Antialiased Soft Clipping using an Integrated Bandlimited Ramp

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Abstract—A new method for aliasing reduction in soft-clipping nonlinearities is proposed. Digital implementations of saturating systems introduce harmonic distortion which, if untreated, gets reflected at the Nyquist limit and is mixed with the signal. This is called aliasing and is heard as a disturbance. A new correction function, derived by integrating the bandlimited ramp function, is presented. This function reduces the level of aliasing distortion seen at the output of soft clippers by quasi-bandlimiting the discontinuities introduced in the second derivative of the signal. The proposed method increases the quality of the signal by attenuating those aliased components that lie on the lower end of the spectrum, which are known to be perceptually important. The four-point version of the algorithm reduces aliasing at low frequencies by up to about 50 dB. This work extends our understanding of aliasing in nonlinear systems and provides a new tool for its suppression in virtual analog models.

I. INTRODUCTION

Analog audio systems such as musical instruments, tube amplifiers, and loudspeakers exhibit highly nonlinear behavior. A conventional approach when modeling audio systems in the digital domain has been to assume linearity and time-invariance, and proceed using traditional digital signal processing techniques [1]. While feasible, systems modeled in this fashion will not exhibit the same performance as their analog counterparts and may be unsuitable for their intended purpose. In digital audio systems, nonlinearities are commonly used to control oscillation dynamics in models with feedback and to provide a natural overdrive response [2]–[4]. Nonlinear audio processing is particularly relevant in systems such as virtual analog models of effects devices and guitar amplifiers [4], and in peak limiting [5]. The latter includes tools like compressors and limiters which are commonly used in music production and sound reproduction systems.

An inherent property of nonlinear audio processing systems is that they introduce frequency components not present in the original input signal. This property is responsible for the perceived analog warmth of the system [3], [6]. When the frequency of the new spectral components exceeds the Nyquist limit, or half the sampling frequency, they are reflected into the audio band, causing aliasing [3]. This phenomenon can

severely affect sound quality by producing audible disturbances such as beating and inharmonicity [7]. Nevertheless, if the aliased spectral components are sufficiently attenuated, they become inaudible, as they fall below the hearing threshold or are masked by other spectral peaks [7], [8]. This paper presents a novel method to suppress aliasing distortion in nonlinear audio processing systems.

Oversampling is currently a standard antialiasing method in nonlinear audio processing [3], [9]. In this approach, the input signal is first upsampled by an integer factor (e.g. eight). Following the nonlinearity, the oversampled signal is lowpass filtered and downsampled back to the original rate. This is a means whereby aliasing can be largely avoided. The main drawback of oversampling is the increased computational cost of operating at a high sample rate and of implementing the necessary up- and downsampling filters.

Other existing alias-reduction approaches include lowpass filtering the signal before it is fed to the nonlinear stage [10] and modification of the nonlinearity [2]. The first of these can be expanded into a parallel bank of antialiasing filters and increasing-order nonlinearities, a configuration known as the harmonic mixer [1], [10]. The harmonic mixer can completely eliminate aliasing, but its computational cost is proportional to the order of the implemented nonlinearity. When the nonlinearity is of high order the harmonic mixer becomes wasteful. Modifying the nonlinearity, as suggested by Thornburg [2], involves reducing the order of the system with the intention of limiting the bandwidth of the resulting spectrum. However, doing so can compromise the overall quality and behavior of the system, as high-order harmonic distortion is not reproduced.

Recently, we introduced the idea of reducing aliasing produced by hard clipping using a bandlimited ramp (BLAMP) function [11]–[13]. This method can be cascaded with a low-order approximation of a nonlinear function to arrive at an alias-suppressed soft-clipping system. It turns out that the BLAMP method works best for the hard-clipping stage only [12], and for other waveshaping devices new antialiasing methods are still needed.

