

OFF-GRID TARGET DETECTION WITH NORMALIZED MATCHED SUBSPACE FILTER

Olivier Rabaste, Jonathan Bosse and Jean-Philippe Ovarlez

ONERA, The French Aerospace Lab, BP 80100 91123 Palaiseau Cedex, France

Emails: surname.name@onera.fr

ABSTRACT

The problem of off-grid target detection with the normalized matched filter (NMF) detector is considered. We show that this detector is highly sensitive to off-grid targets. In particular its mean asymptotic detection probability may not converge to 1. We then consider two solutions to solve this off-grid problem. The first solution approximates the Generalized Likelihood Ratio Test (GLRT) by oversampling the resolution cell; this solution may be computationally heavy and does not permit to compute a theoretical detection threshold. We then propose a second solution based on the matched subspace detection framework. For Doppler steering vectors, the subspace considered is deduced from Discrete Prolate Spheroidal Sequence vectors. Simulation results permit to demonstrate interesting performance for off-grid targets.

Index Terms— Off-grid, normalized matched filter, matched subspace detector, discrete prolate spheroidal sequences

1. INTRODUCTION

In classic radar processing, since the target parameters (for instance range, Doppler or angle) are unknown, different matched filter operations (range matched filter, Doppler processing, array beamforming) are applied for different parameter hypotheses in order to retrieve the target. These hypotheses define a grid in the parameter space. However target parameters never lie exactly on the resolution grid, and some processing loss is thus observed due to a mismatch between the closest grid point and the true target steering vector. For the famous matched filter detector - optimum in the Gaussian noise case -, this loss may reach 3dB near the edge of the resolution cell for one single dimension when grid samples are separated by the radar resolution. It is nonetheless generally accepted as such during the radar design stage.

Although the matched filter detector is optimal in Gaussian noise, the need for a different detector has grown to deal with impulsive non Gaussian noise or partially homogeneous Gaussian noise. This has led to the Normalized Matched Filter (NMF) detector, that is well adapted to signal detection in non-Gaussian clutter. This detector, extensively studied in the past years [1,2], is CFAR-texture (Constant False Alarm Rate) and thus robust to strong clutter spikes, but has been proved to be dramatically sensitive to angle mismatch [3]. To our knowledge, the problem of off-grid targets has not been raised

in the literature. This paper intends to fill this gap by studying the behavior of the NMF for off-grid targets. We show in section 2 of this paper that the angle mismatch between the off-grid target and the steering vector under test may be quite important at the edge of the resolution cell, compared to the detection threshold value. This phenomenon may lead to strong performance loss: assuming the target is uniformly distributed in the resolution cell, the mean asymptotic detection probability of the NMF may not be equal to 1! Thus the NMF detector cannot insure good detection performance even for an arbitrarily high SNR.

This study thus requires to modify the NMF detection strategy to solve the off-grid target problem. We consider two possible solutions. The first solution presented in section 3.1 approximates the GLRT by oversampling the resolution cell and taking the maximum output over the oversampled grid. This natural strategy, despite its good performance, raises two drawbacks. First it requires a sufficiently high factor of oversampling to guarantee good performance (especially for small false alarm probabilities), thus increasing the computational cost. Second analytical computation of the detection threshold is difficult.

We thus propose in section 3.2 a different solution based on the matched subspace detection framework [1,4]. The idea is to approximate the manifold spanned by the target steering vectors over the resolution cell by a linear subspace and to consider the normalized matched subspace detector corresponding to that subspace. One advantage of this approach is the possibility to derive the analytic detection threshold for a given false alarm probability. We show here that the best subspace for the off-grid target problem with Doppler steering vectors is deduced from Discrete Prolate Spheroidal Sequence vectors [5]. Simulations show that this new approach provides interesting performance, better than the simple on-the-grid NMF, and that it can even compete with the GLRT solution for small oversampling size.

