A DUAL-FUNCTION MIMO RADAR-COMMUNICATIONS SYSTEM USING PSK MODULATION

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ABSTRACT

In this paper, we develop a new technique for information embedding into the emission of multiple-input multiple-output (MIMO) radar using dual-functionality platforms. A set of orthogonal waveforms occupying the same band is used to implement the primary MIMO radar operation. The secondary communication function is implemented by embedding one phase-shift keying (PSK) communication symbol in each orthogonal waveform, i.e., the number of embedded communication symbols during each radar pulse equals the number of transmit antennas. We show that the communication operation is transparent to the MIMO radar operation. The communication receiver detects the embedded PSK symbols using standard ratio testing. The achievable data rate is proportional to the pulse repetition frequency, the number of transmit elements, and the size of the PSK constellation. The performance of the proposed technique is investigated in terms of the symbol error rate. Simulations examples demonstrate that data rates in the range of several Mbps can be embedded and reliably detected.

I. INTRODUCTION

Recently, the coexistence of radar and communications as a solution to the radio frequency spectrum congestion problem has been the focus of intensive research [1], [2]. Competition over frequency spectrum between radar and communications could be directly alleviated when both systems are allowed to share the same resources and be deployed from a single platform [3]–[17]. This requires the establishment of dual system functionality where identical signals and a common antenna array are used for both operations. Dual-function radar-communication (DFRC) systems have been recently introduced in a number of papers [18]–[27].

In this paper, we propose a novel signaling scheme for information embedding in multiple-input multiple-output (MIMO) radar. The basic idea behind this scheme is embedding phase-shift keying (PSK) communication symbols by phase-rotating the orthogonal transmit waveforms of the MIMO radar. The phase-rotation is transparent to the radar operation and does not alter or compromise the offerings of the MIMO radar functionality. We derive the achievable data rate under the proposed signaling scheme and show that the number of symbols that can be embedded equals the product of the number of transmit antennas and the pulse repetition frequency (PRF). The symbol error rate (SER) performance is investigated as a function of signal-to-noise ratio (SNR) and the direction of the communication receiver for different PSK constellation sizes.

The proposed signaling scheme preserves the MIMO functionality of the radar system. It does not, however, establish a dual MIMO radar MIMO communication system. This is because, although, space and time dimensions are both involved in communication signal transmissions, we have not attempted to re-transmit the same symbol from different antennas or code the signal at reduced rate to combat the channel. Nevertheless, the paper paves the way toward this objective and develops the relationships between achievable communication data rate and existing MIMO radar resources.

The paper is organized as follows. Sec. II presents the MIMO radar signal model. In Sec. III, the proposed PSK signaling scheme is developed. Simulation results are given in Sec. IV and conclusions are drawn in Sec. V.

II. MIMO RADAR SIGNAL MODEL

Consider a dual-function system equipped with a joint transmit array comprising M omnidirectional transmit antennas. Without loss of generality, we assumed that the transmit antennas are co-located and arranged in an arbitrary linear array. The radar receiver has an array of N colocated antennas. It is assumed that both the transmit and receive arrays are closely spaced such that a target in the far-field would be seen from the same direction by both arrays. Let $\phi_m(t)$, $m = 1 \dots M$, be M orthonormal waveforms, that is, $\int_{T_0} \phi_m(t)\phi^*_{m'}(t)dt = \delta(m - m')$, where t is the fast time index, T_0 is the waveform duration, $(\cdot)^*$ denotes the conjugate, and $\delta(\cdot)$ is the Kronecker delta function.

Assume that Q targets are located in the far-field. Then, the $N \times 1$ complex vector of the received observations can

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be expressed as [28]–[30]

$$\mathbf{x}(t,\tau) = \sum_{q=1}^{Q} \alpha_q(\tau)\left[\mathbf{a}^T(\theta_q)\,\boldsymbol{\Phi}(t)\right]\mathbf{b}(\theta_q) + \mathbf{n}(t,\tau), \quad (1)$$

where τ is the pulse number, $\alpha_q(\tau)$ is the reflection coefficient of the q-th target, θ_q is the spatial angle associated with the q-th target, $\mathbf{a}(\theta)$ and $\mathbf{b}(\theta)$ are the steering vectors of the transmit and receive arrays, respectively, $(\cdot)^T$ stands for the transpose, $\boldsymbol{\Phi}(t) \triangleq [\phi_1(t),\ldots,\phi_M(t)]^T$ is the $M\times 1$ vector of orthogonal waveforms, and $\mathbf{n}(t,\tau)$ is the $N\times 1$ vector of zero-mean white Gaussian noise. In (1), the reflection coefficients $\alpha_q(\tau)$, $q = 1,\ldots,Q$, are assumed to obey the Swerling II target model [31], i.e., they remain constant during the entire pulse duration, but vary independently from pulse to pulse.