This paper presents a new technique to reduce aliasing caused by a memoryless waveshaper. By analyzing a piecewise saturating function, it was observed that it generated disconti-
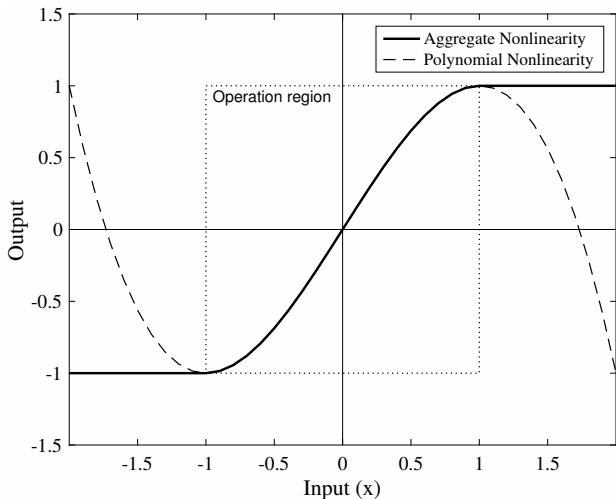
Fig. 1. Example of a memoryless waveshaping function, which maps the input signal values nonlinearly to its output. For input signal values between -1 and 1, a continuous polynomial function is used. Larger values are clipped.

nuities in the second derivative of the signal. A correction function derived by integrating the BLAMP function was designed to tackle these discontinuities directly on the soft-clipped signal. When combined with oversampling by factor 2, the proposed method reduces the level of aliasing distortion for input signals with fundamental frequency below 8 kHz. As aliasing is more audible at low and mid frequencies than at high frequencies, and especially below the lowest fundamental frequency appearing in the audio signal [8], this approach leads to a perceptually enhanced signal.

This paper is organized as follows. Section 2 discusses aliasing caused by waveshaping using a memoryless nonlinearity. Section 3 introduces the new method which is based on the integrated BLAMP function. Section 4 evaluates the performance of the proposed algorithm. Finally, Section 5 provides concluding remarks.

II. ALIASING IN MEMORYLESS NONLINEARITIES

In the digital domain, the saturating behavior of analog audio systems is usually assumed to be memoryless and can be emulated using piecewise continuous functions [3]. These functions typically consist of a low-order polynomial that models the non-saturating portion of the system followed by a saturation. Using a first-order polynomial to model the non-saturating behavior results in the special case of a hard-clipper. In hard clipping, signal values that exceed a predetermined threshold are set to this threshold, while the remaining signal values are unaltered. This process introduces discontinuities in the first derivative of the clipped signal, which require infinite bandwidth and are a major source of aliasing [5], [11], [12]. When higher-order polynomials are used, the transition from non-clipping to clipping samples is made gradual (i.e. softened) rather than abrupt, and as a result aliasing is reduced. However, due to their piecewise nature

these functions are not completely exempt from aliasing, as shown in this work.

We begin our study of aliasing in soft piecewise nonlinearities by considering a soft-clipping algorithm designed by Yamaha and included in one of their early digital multi-effects processors [14]. This algorithm is defined as

$$c(x) = \begin{cases} \frac{3x}{2} - \frac{x^3}{2} & \text{when } |x| < 1 \\ \text{sgn}(x) & \text{otherwise ,} \end{cases} \quad (1)$$

where x is the input signal value and $\text{sgn}(\cdot)$ is the sign function; $c(x)$ is the soft-clipped output signal. Implementing an arbitrary clipping threshold $L \in (0, 1]$, requires x to be scaled by $1/L$ and L before and after processing, respectively.

This soft-clipping function uses a third-order polynomial to model the non-saturating behavior of the system. This means that the spectrum of signals that do not exceed the clipping threshold will be expanded threefold in bandwidth. We isolate this cubic polynomial $c_s(x)$ as

$$c_s(x) = \frac{3x}{2} - \frac{x^3}{2}, \quad (2)$$

and refer to it as the polynomial nonlinearity. Fig. 1 compares the input–output relationships of (2) and (1). As shown by this figure, $c_s(x)$ exhibits fairly linear behavior at low input levels and smoothly approaches the clipping threshold. The antialiasing approach proposed in this work tackles the unwanted distortion components introduced by the saturating behavior of the aggregate nonlinearity (1), not by the polynomial nonlinearity. In this case, oversampling by factor 2 will be sufficient to account for the latter, since the downsampling filter will prevent harmonics above one third of the Nyquist limit from appearing.