2. NMF DETECTOR IN PRESENCE OF AN OFF-GRID TARGET

2.1. Off-grid target detection problem

In this paper, we will only consider for simplicity the common Doppler case. Considering a transmitted signal composed of a train of N_p pulses with pulse repetition interval T_r , the re-

ceived signal \mathbf{y} for a given time delay can be represented by a vector of size N_p , and the corresponding Doppler steering vector is given by

$$\mathbf{s}(\nu) = \left[1, e^{j2\pi\nu T_r}, e^{j2\pi\nu 2T_r}, \dots, e^{j2\pi\nu(N_p-1)T_r} \right]^T$$

for Doppler parameter ν . In such a setting, the size of the Doppler resolution cell is provided by $\Delta\nu = 1/(N_p T_r)$ and the Doppler ambiguity is $\nu_{ambig} = 1/T_r$; there are then exactly N_p resolution cells in the Doppler ambiguity. Without loss of generality, we will consider that the centers of the resolution cells are located at grid values $\{0, \Delta\nu, 2\Delta\nu, \dots, (N_p-1)\Delta\nu\}$. Then, the Doppler parameter ν of a hypothetical target located in a given Doppler cell $n \in \{0, \dots, (N_p-1)\}$ can take any value in the interval $\mathcal{V}_n = [n\Delta\nu - \Delta\nu/2, n\Delta\nu + \Delta\nu/2]$. As long as $\nu \neq n\Delta\nu$, the target is located off-grid.

The detection problem in presence of an off-grid target can be described by the following hypothesis testing problem:

$$\begin{cases} \mathcal{H}_0 : \mathbf{y} = \mathbf{n}, \\ \mathcal{H}_1 : \mathbf{y} = A\mathbf{s}(\nu_t) + \mathbf{n}, \quad \nu_t \sim \mathcal{U}(\mathcal{V}_n), \end{cases}$$

where \mathbf{y} is the received signal for a given delay, $\mathbf{s}(\nu_t)$ is the signal contribution with ν_t the target Doppler shift assumed uniformly distributed in the n -th resolution cell \mathcal{V}_n , A is the unknown complex signal amplitude and \mathbf{n} is a zero-mean compound Gaussian noise [2] with covariance matrix $\mathbf{\Gamma}$. Since adaptive detection is beyond the scope of this paper, we will assume throughout this article that $\mathbf{\Gamma}$ is known. Of course, two-step detection will be simply obtained by replacing $\mathbf{\Gamma}$ in the proposed detectors by its estimated counterpart, for instance the classic Sample Covariance Matrix for Gaussian noise or the Fixed Point estimator for Spherically Invariant Random Vector (SIRV) noise [6].

2.2. NMF detector

Let us first assume that the parameter ν is known. When the noise variance is unknown, the associated GLRT is given by the normalized matched filter (NMF):

$$\mathcal{T}_{NMF} = \frac{|\mathbf{s}^H(\nu)\mathbf{\Gamma}^{-1}\mathbf{y}|^2}{(\mathbf{s}^H(\nu)\mathbf{\Gamma}^{-1}\mathbf{s}(\nu))(\mathbf{y}^H\mathbf{\Gamma}^{-1}\mathbf{y})} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \tau = \cos^2 \theta_S$$

Interestingly, this detector also corresponds to the asymptotic solution in the SIRV case [2]. It presents some interesting features. First it is CFAR-texture, i.e. it is invariant to the noise power density for SIRV noise (or to the variance in the Gaussian case with unknown variance). Second, since the NMF test statistic can be written as $\cos^2 \theta$ where θ is the angle between vectors $\mathbf{s}(\nu)$ and \mathbf{y} , it can be geometrically represented by a double cone with aperture angle θ_S ; any points falling inside this cone are detected by the NMF.

The NMF specific geometry also implies some drawbacks: it has been shown in [3] that the NMF detector is not very robust to signal mismatch. In particular, if the true steering vector $\mathbf{s}(\nu)$ differs from the steering vector under test

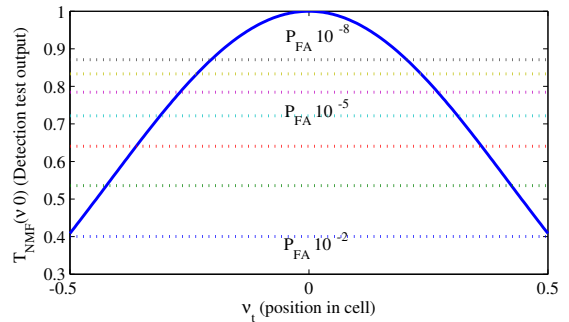


Fig. 1. On-grid NMF output testing $\nu = 0$ versus true Doppler parameter ν_t , and detection thresholds for various P_{FA} ranging from 10^{-2} to 10^{-8} . $\mathbf{\Gamma} = \mathbf{I}_{N_p}$.