Matched filtering is employed at the radar receiver to extract the received signal components associated with the individual transmitted waveforms. The M transmit waveforms are pre-designed according to the requirements of the radar operation such that the orthogonality condition is satisfied at all time delays and Doppler shifts within the range and velocity specifications of the radar. Matched filtering the signals at the output of the radar receiver to the waveforms yields the $MN\times 1$ extended virtual data vector

$$\begin{aligned}\mathbf{y}(\tau) &= \mathrm{vec}\left(\int_{T_0}\mathbf{x}(t,\tau)\,\boldsymbol{\Phi}^H(t)\,dt\right)\\ &= \sum_{q=1}^{Q}\alpha_q(\tau)\left[\mathbf{a}(\theta_q)\otimes\mathbf{b}(\theta_q)\right] + \tilde{\mathbf{n}}(\tau), \end{aligned}\quad (2)$$

where $\mathrm{vec}(\cdot)$ is the operator that stacks the columns of a matrix into one column vector, $\otimes$ denotes the Kronecker product, $(\cdot)^H$ stands for the Hermitian transpose, and

$$\tilde{\mathbf{n}}(\tau) = \mathrm{vec}\left(\int_{T_0}\mathbf{n}(t,\tau)\boldsymbol{\Phi}^H(t)dt\right) \quad (3)$$

is the $MN\times 1$ additive zero-mean noise term with covariance $\sigma_z^2\mathbf{I}_{MN}$, and $\mathbf{I}_{MN}$ is the identity matrix of size $MN\times MN$. It is worth noting that, in practice, perfectly orthogonal waveforms occupying the same bandwidth cannot be achieved; however, those with low cross-correlations can be used. The problem of waveform design with low cross-correlations has been extensively studied in the literature (see [32]–[37] and references therein).

III. PROPOSED FORMULATIONS AND INFORMATION EMBEDDING SCHEME

In this section, we show that MIMO radar with PSK-embedded symbols yields the same extended virtual data model at the radar receiver. The section also presents the proposed PSK information embedding scheme and the corresponding data rate.

III-A. MIMO Radar With Waveform Phase Rotation

Let $\boldsymbol{\Omega} = [e^{j\Omega_1},\ldots,e^{j\Omega_M}]^T$ be the $M\times 1$ vector of phase rotations, where $\Omega_m \in [0,\ 2\pi]$, $m = 1,\ldots,M$ are arbitrary phases. Define the $M\times 1$ vector of phase rotated waveforms $\boldsymbol{\Psi}(t)$ as

$$\boldsymbol{\Psi}(t) = \boldsymbol{\Pi}\boldsymbol{\Phi}(t), \quad (4)$$

where $\boldsymbol{\Pi} = \mathrm{diag}\{\boldsymbol{\Omega}\}$ is an $M\times M$ diagonal matrix and $\mathrm{diag}\{\cdot\}$ stands for the operator that builds a diagonal matrix using the elements of a vector. Note that it can be easily verified that phase rotated waveforms $\boldsymbol{\Psi}(t)$ are also orthogonal, that is $\boldsymbol{\Psi}(t)\boldsymbol{\Psi}^H(t) = \mathbf{I}_M$.

Assume that during the τ-th radar pulse, the vector of phase rotated waveforms $\boldsymbol{\Psi}(t)$ is transmitted. Then, the $N\times 1$ complex vector of the radar received observations can be rewritten as

$$\tilde{\mathbf{x}}(t,\tau) = \sum_{q=1}^{Q}\alpha_q(\tau)\left[\mathbf{a}^T(\theta_q)\boldsymbol{\Psi}(t)\right]\mathbf{b}(\theta_q) + \mathbf{n}(t,\tau). \quad (5)$$