To illustrate the issue of aliasing introduced by (1), we consider the simple case of a sinusoidal input with fundamental frequency $f_0 = 2490$ Hz. Fig. 2 shows the waveforms and magnitude spectra that result from clipping this signal with two different thresholds and an oversampling factor of 2. A second-order linear interpolation filter was used to upsample the input signal. The transfer function of this filter is

$$H(z) = 0.25 + 0.5z^{-1} + 0.25z^{-2}. \quad (3)$$

An input sampling frequency $f_s = 44.1$ kHz was used for this and the rest of the examples included in this study.

As can be observed in Fig. 2, the level of aliasing introduced by the nonlinearity increases as the clipping threshold L decreases. For the case of $L = 0.95$, most aliased components fall below -100 dB and can therefore be neglected. Overall, the signals in Figs. 2(a) and (c) exhibit a signal-to-noise ratio (SNR) of approx. 48 dB and 26 dB, respectively. In this case, we considered the SNR of a clipped signal as the power ratio between harmonics and aliased components.

In the previous example, both signals exceeded the clipping threshold into the saturating portion of the nonlinearity with different results in terms of aliasing. In order to appreciate the cause behind this, we can analytically evaluate the first derivative of the two signals depicted in Figs. 2(a) and (c).
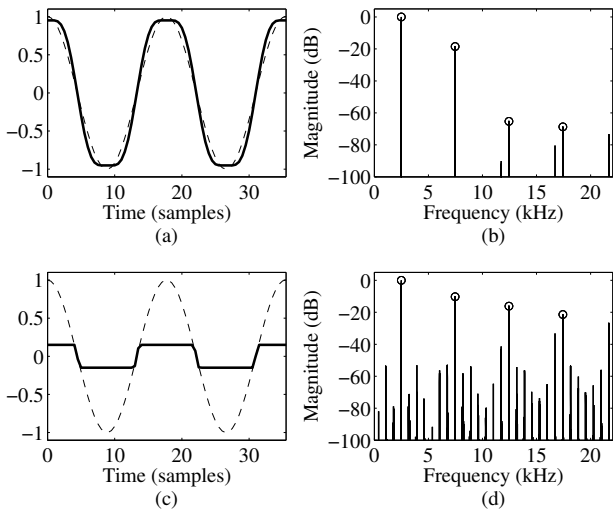
Fig. 2. Waveform and magnitude spectrum of a 2490-Hz sinusoid clipped with oversamplibng by factor 2, and (a)–(b) clipping thresholds $L = 0.95$ and (c)–(d) $L = 0.15$. In both cases, the dashed line shows the original undistorted waveform. Circles indicate non-aliased spectral components.

Fig. 3. Waveform for the first derivative of a 2490-Hz sinewave soft-clipped with threshold (a) $L = 0.95$ and (b) $L = 0.15$. Cf. Fig. 2.

The resulting waveforms are shown in Fig. 3. In these plots, we observe that a clipping threshold of $L = 0.15$ introduces a hard edge similar to that of hard clipping in the derivative of the signal at the points were the scaled input signal enters the saturating portion of the aggregate nonlinearity, i.e. when $|x| = 1$. This edge or corner translates into a discontinuity in the second derivative of the signal, which is responsible for the large amount of aliasing introduced. On the other hand, for a threshold of $L = 0.95$ this edge appears to be smooth.

To better understand this behavior we can consider the continuous-time version of the proposed memoryless soft-clipping algorithm. This process can be defined as

$$y(t) = c(x(t)), \tag{4}$$

where $y(t)$ is the clipped signal as a function of time. Using the chain and product rules we define the second derivative of this function with respect to time as

$$\frac{d^2y}{dt^2} = \frac{d^2c}{dx^2}\left(\frac{dx}{dt}\right)^2 + \frac{dc}{dx}\frac{d^2x}{dt^2}. \tag{5}$$

For an arbitrary clipping point, we wish to know the value of this expression at the exact point when $|x(t)| = 1$. Since we know $\frac{dc}{dx}$ at this point will be zero, the second term in the derivative can be eliminated, resulting in

$$\left.\frac{d^2y}{dt^2}\right|_{|x|=1} = \frac{d^2c}{dx^2}\left(\frac{dx}{dt}\right)^2. \tag{6}$$

