Localization of radar emitters from a single sensor using multipath and TDOA-AOA measurements in a naval context

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***Abstract*—This paper investigates the problem of source location from a single sensor using direct and indirect signals in a maritime context. To localize the emitters, we propose to exploit time difference of arrival (TDOA) and angle of arrival (AOA) measurements. The proposed approach uses any *a priori* knowledge of the reflectors positions. However, in practice, it is necessary to solve an assignment problem. It consists in associating each pair of TDOA-AOA measurement to a given reflector, this pair being already associated to a given emitter. In order to show the potential of our approach, numerical results using simulated and real data are proposed.**

***Index Terms*—Source localization, single sensor, multipath signals, TDOA, AOA, assignment problem, data association.**

I. Introduction

The localization of emitting sources is an important problem in several applications. It has been shown that passive systems, based on sensor networks or a single sensor, can estimate these locations from various measurements like angle of arrival (AOA), time difference of arrival (TDOA), time of arrival (TOA), frequency difference of arrival (FDOA), received signal energy measurements and/or time of intercept (TOI) of the emitter beam.

Various methods have been already proposed for sensor networks. One of the main method, known as triangulation, uses the AOA measurement from each sensor to calculate the most likely source location from the intersections of the lines of bearing [1], [2]. The second main approach takes advantage of the TDOA or TOA [3], [4], [5]. Each TDOA (computed from pair of sensors) localizes the emitter on a hyperboloid. The intersection between the hyperboloids from at least three independent TDOA (thanks to four different sensors) gives the location of the target. An equivalent method can be done using TOA. Approaches combining bearing (i.e. AOA) and TDOA have also been proposed in [6]. To avoid the use of sensor networks, a solution consists in moving a single sensor on a carrier having a relatively high speed compared to the sources to be located. Then a set of AOA can be measured and used to estimate the emitter positions [2].

When considering a single sensor at a stationary position, multipath must be taken into account. The proposed methods are mostly used in urban or indoor contexts and exploit TOA measurements [7], [8], [9]. There exists other methods exploiting for example FDOA measurements for moving emitters [10]. In [11], it is shown that, for scanning emitters, the emitter-sensor-reflector triangle is perfectly solved when using measurements of AOA, TDOA and emitter pointing angle.

In this paper, we address the problem of source localization using a single sensor, assumed to be at a stationary spatial position, and multipath propagation. In naval application the sources can also be considered fixed during the localization procedure. The proposed approach, based on geometric calculations, uses AOA and TDOA measurements. In practice, an assignment problem must be solved before localizing the emitters and the reflectors. It consist in assigning each pair of TDOA-AOA measurement to a given reflector, assuming this pair being already associated to a given emitter.

Assignment algorithms have already been proposed for other applications. For example, in [12] an assignment algorithm for the multisensor-multitarget state estimation is proposed. The goal is to determine from which target, a particular measurement is originated. [13] has adapted this algorithm for a multi-input single-output passive system. These approaches can not be directly used in our context. Thus we propose an new assignment algorithm.

This paper is organized as follows. Section II describes the problem considered in this paper. In Section III the localization problem is introduced. Section IV describes the assignment method. Section V gives a numerical results using simulated and real data. Finally, Section VI gives some concluding remarks and prospects.

II. Problem formulation

In order to localize radar emitters, passive measurements from a single sensor are considered. To solve this problem a TDOA-AOA method is investigated within a two-dimensional configuration. It uses the measurement of the direct and indirect paths received by the sensor.

Assume that there are an unknown number of emitters N_E and an unknown number of point reflectors N_R. Each emitter illuminates each reflector at the receiver with the probability

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P_d. It is also assumed a false alarm rate P_{fa}. From the detected signals, the sensor provides TDOA and AOA measurements. It is assumed that the direct signals can be separated from the indirect signals by the sensor system. It is also assumed that the direct and direct paths can be associated to the emitters that generated them. The corresponding set of information are put together in a measurement vector $\mathbf{z}$:

$$\mathbf{z} = [\mathbf{z_E}, \mathbf{z_{R_1}}, \ldots, \mathbf{z_{R_{N_S}}}]^T, \quad (1)$$

with

$$\begin{aligned}\mathbf{z_E} &= [\theta_{E_1}, \ldots, \theta_{E_{N_E}}]^T, \\ \mathbf{z_{R_i}} &= [\theta_{R_i/E_1}, \tau_{R_i/E_1}, \ldots, \theta_{R_i/E_{N_E}}, \tau_{R_i/E_{N_E}}]^T.\end{aligned}$$

θ_{E_i} and θ_{R_j/E_i} are respectively the measured AOA associated to the emitter E_i and to the reflector R_j illuminated by E_i. τ_{R_j/E_i} is the TDOA associated to the emitter E_i and reflector R_j (see Fig. 1).

