. It should be large enough to confine the satellite-user dynamics along with other slowly varying error terms in the first principal component. However, it should be small enough so as to ensure that no energy leaks from the first principal component into the subsequent ones. Thus there is a trade-off between the value of N_w and the bandwidth of the range dynamics. The two quantities are inversely related. The value of N_w is also related to the computational complexity and the delay to detection of the proposed algorithm. Smaller values of N_w result in a smaller size matrix for the SVD process and hence are computationally efficient. The detection process roughly suffers a delay of $\frac{N_w}{2}$, as explained in next section. So, choosing a smaller N_w also shortens this delay. Keeping computational complexity and delay in detection process as parameters of interest we decide in favor of a smaller value of N_w and hence choose $N_w = 50$ epochs, for our simulations.

The second parameter that affects the separability of the actual dynamics, random errors and anomalies is the size of the sub-window L of the Hankel matrix H_t. Defining the optimal value of L as the one which separates different trends present in the underlying time series, the complete separability is obtained for $L = \frac{N_w}{2}$ [7]. Thus we chose $L = 25$ epochs for our simulation results.

2) Artificially Generated Cycle Slip and Behavior of Λ_t: A cycle slip can be thought of as a change in the range that was missed by the receiver for a number of possible reasons. Thus a cycle when missed, will persist from that moment onwards, unless we solve for the new integer ambiguity between the satellite and the user. We model a cycle slip as a jump of $q\lambda$ meters in the carrier phase measurement in (1), where $q \in \mathbb{Z}$ is the number of missed integer cycles of carrier phase. Now if a cycle slip has to be inserted at epoch $n = k$, we can write

$$\rho_\phi(n) = \begin{cases} \rho_\phi(n) & \text{when } n < k \\ \rho_\phi(n) + q\lambda & \text{when } n \geq k \end{cases}$$

Fig. 1 shows the trend of Λ_t for the considered pseudorange model for $\sigma_\phi^2 = 0.0001\ m^2$, $N_w = 50$ epochs and $L = 25$ epochs for two cases: when there is no cycle slip on the carrier phase measurements and when there is only a single cycle slip i.e. $q = 1$. Note that small cycle slip detection is very difficult for the traditional techniques as time-differencing increases the amount of noise. The change in Λ_t trend is clearly visible in case when a single cycle slip is introduced. This change can easily be detected by setting a threshold using (4). Note that the peak in Λ_t trend occurs when the sample at which the cycle slip is inserted is around the center of the current windowed phase measurements X_t. This is the reason for the delay of $\frac{N_w}{2}$ in the detection process. Whenever an anomaly occurs it results in the forming of a plateau like shape in Λ_t trend which can be easily detected. The peak of the plateau is dependent on the noise statistics. With increasing amount of noise, the detection of peak becomes difficult.

3) Monte-Carlo Results: In this section, we compare the cycle slip detection performance of the proposed algorithm with OTD based method. A Monte Carlo simulation was set up by varying σ_ϕ^2 from 0.0001 to $0.003\ m^2$ and counting the detection events for both algorithms at each noise variance. We define the detection event as when the peak occurs within ± 5 epochs of the actual cycle slip insertion point. The classification of any point other than the correct insertion point is counted as a false positive irrespective of whether the true point was detected or not. In this way, our detection process automatically suppresses the false alarm rate. At each carrier noise variance level, a number of iterations are run and the probability of correct detection P_d is computed at the end. For

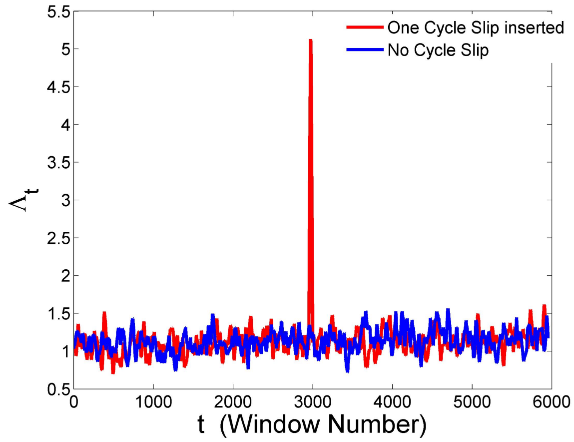

Fig. 1. Trend of Λ_t in the presence and absence of cycle slip

the proposed method, γ was chosen to be $1.5\hat{\Lambda}_t$. Fig. 2 shows the detection probability comparison of the two algorithms against the varying carrier noise variance.