$\mathbf{s}(\nu_t)$ by an angle $\theta(\nu) > \theta_S$, then the asymptotic detection probability when the target SNR tends to infinity becomes equal to 0 [3].

2.3. On-grid NMF detector in presence of off-grid target

In the off-grid target detection problem, testing on-grid steering vector $\mathbf{s}(\nu_t)$ whereas the true Doppler parameter is ν corresponds to a mismatch situation, that may lead to strong degradation of the detection probability depending on the mismatch angle between $\mathbf{s}(\nu)$ and $\mathbf{s}(\nu_t)$ compared to the threshold. Figure 1 presents the evolution of the detection test for true steering vectors inside the resolution cell with respect to the detection thresholds for different false alarm probabilities. Clearly the detection test output decreases on the edge of the Doppler cell below the detection threshold for false alarm probabilities smaller than $P_{FA} = 10^{-3}$. This has a dramatic impact on the mean (over the resolution cell) asymptotic detection probability.

Let us denote by $P_D(|A|, \nu)$ the detection probability provided for a given false alarm probability by the NMF detector testing $\mathbf{s}(\nu_t)$ in presence of a target with steering vector $\mathbf{s}(\nu)$ and amplitude A . Assuming that the target is uniformly distributed in the resolution cell, and using results on asymptotic detection probability of the NMF in presence of signal mismatch stated above [3], the mean (over the resolution cell) asymptotic detection probability can be computed as:

$$\begin{aligned} \lim_{|A| \rightarrow +\infty} P_D^{mean} &= \lim_{|A| \rightarrow +\infty} \int_{-\frac{\Delta\nu}{2}}^{\frac{\Delta\nu}{2}} P_D(|A|, \nu) d\nu \\ &= \frac{1}{\Delta\nu} \int_{-\frac{\Delta\nu}{2}}^{\frac{\Delta\nu}{2}} \mathbb{I}_{\theta(\nu) \leq \theta_S} d\nu, \end{aligned} \quad (1)$$

where \mathbb{I}_Ω is the characteristic function of the set Ω , i.e. $\mathbb{I}_\Omega(x) = 1$ if $x \in \Omega$, 0 otherwise, and $\theta(\nu)$ is the mismatch angle between $\mathbf{s}(\nu)$ and $\mathbf{s}(\nu_t)$. Let us simply notice here that for the case considered in Figure 1, the mean asymptotic detection probability is not equal to 1 for false alarm probabilities lower than $P_{FA} = 10^{-2}$ since on the edge of the resolution cell the mismatch angle becomes larger than

the detection threshold. On the contrary the classic Matched Filter detector would present a mean asymptotic detection probability equal to 1 for any false alarm probability. Thus the on-grid NMF detector is much more sensitive to the grid problem than the on-grid MF detector, and solutions must be proposed to deal with off-grid targets.

3. SOLVING THE OFF-GRID TARGET PROBLEM

In this section, we consider two solutions for dealing with the off-grid target problem. The first one is based on the classic GLRT and tries to estimate the Doppler of the target. The second one is based on a matched subspace detector strategy.