Matched-filtering the data (5) to the shuffled vector of orthogonal waveform $\boldsymbol{\Psi}(t)$ yields

$$\begin{aligned}\tilde{\mathbf{y}}(\tau) &= \mathrm{vec}\left(\int_{T_0}\tilde{\mathbf{x}}(t,\tau)\boldsymbol{\Phi}^H(t)\boldsymbol{\Pi}^H dt\right)\\ &= \sum_{q=1}^{Q}\alpha_q(\tau)\left[\mathbf{a}(\theta_q)\otimes\mathbf{b}(\theta_q)\right] + \check{\mathbf{n}}(\tau), \end{aligned}\quad (6)$$

where

$$\check{\mathbf{n}}(\tau) = \mathrm{vec}\left(\left[\int_{T_0}\mathbf{n}(t,\tau)\boldsymbol{\Phi}^H(t)dt\right]\boldsymbol{\Pi}^H\right) \quad (7)$$

is the $MN\times 1$ additive noise term with zero mean and covariance $\sigma_n^2\mathbf{I}_{MN}$.

The comparison between the extended virtual signal models (2) and (6) reveals that applying phase rotation to the transmit waveform yields the same data model at the radar receiver except for a phase-rotated additive noise term. In this respect, the phase rotated noise term (7) can be expressed in terms of the noise term (3) using the following relationship

$$\check{\mathbf{n}}(\tau) = [\boldsymbol{\Pi}^* \otimes \mathbf{I}_M]\tilde{\mathbf{n}}(\tau). \quad (8)$$

It is worth noting that both sides in (8) have the same statistics. Therefore, we conclude that applying phase rotation to the orthogonal transmit waveforms as in (4) does not disturb the radar receiver operation.

III-B. Proposed Information Embedding

In order to embed information into the MIMO radar emission, the phases Ω_m can be used as communications symbol. Specifically, during each radar pulse, M of such symbols can be embedded. Assume that each communication symbol represents B bits of binary information. Therefore, depending on the actual binary sequence of information that need to be embedded, during the τ-th pulse, the phase symbols $\Omega_m(\tau),\ m=1,\ldots,M$, can be selected from a predefined a dictionary of $K=2^B$ unique symbols. Without loss of generality, we assume that the dictionary is uniformly distributed within the interval $[0,\ 2\pi]$, that is,

$$\mathbb{D}_{\text{PSK}}=\left\{0,\frac{2\pi}{K},\ldots,\frac{(K-1)2\pi}{K}\right\}. \tag{9}$$

During the τ-th pulse, the phase rotated set of orthogonal waveforms is expressed as

$$\mathbf{\Psi}(t,\tau)=\mathbf{\Pi}(\tau)\mathbf{\Phi}(t), \tag{10}$$

where

$$\begin{aligned}\mathbf{\Pi}(\tau)&=\text{diag}\left\{\left[e^{j\Omega_1(\tau)},\ldots,e^{j\Omega_M(\tau)}\right]^T\right\} \qquad (11)\\ \Omega_m(\tau)&\in\mathbb{D}_{\text{PSK}},\qquad m=1,\ldots,M.\end{aligned}$$

Assume that a single antenna communication receiver is located at an arbitrary direction θ_c. The direction θ_c is assumed to be known to the communication receiver. Then, the signal at the output of the communication receiver can be modelled as

$$\begin{aligned}r(t,\tau)&=\alpha_{\text{ch}}\mathbf{a}^T(\theta_c)\mathbf{\Psi}(t,\tau)+w(t,\tau)\\ &=\alpha_{\text{ch}}\mathbf{a}^T(\theta_c)\mathbf{\Pi}(\tau)\mathbf{\Phi}(t)+w(t,\tau),\end{aligned} \tag{12}$$

where α_{ch} is the channel coefficient which summarizes the propagation environment between the MIMO radar transmit array and the communication receiver and $w(t,\tau)$ is interference plus noise additive term which is assumed to be Gaussian with zero mean and variance σ_w^2. In (12), α_{ch} is assumed to be constant during the entire coherent processing interval. Moreover, an accurate estimate of the channel coefficient is assumed to be known at he communication receiver.