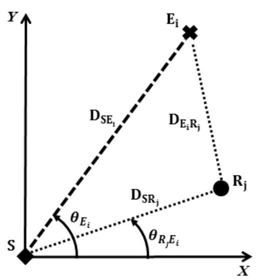


Fig. 1. Geometric representation of the considered quantities.

From these measurements, the goal is to locate the emitters and, possibly, the reflectors. As we have access to the AOA, the localization problem reduce to the estimation of the distances between the sensor and the emitters D_{SE_i}, and the sensor and the reflectors D_{SR_j}.

However, in practice, a data association problem must be solved before the localization step. Indeed, the sensor provides N_E estimated AOA for the emitters and N_E lists of measurements noted $\mathcal{L}_i$, $i = 1, ...N_E$. Each list is made of n_i pairs of variables $(\theta_{m_i/E_j}, \tau_{m_i/E_j})$, $m_i = [1, ..., n_i]$ and $j = 1, ..., N_E$, according to the P_d associated to each reflector illuminated by a given emitter, noted $P_{d_{R_i/E_j}}$. The assignment problem consists in associating these pairs of variables to the appropriate reflectors.

Finally, we would like to emphasize that the goal of this paper is not to identify the emitters or the reflectors but to localize them. Thus, the numbering of these objects in the obtained localization maps are not necessary the same to those in the simulated configurations. Indeed, they are numbered according to some considerations in the global algorithm, like for example the increasing values of the estimated AOA for the emitter numbering.

III. Source localization

In this section the source location problem is described and the proposed solution is introduced. It is assumed that the assignment problem, presented in the next section, has been successfully solved.

A. Basic configuration

It can be shown that the minimal (or elementary) configuration requires at least $N_E = 2$ emitters and $N_R = 2$ reflectors. Fig. 2 shows this elementary setup. The sensor (S) position is given by a black disk. A color (red or blue) cross stands for the emitter (E_i) position. A color-coded disk stands for a reflector (R_i) position: the color indicates that there exist a reflection on this point due to the emitter with the same color. This disk can include several colors (i.e. there are several reflections).

This configuration (with $S \neq E_i \neq R_j$, $E_i \neq E_j$ and $R_i \neq R_j$ for any i and j) assumes that there is not indirect path detected at the receiver coming from the emitter (i.e. $P_{d_{E_i/E_j}} = 0$, with $i \neq j$, and $P_{d_{E_i/R_j}} = 0$, for i and $j = \{1, 2\}$), and $P_{d_{R_i/E_j}} = 1$, for i and $j = \{1, 2\}$, and $P_{d_{R_i/R_j}} = 0$ for $i \neq j$.

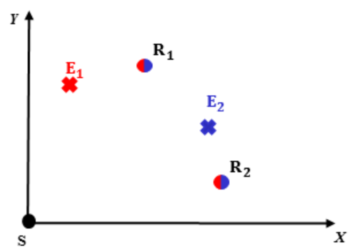


Fig. 2. Basic configuration for passive TDOA-AOA localization.

Using a geometric description of the problem, an analytical solution of the emitters locations can be easily obtained by solving a set of two equations with two unknowns (the distances D_{SE_j} between the sensor and the two sources). Then, we can also easily find the distances D_{SR_j} between the sensor and the two reflectors. These equations are not presented in this paper due to the page limitation.

B. General configuration

Let consider a scenario with $N_E \geq 2$ and $N_R \geq 2$. To solve the localization problem, we propose to split the problem by considering a first pair of emitters and their associated reflections. Thus it is possible to solve the corresponding localization problem which is like the basic configuration but with $N_R \geq 2$. Once this problem is solved, we can consider another pair or use one localized emitter with a non localized one. Thus, in what follows we just describe the localization problem for $N_E = 2$ and $N_R \geq 2$. An example of this configuration is given in Figure 3 with $N_R = 6$. In this figure, one can note that the receiver do not detect all the reflecting signals (for R_1 and R_5).