3.1. GLRT solution

The classic GLRT strategy consists in injecting the Maximum Likelihood estimate of the target Doppler shift into the NMF detector. Since the Doppler parameter only arises under hypothesis \mathcal{H}_1 , this resorts to considering the following test for the n -th resolution cell $\mathcal{V}_n = [n\Delta\nu - \Delta\nu/2, n\Delta\nu + \Delta\nu/2]$:

$$\mathcal{T}_{NMF-GLRT} = \max_{\nu_t \in \mathcal{V}_n} \frac{|\mathbf{s}^H(\nu_t)\mathbf{\Gamma}^{-1}\mathbf{y}|^2}{(\mathbf{s}^H(\nu_t)\mathbf{\Gamma}^{-1}\mathbf{s}(\nu_t))(\mathbf{y}^H\mathbf{\Gamma}^{-1}\mathbf{y})} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \tau.$$

Geometrically speaking, this detection test projects the received signal \mathbf{y} onto the 2D-manifold \mathcal{D}_n defined by any vector of the form $A\mathbf{s}(\nu)$ with $\nu \in \mathcal{V}_n$. However this projection cannot be performed analytically. Thus in practice, it will generally be solved numerically, by oversampling the doppler cell under test and maximizing the detection test over the oversampled grid.

This strategy, that approximates the theoretical GLRT solution, raises two problems:

- The oversampling factor should be based on the following trade-off: a large oversampling factor will lead to good performance at the expense of a high computational cost while a small oversampling factor will present a lighter computational cost but poorer performance.
- Since the steering vectors provided by the oversampled grid are necessarily correlated (since they are separated by less than the Doppler resolution cell), the detection tests for the oversampled steering vectors are not independent, and it is thus difficult to compute the theoretical detection threshold assuring a given false alarm probability. This implies that the detection threshold can only be obtained by testing over a sufficiently large set of secondary data identically distributed as the data under test. Such a set may not always be available.

3.2. Matched subspace solution

We propose here a different solution to solve the problem of off-grid targets based on a matched subspace detector [1]. The idea here is to approximate the nonlinear 2D-manifold \mathcal{D}_n by a linear subspace of dimension m .

3.2.1. Normalized Matched Subspace Filter

For simplicity, let us first consider the equivalent ‘‘whitened’’ formulation of the hypothesis testing problem:

$$\begin{cases} \mathcal{H}_0 : \mathbf{y}_w = \mathbf{n}_w & \text{(noise only),} \\ \mathcal{H}_1 : \mathbf{y}_w = A\mathbf{s}_w(\nu) + \mathbf{n}_w & \text{(signal + noise),} \end{cases}$$

where $\mathbf{y}_w = \mathbf{\Gamma}^{-1/2}\mathbf{y}$, $\mathbf{s}_w(\nu) = \mathbf{\Gamma}^{-1/2}\mathbf{s}(\nu)$ and $\mathbf{n}_w = \mathbf{\Gamma}^{-1/2}\mathbf{n}$ are respectively the whitened measurement vector, whitened signal and whitened noise.

Instead of considering the steering vector $\mathbf{s}_w(\nu)$ with unknown ν , we propose to consider the subspace of dimension m that best represents the 2D-manifold defined by the set of steering vectors $\mathbf{s}_w(\nu)$ for ν in the resolution cell. The choice of this subspace and its dimension will be detailed in the next subsections. Let us for now simply reformulate the hypothesis testing problem as a subspace detection problem in the considered resolution cell:

$$\begin{cases} \mathcal{H}_0 : \mathbf{y}_w = \mathbf{n}_w & \text{(noise only),} \\ \mathcal{H}_1 : \mathbf{y}_w = \mathbf{U}_{n,w}\boldsymbol{\beta} + \mathbf{n}_w & \text{(signal + noise),} \end{cases}$$

where the columns of the matrix $\mathbf{U}_{n,w}$ of size $N_p \times m$ define an orthonormal basis for the chosen linear subspace (i.e. $\mathbf{U}_{n,w}^H\mathbf{U}_{n,w} = \mathbf{I}_m$), and $\boldsymbol{\beta}$ is a vector representing both the target amplitude and the decomposition of the target steering vector along the subspace dimensions. For such an hypothesis test, the normalized matched subspace filter (i.e. the subspace counterpart of the NMF detector) is given by [1, 7]:

$$\mathcal{T}_{NMSF} = \frac{\|\mathbf{U}_{n,w}^H\mathbf{y}_w\|^2}{\mathbf{y}_w^H\mathbf{y}_w} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\geq}} \tau.$$

Under \mathcal{H}_0 , in the compound Gaussian case, this test statistic can be written as a ratio between two independent centered chi-square variables with $2m$ and $2(N_p - m)$ degrees of freedom respectively, so that the false alarm probability associated to this detector is the false alarm probability for the normalized matched subspace filter provided by [1]:

$$P_{FA}^{NMSF} = 1 - B_\tau(m, N_p - m),$$

where $B_x(a, b)$ is the incomplete beta function. This false alarm probability depends on the threshold τ , the dimension of the steering vector N_p and the dimension m of the considered linear subspace.