The communication receiver is assumed to have perfect knowledge of the orthogonal waveforms $\phi_m(t),\ m=1,\ldots,M$. Moreover, it is assumed that the phase synchronization between the transmit array and the communication receiver is adjusted. Matched-filtering the received data (12) to $\phi_m(t)$ yields

$$\begin{aligned}y_m(\tau)&=\int_{T_0}r(t)\phi_m^*(t)dt\\ &=\alpha_{\text{ch}}\mathbf{a}_{[m]}e^{\Omega_m(\tau)}+w_m(\tau),\ m=1,\ldots,M,\end{aligned} \tag{13}$$

where $\mathbf{a}_{[m]}\triangleq e^{-j2\pi d_m\sin\theta_c}$ stands for the m-th entry of $\mathbf{a}^T(\theta_c)$, d_m is the displacement between the first and the m-th elements of the transmit array measured in wavelength, and $w_m(\tau)\triangleq\int_{T_0}w(t,\tau)\phi^*(t)dt$ is the additive noise term at the output of the m-th matched filter with zero mean and variance σ_w^2. Thus, the received communication signal at the output of the m-th matched filter is a phase-shifted and noisy version of the m-th entry of the steering vector $\mathbf{a}(\theta_c)$, meaning that the phase shift $\Omega_m(\tau)$ can be recovered from the received signal $y_m(\tau)$.

The following two remarks are in order:

Remark 1: The communication receiver knows its direction with respect to the MIMO radar transmit array θ_c. Moreover, the communication receiver knows the physical arrangement of the transmit array elements, i.e., it has perfect knowledge of the displacement of the transmit array elements from the reference element. Therefore, it is expected that the communication receiver's ability to cancel the phase term $e^{-j2\pi d_m\sin\theta_c}$ before it proceeds to detect the embedded phase symbol $\Omega_m(\tau)$.

Remark 2: During each radar pulse, the MIMO radar receiver has perfect knowledge of the vector of phase-rotated waveforms $\mathbf{\Psi}(t,\tau)$ and, consequently, it is able to apply the correct matched filters to undo the phase shift from the signal at the output of each matched filter, as explained earlier in Sec. III-A. This, therefore, does not impact the radar's primary task of estimating the unknown directions of the scatterers (targets) from the received signal.

III-C. Symbol Detection and Data Rate

Let us assume that the channel is estimated accurately. In practice, training sequences can be periodically transmitted to update the channel estimate and adjust phase synchronization between the transmit array and the communication receiver. The embedded phase symbols can be estimated as

$$\hat{\Omega}_m(\tau)=\text{angle}\big(y_m(\tau)\big)-\varphi_{\text{ch}}+2\pi d_m\sin\theta_c, \tag{14}$$

where $\text{angle}(\cdot)$ stands for the angle of a complex number and φ_{ch} is the phase of the channel coefficient. Once the embedded phase is estimated, the communication receiver compares the estimates to the dictionary $\mathbb{D}_{\text{PSK}}$ to find the embedded communication symbols and convert them into the corresponding binary sequence.

It is worth noting that the number of phase symbols which can be embedded during a single pulse repetition interval (PRI) equals the number of transmit antennas. Therefore, the achievable data rate in bits per second (bps) can be expressed as

$$R=B\times M\times\text{PRF}. \tag{15}$$

In radar applications with a high PRF, such as in X-band radar, a data rate in the range of Mbps can be easily achieved as will be discussed in Sec. IV below.

IV. SIMULATION RESULTS

In the simulations, we consider a MIMO radar operating in the X-band with carrier frequency $f=8$ GHz and bandwidth 500 MHz. The sampling frequency is taken as the Nyquist rate, i.e., $f_s=5\times10^8$ sample/sec. The PRI is taken as $T_o=10\ \mu s$, i.e., the PRF is 100 KHz. The transmit array is considered to be a uniform linear array consisting of $M=12$ antennas spaced half a wavelength apart. To implement the MIMO radar, we generate two types of waveforms which are referred to, hereafter, as random

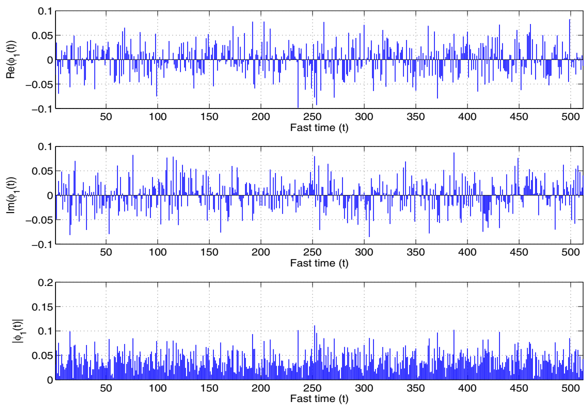

Fig. 1. Real-part, imaginary-part, and magnitude of $\phi_1(t)$; random waveform.