Let $\mathbf{s}$ be the state vector made of the positions of the two emitters and of N_{dR} detected reflectors ($N_{dR} \geq N_R$), and $\mathbf{z}$ the measurement vector:

$$\begin{aligned}\mathbf{s} &= [x_{E_1}, y_{E_1}, x_{E_2}, y_{E_2}, x_{R_1}, y_{R_1}, \ldots, x_{R_{N_{dR}}}, y_{R_{N_{dR}}}]^T, \\ \mathbf{z} &= [\mathbf{z_E}, \mathbf{z_{R_1}}, \ldots, \mathbf{z_{R_{N_{dR}}}}]^T.\end{aligned}$$

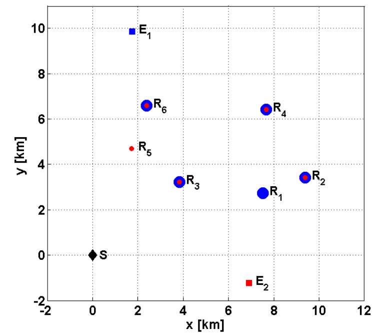


Fig. 3. Example of source localization scenario.

The Cartesian coordinates of the emitters and reflectors in $\mathbf{s}$ are obtained from their polar coordinates given by the AOA and the distances.

The measurement equation is given by

$$\mathbf{z} = h(\mathbf{s}) + \mathbf{w} \tag{2}$$

where $\mathbf{w}$ is the noise vector and h stands for the transfer function.

The maximum likelihood (ML) estimation of the state vector is given by

$$\hat{\mathbf{s}} = \underset{\mathbf{s}}{\operatorname{argmin}}((\mathbf{z} - h(\mathbf{s}))^T R^{-1}(\mathbf{z} - h(\mathbf{s}))), \tag{3}$$

where R is the covariance matrix of measurements. The ML estimation can be solved using for example the matlab function *fminsearch*. It is well known that this kind of iteration method is sensitive to the initialization. We will see in what follows that a first estimate $\hat{\mathbf{s}}_0$ can be obtained from the assignment algorithm.

IV. Assignment problem

Due to the page limitation, it is not possible to fully detail our assignment algorithm. Thus, in what follows we introduce the basic ideas and the main parts of our approach. We also consider the basic configuration with $N_E = 2$ and $N_R \geq 2$.

A. Problem description

The assignment problem consists in associating the measurements provided by the receiver and made of pairs of values: $(\theta_{m_j/E_j}, \tau_{m_j/E_j})$, with $m_j = [1, ..., n_j]$ and $j = 1, 2$, to the appropriate reflectors. These measurements are given through two lists $\mathcal{L}_1$ and $\mathcal{L}_2$ associated to each emitter.

To solve this problem, an $[n_1 \times n_2]$ assignment matrix, noted X, must be constructed. Each element of this matrix represents a valid ($x_{ij} = 1$) or invalid ($x_{ij} = 0$) association between one reflection of $\mathcal{L}_1$ and one reflection of $\mathcal{L}_2$. Note that one reflection for each list shall only be associated to one couple of emitter/reflector. Thus we can define the following assignment constraints

$$\begin{cases} \sum_{i=1}^{n_1} x_{ij} \leq 1 \text{ for all } j = 1, \cdots, n_2 \\ \sum_{j=1}^{n_2} x_{ij} \leq 1 \text{ for all } i = 1, \cdots, n_1 \end{cases} \tag{4}$$

The assigned measurements also provide the number of reflectors that are illuminated by the two emitters and whose reflecting signals are detected by the receiver. In what follows N_{jR} stands for the theoretical number of joint reflectors.

Some reflections of $\mathcal{L}_1$ (respectively $\mathcal{L}_2$) can not be associated with reflections of $\mathcal{L}_2$ (respectively $\mathcal{L}_1$). In this case, the receiver detects only one of the two possible reflections of the corresponding reflectors. Corresponding rows and/or columns of the assignment matrix are equal to zeros. N_{nR}^1 and N_{nR}^2 stand for the numbers of these so-called unjoint reflectors illuminated respectively by E_1 and E_2. Thus $n_1 = N_{jR} + N_{nR}^1$, $n_2 = N_{jR} + N_{nR}^2$ and the number of detected reflectors $N_{dR} = N_{jR} + N_{nR}^1 + N_{nR}^2$.