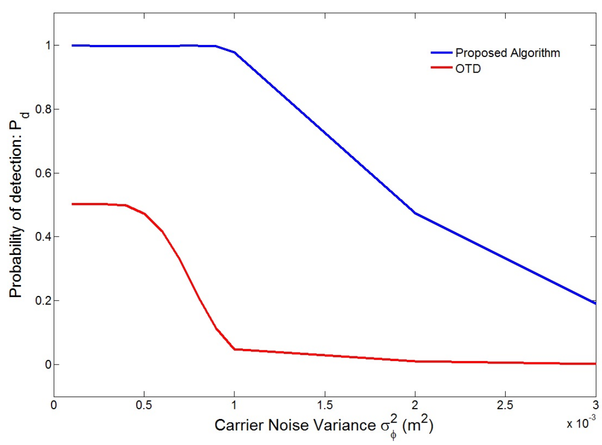

Fig. 2. Cycle slip detection performance of the proposed method and OTD method

The simulation results clearly indicate that as the noise variance increases the performance of the differencing algorithm deteriorates very quickly. This is attributed to the fact that the differencing algorithm uses a time series with enhanced noise for detection. If the algorithm decides that the optimum differencing order is 2 or 3, then the noise level on difference time series is doubled or tripled respectively. Consequently the algorithm generates a lot of false positives thus degenerating in performance. On the other hand, the proposed method is able to maintain the good detection performance even for enhanced noise levels.

B. Real Data Results

In order to further validate the proposed algorithm, we test it on real GPS data collected from a radio-frequency front-end mounted on road vehicle. The vehicle was moved in an urban environment on a trajectory shown in Fig. 3. Note that the path of the vehicle was selected to introduce irregular dynamics contributed to the range due to the motion of the receiver in a circle multiple times during the trajectory. Note that the purpose of this experiment is to illustrate the fact that almost all satellite-user dynamics are always captured in the largest principle components and hence have almost negligible effect on Λ_t. The results are shown in Fig. 4 for a single satellite

Fig. 3. Trajectory of moving vehicle

of PRN 22. The region corresponding to circular motion of the vehicle contains sinusoidal variations in the carrier phase measurements as evident from single difference of carrier phase measurements shown in Fig. 4. In order to make sure that the variations in trend of Λ_t due to satellite-user dynamics are negligible as compared to some anomaly such as cycle slips, we have deliberately introduced an artificial cycle slip of one cycle at epoch number of 1600. It is clear from Fig. 4 that the variations in trend of Λ_t due to user dynamics are negligible as compared to even very small cycle slips. Hence, the direct use of carrier phase measurements as a time series for detection of anomalies such as cycle slips is justified.

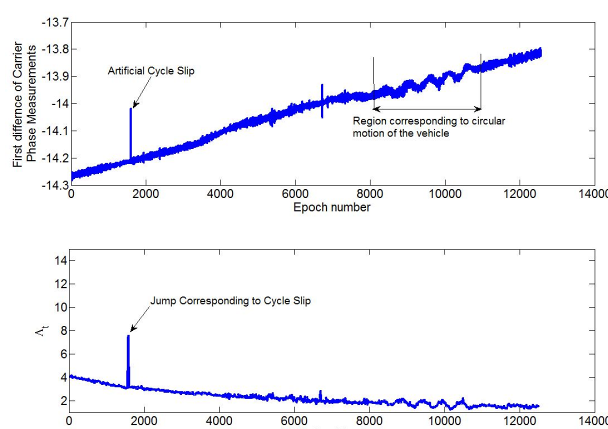

Fig. 4. Single difference of carrier phase measurements and corresponding trend of Λ_t in case of moving vehicle

V. Conclusion

We presented a new approach for detection of anomalies on carrier phase measurements based on singular spectrum analysis. The presented approach is based upon the separability of various components of the carrier phase measurement. The superiority of the proposed approach over traditional approaches in terms of accurate detection and robustness to noise is established through simulations. This is supported by the estimation of cycle slips on real GPS data. Future work in this direction consists of utilizing the reconstructed principal components for detecting continuous cycle slips as opposed to considering only separated cycle slips.

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