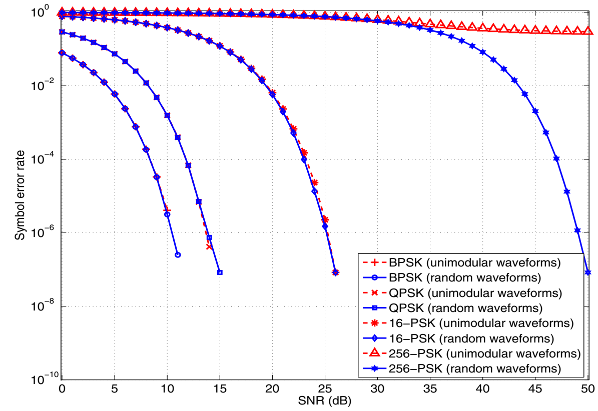

Fig. 2. SER versus SNR.

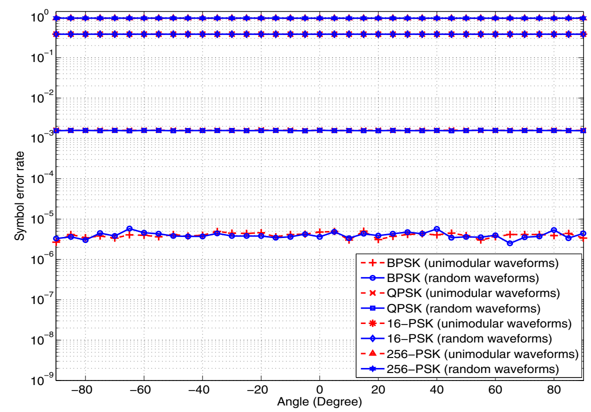

Fig. 3. SER versus communication direction.

and unimodular waveforms. The former enables realizing orthogonal waveforms with ultra low cross-correlations but practically unattractive because of their high peak to average ratio which means poor transmit power efficiency. The latter enables the transmit array to use the highest transmit power rating but may exhibit high cross-correlation levels. Random realizations of 12 Gaussian noise signals are used to obtain random signals. Unimodular signals are generated as random realization of phase noise waveforms with uniform amplitude. Although the used types of waveforms are not necessarily optimal, we use them as it is straightforward to generate multiple but dissimilar signals. Fig. 1 shows the real-part, imaginary-part, and the magnitude of $\phi_1(t)$ for the used random signal. The random waveforms are used without considering transmit power amplifier distortion effects. We investigate the SER performance of the proposed information embedding technique using BPSK, QPSK, 16-PSK, and 256-PSK constellations. This corresponds to data rate of $R = 1.2,\ 2.4,\ 4.8,$ and 9.6 Mbps, respectively. To test the SER performance, a total number of 12×10^7 random symbols are used. Fig. 2 shows the SER versus signal-to-noise ratio (SNR) for all constellation sizes considered. The figure shows that the smaller the constellation size is, the better the SER performance will be. The figure also depicts that for BPSK, QPSK, and 16-PSK constellations, the SER associated with unimodular waveforms is almost the same as that of the random waveforms. However, for the 256-PSK constellation, the performance of unimodular waveforms is worse than random waveforms due to the high cross-correlation levels between the unimodular waveforms. Fig. 3 shows the SER versus angle for all constellation sizes considered while the SNR is fixed to 10 dB. The figure shows that the SER performance is flat for all constellation, i.e., the detection performance is not sensitive to the direction of the communication receiver. This can be attributed to the omni-directional transmission of MIMO radar and, as a result, the communication user receives the same power regardless of its direction.

V. CONCLUSION

A new technique for information embedding into the emission of MIMO radar using dual-functionality platforms was developed. The secondary communication function is achieved by embedding one PSK communication symbol in each orthogonal waveform. The number of communication symbols that can be embedded during each radar pulse equals the number of transmit antennas. It was shown that the secondary communication operation is transparent to the primary MIMO radar operation of the dual-functionality system. The achievable data rate is proportional to the PRF, the number of transmit elements, and the size of the PSK constellation bits, leading to a rate in the range of several Mbps. The performance of the proposed technique was investigated in terms of the symbol error rate.

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