B. Assignment algorithm

The first part (called AOA assignment algorithm) of the proposed method is based on similarity between the AOA values of the reflected signals from the same reflector. There are $n_1 n_2$ candidates. It is possible to simply compare the AOA values according to a threshold ϵ ($|\theta_{i/E_1} - \theta_{j/E_2}| < \epsilon$). This threshold can be defined from the estimated standard deviation of the AOA measurement (i.e. the measuring resolution): $\epsilon = 2\sigma_{AOA}$. Thus, if the AOA are similar then the corresponding matrix element (x_{ij}) is set to 1 otherwise it is set to 0. Let N_a the number of assignments. If the assignment matrix verify the assignment constraints then the problem is solved and $N_a = N_{jS}$.

However, if at least two reflectors are in the same angular area, the matrix X obtained does not verify the assignment constraints and so we have to go further in the assignment process. As a result, at this point, we have $N_a > N_{jS}$ and we are looking for the N_{jS} assignments that correspond to actual joint reflectors. This is the goal of the second part of the method.

We construct the set of $N_C = \binom{N_a}{2}$ measurement vectors

$$\mathbf{z} = [\theta_{E_1}, \theta_{E_2}, \theta_{i/E_1}, \tau_{i/E_2}, \theta_{j/E_2}, \tau_{j/E_1}]^T, \tag{5}$$

with $i, j = 1, ..., N_a$. Thus, we compute all the possible pairs of distances ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$) corresponding to the configuration combinations made of $N_E = 2$ and $N_R = 2$. The combinations leading to a negative values of at least one of the estimated distances are removed.

Then, a bivariate probability density f of these pairs of distances ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$) is estimated. The maximal value of this density, noted f_{max}, is assumed to provide a good candidate for the cartesian coordinates of the emitters.

We select afterward the pairs ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$) around f_{max}, noted $\{\mathbf{d}_{SE_1}, \mathbf{d}_{SE_2}\}$ and we iterate a procedure until an assignment matrix, noted X^*, verifying the assignment constraints is found. This procedure consists in setting X^* to zero and then allocating 1 at the rows and the columns of X^* associated to the pairs $\{\mathbf{d}_{SE_1}, \mathbf{d}_{SE_2}\}$. If X^* does not verify the assignment constraints then the pair ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$) associate to the minimal value $f_{min} = \min(f\{\mathbf{d}_{SE_1}, \mathbf{d}_{SE_2}\})$ is removed from $\{\mathbf{d}_{SE_1}, \mathbf{d}_{SE_2}\}$ (see the pseudo-code in Algorithm 1). The initial state vector $\hat{\mathbf{s}}_0$ for the ML estimation in Equation (3) is obtained at the end of the assignment process.

It's important to note that for some configurations, the algorithm converges to an inappropriate solution or gives no solution. Future works will try to sort out these cases.

Algorithm 1 Assignment algorithm - pseudo code

Inputs: $\mathcal{L}_1$ and $\mathcal{L}_2$
Run AOA assignment algorithm;
Compute the $\binom{N_a}{2}$ pairs of distances ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$);
Suppress the pairs with negative values;
Calculate the bivariate kernel density estimation f of the selected distances ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$);
From $\max(f)$ compute $(\hat{x}_{E_1}, \hat{y}_{E_1}, \hat{x}_{E_2}, \hat{y}_{E_2})$;
Select the pairs ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$) around $\max(f)$
while constraints in (4) are not respected by X^* **do**
  $X^* \leftarrow 0$;
  x^*_{ij} associated to $\{\mathbf{d}_{SE_1}, \mathbf{d}_{SE_2}\} \leftarrow 1$;
  Remove the pairs ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$) associate to the minimal density value from the set $\{\mathbf{d}_{SE_1}, \mathbf{d}_{SE_2}\}$;
end while
From X compute $(\hat{x}_{R_i}, \hat{y}_{R_i})$ with $i = 1, \ldots, N_{dR}$;
Outputs: Assignment matrix X and the initial state vector $\hat{\mathbf{s}}_0$.

V. Numerical results

A. Simulated data

We consider the scenario depicted in Figure 3. The polar coordinates of the emitters and reflectors are given in Table I.

	E_1	E_2	R_1	R_2	R_3	R_4	R_5	R_6
Angle θ (deg)	80	-10	20	20	40	40	70	70
Distance D (km)	10	7	8	10	6	12	4	7

TABLE I
Location (polar coordinates) of the emitters and reflectors.

From these coordinates, it is possible to calculate the exact AOA and TDOA. To simulate more realistic values, we consider an additive Gaussian noise with zero mean and standard deviation $\sigma_{AOA} = 0.5°$ and $\sigma_{TDOA} = 50$ns for the AOA and the TDOA, respectively. These values appear to be realistic in practice. Thus, we can generate the two lists of measurements $\mathcal{L}_1$ and $\mathcal{L}_2$.