From (6) we can observe the relation between the input signal and the size of the discontinuity introduced in the second derivative of $y(t)$, determined by $\frac{d^2y}{dt^2}$. When a clipping point occurs at a signal value with small slope [e.g. at the tips of a sinusoid, as shown in Fig. 2(a)], the curvature at that point will also be small. At low clipping thresholds, clipping will most likely occur at signal portions with relatively large slope
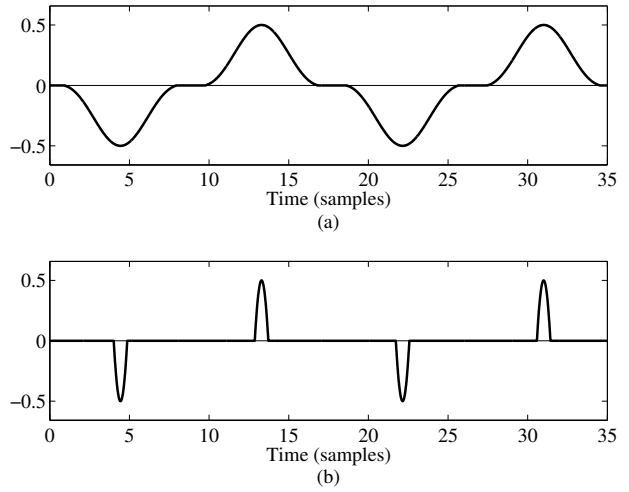
values. This will cause the curvature at the clipping points, and hence the discontinuities introduced, to be higher than for large clipping thresholds, introducing significant aliasing [15].

To reduce the aliasing in the signal depicted in 3(b), we could apply the BLAMP method to correct the hard edge and then integrate the signal. However, due to the difficulties associated with computing the necessary parameters for such small signal levels and performing the integration process, we instead propose implementing the correction process directly on the actual signal using the integral of the BLAMP function.

III. INTEGRATED BANDLIMITED RAMP FUNCTION

Our previous work has proved the effectiveness of using the BLAMP function to treat aliasing caused by discontinuities in the first derivative of a signal [11]–[13]. The closed-form expression for the BLAMP function is derived by integrating the bandlimited step (BLEP) function, defined as

$$h^{(1)}(t) = \frac{1}{2} + \frac{1}{\pi}\mathrm{Si}(\pi f_s t), \tag{7}$$

where t is time, the superscript (1) denotes this function is the first integral of the bandlimited impulse [16], and $\mathrm{Si}(t)$ is the sine integral

$$\mathrm{Si}(t) = \int_0^t \frac{\sin(\tau)}{\tau}d\tau. \tag{8}$$

The BLAMP function is then defined as [16]

$$h^{(2)}(t) = t\left(\frac{1}{2} + \frac{1}{\pi}\mathrm{Si}(\pi f_s t)\right) + \frac{\cos(\pi f_s t)}{\pi^2 f_s}. \tag{9}$$

Further integration yields the third integral of the bandlimited impulse or integrated BLAMP:

$$h^{(3)}(t) = \frac{t^2}{2}\left(\frac{1}{2} + \frac{1}{\pi}\mathrm{Si}(\pi f_s t)\right) + t\frac{\cos(\pi f_s t)}{2\pi^2 f_s} + \frac{\sin(f_s \pi t)}{2\pi^3 f_s^2}. \tag{10}$$
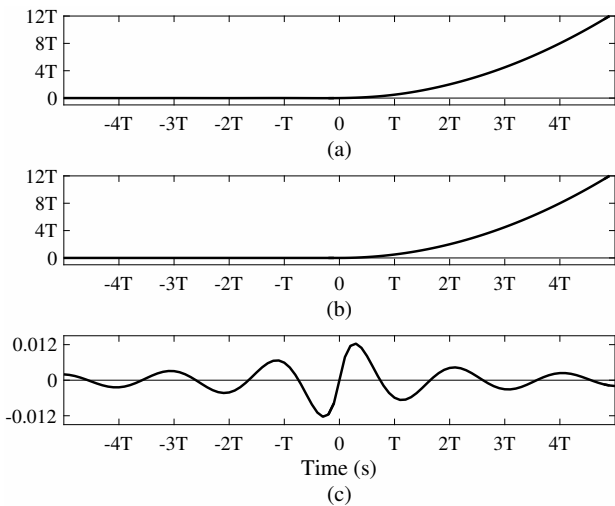
Fig. 4. Waveform for the continuous (a) integrated BLAMP function, (b) trivial parabolic function, and (c) the integrated BLAMP residual function. Parameter $T = 1/f_s$ is the sampling period of the system.
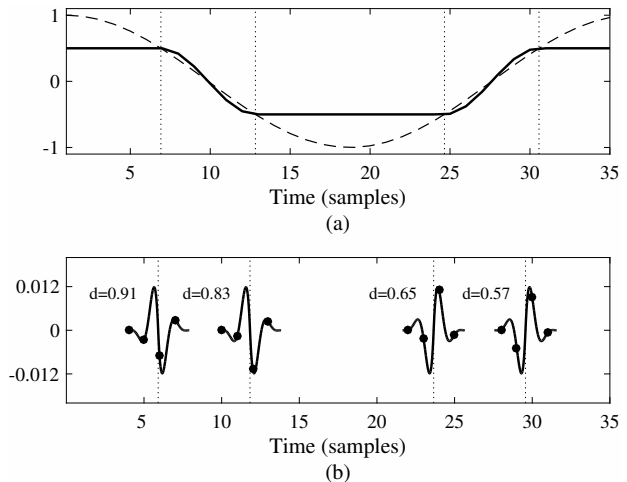

Fig. 5. One cycle of (a) a 1245-Hz sinewave soft-clipped with $L = 0.5$ and (b) continuous-time (solid) and sampled (dots) windowed integrated BLAMP residuals centered around every clipping point. The dotted vertical lines indicate the points where $|x| = 1$.

The integrated BLAMP function (10) is the closed-form expression for a bandlimited parabolic ramp with unity curvature, as shown in Fig. 4(a). We can then define its trivial non-bandlimited counterpart [see Fig. 4(b)] as

$$p(t) = \begin{cases} 0 & \text{when } t < 0 \\ \frac{t^2}{2} & \text{when } t \geq 0. \end{cases} \tag{11}$$

These functions appear to be very similar, but evaluating the difference between (10) and (11) reveals that they are not identical, as seen in Fig. 4(c). The resulting function exhibits odd symmetry and we will refer to it as the integrated BLAMP residual function.

This residual function can be used to reduce aliasing caused by the soft clipper by centering it around every clipping point, scaling it by the second derivative of the output signal at that point, sampling it at the nearest integer locations (e.g. one or two on each side), and adding the corresponding samples. To avoid the computational costs of evaluating the residual function at every clipping point, the function can be precomputed and stored in a lookup table. Fig. 5 illustrates the process of centering the integrated BLAMP residual function at each clipping point and sampling it at the four neighboring sample points for a 1245-Hz sinusoid processed with a clipping threshold $L = 0.5$. A symmetrical window function has been applied to these table-based residuals to account for the trivial truncation of the waveform. As depicted in the figure, the polarity and orientation of the correction function has to be adjusted for each clipping point. Parameter d represents the fractional delay which has to be introduced to center these residual functions, since in practical implementations clipping points will most likely not coincide with the system sample points.

To avoid processing clipping points that occur when the peak level of the signal is too close to the clipping threshold L [e.g. Figs. 3(a)–(b)], we can estimate the slope μ of the input signal at that point and determine whether to proceed or not. A small μ will mean that the input signal has low-frequency content or that we are dealing with a peak in the waveform that is very close to L. In either case, aliases will be sufficiently low and can be neglected.

IV. EVALUATION OF THE PROPOSED METHOD

The proposed method was tested on the sinusoidal input described by Fig. 2. As full knowledge of the input signal is available in this case, the parameters necessary to implement the correction were computed analytically to simulate a best-case correction scenario. Fig. 6 shows the spectra of this signal after two- and four-point integrated BLAMP correction, i.e. one and two samples corrected on each side of a clipping point, respectively. These results show clear improvements in terms of aliasing reduction [cf. Figs. 2(c)–(d)].