In contrast, under \mathcal{H}_1 , the exact detection probability for the proposed detector in presence of an off-grid target differs from the classic detection probability for a subspace detector, due to the mismatch between the true signal steering vector and the considered subspace. Decomposing $\mathbf{s}_w(\nu)$ and \mathbf{n}_w in the basis defined by $\mathbf{U}_{n,w}$ and $\mathbf{U}_{n,w}^\perp$ where the columns of $\mathbf{U}_{n,w}^\perp$ defines a basis of the subspace orthogonal to the subspace spanned by $\mathbf{U}_{n,w}$ (thus $\mathbf{U}_{n,w}^H\mathbf{U}_{n,w}^\perp = \mathbf{0}$), we can write $\mathbf{s}_w(\nu) = \mathbf{U}_{n,w}\boldsymbol{\beta}_1(\nu) + \mathbf{U}_{n,w}^\perp\boldsymbol{\beta}_2(\nu)$ and $\mathbf{n}_w =$

$\mathbf{U}_{n,w}\mathbf{n}_1 + \mathbf{U}_{n,w}^\perp\mathbf{n}_2$, where β_1 and \mathbf{n}_1 are vectors of length m and β_2 and \mathbf{n}_2 are vectors of length $N_p - m$. Besides, in the compound Gaussian case with texture α , the noise terms can be expressed as $\mathbf{n}_1 = \sqrt{\alpha}\mathbf{x}_1$ and $\mathbf{n}_2 = \sqrt{\alpha}\mathbf{x}_2$ where \mathbf{x}_1 and \mathbf{x}_2 are complex white Gaussian vectors of size m and $N_p - m$ respectively. Then the test statistic can be rewritten as:

$$\mathcal{T}_{NMSF}|\mathcal{H}_1 = \frac{1}{1 + \frac{\xi_2}{\xi_1}} \sum_{\mathcal{H}_0}^{\mathcal{H}_1} \tau \Leftrightarrow \frac{\xi_1}{\xi_2} \sum_{\mathcal{H}_0}^{\mathcal{H}_1} \tau' = \frac{\tau}{1 - \tau} \frac{N_p - m}{m}$$

where $\xi_i = \|\frac{A\beta_i(\nu)}{\sqrt{\alpha}} + \mathbf{n}_i\|^2$ for $i = 1, 2$ is a noncentral chi-square variable conditionally to α with d_i degrees of freedom and noncentral parameter $\mu_i^{\nu,\alpha,A} = |A|^2\|\beta_i(\nu)\|^2/\alpha$; $d_1 = 2m$ and $d_2 = 2(N_p - m)$. The mean (over the resolution cell) detection probability for the proposed off-grid detector is then provided by

$$P_D^{mean}(|A|) = \mathbb{E}_\nu [P_D(|A|, \nu)], \quad (2)$$

where

$$P_D(|A|, \nu) = 1 - \int_0^{+\infty} F(p_1, p_2; \lambda_1, \lambda_2; \tau') p(\alpha) d\alpha,$$

with $F(p_1, p_2; \lambda_1, \lambda_2; x)$ the doubly noncentral F-distribution with degrees of freedom $p_1 = 2m$ and $p_2 = 2(N_p - m)$, noncentral parameters $\lambda_1 = \mu_1^{\nu,\alpha,A}$ and $\lambda_2 = \mu_2^{\nu,\alpha,A}$, computed at value x . This doubly noncentral F-distribution can be numerically evaluated using the saddle point approximation provided in [8].