If we apply the AOA assignment algorithm, the assignment matrix obtained is

$$X = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}. \quad (6)$$

This assignment matrix doesn't respect the constraints described in the previous section. The problem and the main challenge with the considered configuration come from reflectors with same AOA.

Algorithm 1 provides to the assignment matrix

$$X^* = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

which is now in accordance with the assignment constraints.

Figure 4.a shows the pairs ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$) found during the AOA assignment algorithm and the corresponding bivariate density (in gray level). Figure 4.b shows the highest value of the density (red dot). This value is used has the initial state vector $\hat{\mathbf{s}}_0$ in the iterative algorithm for ML estimation. The green dots correspond to the pairs ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$) providing the matrix X^* (which verify the assignment constraints). The number of green dots correspond to the number of joint reflectors ($N_{jS} = 4$).

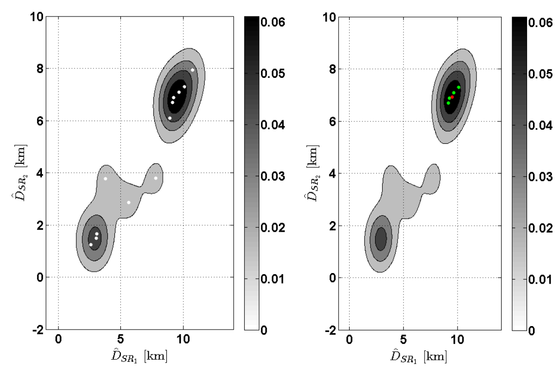


Fig. 4. (a) Density of the estimated pair of distances ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$). (b) the red dot stands for the maximum value of the density and the green dots are the ($\hat{D}_{SE_1}$,$\hat{D}_{SE_2}$) selected.

Figure 5 shows the locations obtained for the two emitters and the four joint reflectors for 200 runs. The black crosses correspond for the estimated positions of the joint reflectors and the magenta crosses show the estimated locations of the emitters. A previously indicated, the numbering of the emitters and reflectors in the final map do not necessary match the numbering in the simulation. Finally, these results illustrate the potential of the proposed approach.

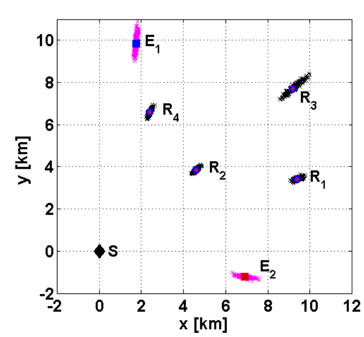


Fig. 5. Results of the proposed approach obtained for 200 runs.

B. Results from real data

In this part, the proposed method is tested using real data. For this first experiment, the receiver is at a fixed location and we have only one emitter that transmitted signals from two different spatial locations, noted P_1 and P_2, at two different times (see Figure 6). For our problem we assume that these signals have been recorded at the same time. We believe that this experiment is realistic and very close to the configuration we are dealing with. The only difference is that we are not recording the signals from two different emitters at the same time. Thus, some (probably limited) reflecting phenomena must not appear.

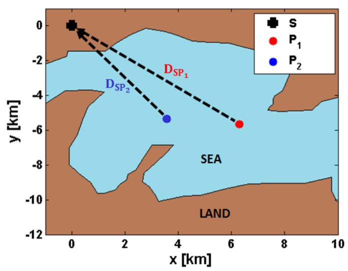
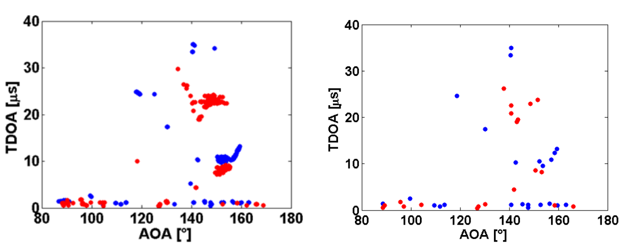


Fig. 6. Experimental setup.

Figure 7.a shows the pairs (AOA,TDOA) measured at P_1 (red points) and at P_2 (blue points). They correspond to the lists $\mathcal{L}_1$ and $\mathcal{L}_2$ that the receiver would provide when considering an experiment with two emitters recorded at the same time. This figure also shows that there are too much cases to be processed. However, it is possible to distinguish some set of points that can be probably associated to the same reflecting area. In order to reduce the number of points, we have applied the well known density-based spatial clustering of applications with noise (DBSCAN) algorithm. Each cluster obtained is replaced by a single point at its centroid coordinates. Figure 7.b shows the obtained result for our data.