Both methods excel at suppressing aliasing at low frequencies, where it is most audible. For instance, the level of the most prominent aliasing component below the fundamental, at 1550 Hz [see Fig. 2(d)] has been reduced by 38 dB and 50 dB using two- and four-point correction, respectively. Additionally, the four-point method exhibits better performance than the two-point method at half the Nyquist limit.

In the previous example, the integrated BLAMP residual table was generated by evaluating the difference between (10) and (11) at 1000 points in the range $t \in [-T, T]$ and applying a Hann window. The windowing is necessary, since otherwise the non-zero end points of the residual function would produce discontinuities in the processed signal. Due to the odd symmetry of this residual, the table size can be reduced by discarding one of its halves.

In a practical implementation of the proposed method, the exact signal clipping points will usually not coincide with the sampling intervals of the system. Therefore, to apply this method on arbitrary input signals, we need a way to estimate
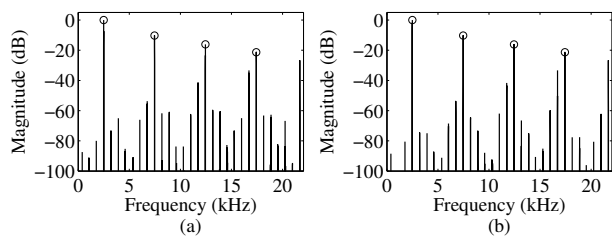
Fig. 6. Magnitude spectra for a 2490-Hz soft-clipped sinewave with clipping threshold $L = 0.15$ with oversampling by a factor 2, and (a) two-point and (b) four-point integrated BLAMP correction.

the two necessary parameters: fractional delay d and slope μ. Previous work recommended the use of inverse linear interpolation using the two samples nearest to the clipping point to estimate the two parameters. This approach, however, limited the operation range of the method to input signals with fundamental frequencies below 3 kHz [11].

In this work, we propose extending the scope of the estimation process to four samples, two on each side of every clipping point and using inverse Lagrangian cubic interpolation. For any given clipping point we consider n_b to be the index of the first sample to enter or exit the saturation part of (1). We define $n_a = n_b - 1$ and n_b as the *clipping boundaries*. Next, the exact fractional clipping point between n_a and n_b is estimated by fitting a cubic polynomial to samples $x[n_a - 1]$, $x[n_a]$, $x[n_b]$, and $x[n_b + 1]$, and using the Newton-Raphson (NR) method to solve the point at which this polynomial curve crosses the clipping threshold. The scaling parameter for the residual function, determined by the curvature, is then derived from (6) as $\gamma = (-3\mu^2)/L$, where slope μ is obtained as a byproduct of the NR method.

The use of the integrated BLAMP function provides several advantages over the previously proposed method [11]. First of all, in this approach the clipped signal is treated directly, rather than using a preprocessing stage. Using oversampling by factor 2 to account for the non-saturating behavior of (1) also extends the range of input signals which can be processed by the algorithm, as it facilitates the parameter estimation process. Additionally, doubling the scope of the correction method, from two to four points provides improved aliasing suppression. For instance, for the case of the 2490-Hz sinusoid with clipping threshold $L = 0.15$ the 1550-Hz alias (i.e. the most prominent one below the fundamental) is only attenuated by 12 dB in the preprocessing method [11].

V. CONCLUSIONS AND FURTHER WORK

This work discussed soft clipping using a piecewise saturating nonlinearity which was shown to introduce discontinuities in the second derivative of a signal for low clipping thresholds. A novel correction method, derived from the integrated BLAMP function, was proposed to suppress the associated aliasing. Additional, inverse cubic interpolation was suggested to estimate the parameters necessary for the correction process. This new method, if combined with oversampling by a factor 2, was shown to exhibit improved performance in terms

of aliasing suppression for input signals with fundamental frequency below 8 kHz.

Further work on the topic includes the derivation of a polynomial approximation of the integrated BLAMP function [15]. The adaptation of the method for other piecewise nonlinear functions is left as future work. Sound examples, along with Matlab code for the proposed method, are available online at research.spa.aalto.fi/publications/papers/eusipco16-intblamp/.

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