(a) (b)

Fig. 7. (a) lists of measure (AOA,TDOA). (b) Selected pairs of (AOA,TDOA) after DBSCAN. Red dots: $\mathcal{L}_1$ and blue dots: $\mathcal{L}_2$.

Finally, using the proposed assignment and localization approach, we obtain the result shown in Figure 8. The estimated distances $\hat{D}_{SP_1} = 8.64$km and $\hat{D}_{SP_2} = 6.52$km fit the distances measured during the experiment $D_{SP_1} = 8.52$km and $D_{SP_2} = 6.42$km. Assuming $\sigma_{AOA} = 0.2°$ and $\sigma_{TDOA} = 25$ns, the Cramer Rao Lower Bounds (CRLB) can be calculated : standard deviations of $\hat{D}_{SP_1}$ and $\hat{D}_{SP_2}$ are bounded by : $\sigma_{SP_1} = 101.79$m and $\sigma_{SP_2} = 102.56$m.

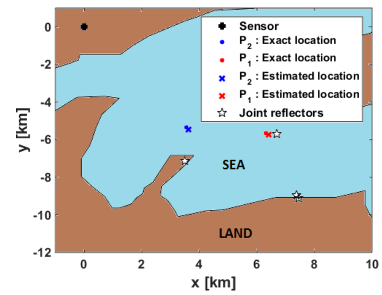


Fig. 8. Estimated locations of the emitters and (joint) reflectors.

VI. Conclusion and Future Works

In this paper, we propose to use passive AOA and TDOA measurements to localize radar emitters with no *a priori* knowledge on the location of the reflectors. To solve this problem an assignment algorithm has been proposed. The numerical results using simulated and experimental data have shown the potential of the proposed approach.

Future works will evaluate the performances of our approach and propose some enhancement in the proposed algorithm. We also plan new experiments to test this approach with more challenging operational situations.

References

[1] R. Poisel, *Electronic Warfare Target Location Methods*. Artech House, 2012, vol. second edition.

[2] M. Gavish and A. Weiss, "Performance analysis of bearing-only target location algorithms," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 28, no. 3, pp. 817–828, 1992.

[3] D. Torrieri, "Statistical theory of passive location systems," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 20, no. 2, pp. 183–198, 1984.

[4] R. Kaune, "Accuracy studies for TDOA and TOA localization," in *15th International Conference on Information Fusion*, 2012, pp. 408–415.

[5] M. Malanowski and K. Kulpa, "Two methods for target localization in multistatic passive radar," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 48, no. 1, pp. 572–580, 2012.

[6] A. Bishopa, B. Fidan, K. Dogançay, B. Andersonb, and P. Pathiranaa, "Exploiting geometry for improved hybrid AOA/TDOA-based localization," *Signal Processing*, vol. 88, pp. 1775–1791, 2008.

[7] A. Muqaibel, M. Amin, and F. Ahmad, "Target localization with a single antenna via directional multipath exploitation," *International Journal of Antennas and Propagation*, 2015.

[8] P. Setlur and N. Devroye, "Multipath exploited bayesian and Cramér-Rao bounds for single-sensor target localization," *EURASIP Journal on Advances in Signal Processing*, vol. 53, 2013.

[9] A. O'connor, P. Setlur, and N. Devroye, "Single-sensor RF emitter localization based on multipath exploitation," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 51, no. 3, pp. 1635–1651, July 2015.

[10] S. Coutts, "Passive localization of moving emitters using out-of-plane multipath," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 36, no. 2, pp. 584–595, Apr 2000.

[11] L. Hammerquist, "Passive monopulse ranging to a non-cooperative emitter and non-emitting object," jan 1994, US Patent 5,280,294. [Online]. Available: http://www.google.com/patents/US5280294

[12] S. Deb, M. Yeddanapudi, K. Pattipati, and Y. Bar-Shalom, "A generalized s-d assignment algorithm for multisensor-multitarget state estimation," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 33, no. 2, pp. 523–538, April 1997.

[13] M. Radmard, S. Karbasi, and M. Nayebi, "Mimo localization by illuminators of opportunity," in *2011 IEEE CIE International Conference on Radar*, vol. 1, Oct 2011, pp. 112–115.