3.2.2. Choice of the subspace basis

The best subspace to consider should be the one maximizing the mean (over the resolution cell) detection probability for a given SNR or set of SNRs. This is a non trivial problem, that also presents the drawback to provide a solution depending on the SNR considered. It seems natural, more reasonable but still legitimate to use the subspace of dimension n minimizing the mean square error of the steering vector projection for a given distribution of ν , i.e. :

$$\mathbf{U}_{n,w} = \arg \min_{\mathbf{U}_{n,w}} \mathbb{E}_\nu [\|\mathbf{s}_w(\nu) - \mathbf{U}_{n,w}\mathbf{U}_{n,w}^H\mathbf{s}_w(\nu)\|^2] \quad (3)$$

In the sequel, we consider that ν is uniformly distributed over the resolution cell. In the white noise case, i.e. $\mathbf{\Gamma} = \mathbf{I}_{N_p}$ so that $\mathbf{s}_w(\nu) = \mathbf{s}(\nu)$, it has been shown for Matching Pursuit applications [9, 10] that the solution of (3) for a given dimension m is provided by the m first eigenvectors of the matrix

$$\mathbf{M}_n = [\mathbf{s}(n\Delta\nu)\mathbf{s}^H(n\Delta\nu)] \odot \mathbf{B}_{N_p, \frac{1}{2N_p}}$$

where \odot represents the component-wise matrix multiplication and the (k, l) entry of matrix $\mathbf{B}_{N,W}$ is given by $(\mathbf{B}_{N,W})_{k,l} = 2W \text{sinc}(2W(k-l))$. Eigenvectors of this matrix are the well-known Discrete Prolate Spheroidal Sequence (DPSS) vectors [5]. Using an approach similar to the one used in [10], it can be shown that for a non-identity

noise correlation matrix, the solution of (3) is provided by the eigenvectors of the following matrix:

$$\mathbf{M}_{n,w} = \mathbf{\Gamma}^{-\frac{1}{2}}\mathbf{M}_n\mathbf{\Gamma}^{-\frac{1}{2}};$$

These eigenvectors can be considered as whitened DPSS. We will use the first m whitened DPSS stacked in matrix $\mathbf{U}_{n,w}$ for off-grid target detection.

Once the general subspace basis has been chosen, it remains to determine the best number of dimensions m to consider. Note that $m = 1$ would already provide better detection performance than the classic on-grid matched filter, although the observed gain is small. It is much preferable to increase the number of dimensions. However if increasing the subspace dimension permits to capture more deeply the signal manifold, it also increases a lot the noise contribution in the signal subspace. Thus there exists a trade-off value for the dimension m . When comparing the detection probabilities provided by (2) for different m around the interesting range (i.e. around 0.9), it appears that the best choice generally corresponds in our case to $m = 2$.

4. SIMULATIONS

In this section, we present detection performance in terms of mean (over the cell) detection probabilities obtained from Monte Carlo simulations. The steering vector considered is of length $N_p = 10$. The noise is distributed according to a K-distribution with shape parameter $\gamma = 5$. Performance of the on-grid NMF is computed and compared to the performance of the NMF-GLRT detector for several oversampling grid where the oversampling factor ranges from 2 to 4, and to the performance of the NMSF detector computed over a DPSS subspace of dimension $m = 2$. Are also considered for comparison the theoretical mean detection probability provided by (2), the mean asymptotic detection probability given by (1), and the detection probability of the NMF detector with ν_t known. This last case provides an upperbound on the detection probability (target is located on the grid). Results are presented in Figure 2 and 3 for false alarm probabilities $P_{FA} = 10^{-4}$ and $P_{FA} = 10^{-6}$ respectively.

Simulation results show that the on-grid NMF presents poor detection performance. In particular, its detection performance does not converge to 1 for the considered false alarm probabilities, as shown above. The two proposed strategies for dealing with the off-grid targets provide better performance: as expected, the best performance is provided by the NMF-GLRT detector with an oversampling factor sufficiently large; indeed for a small oversampling factor, detection performance tends to be degraded by the same phenomena as for the on-grid NMF detector, although the effect is much less dramatic. The NMSF-DPSS detector provides slightly worse performance than the NMF-GLRT with oversampling factor 4. However it appears that it is better than the NMF-GLRT with oversampling factor 2 for sufficiently high SNR, especially for small false alarm probabilities common

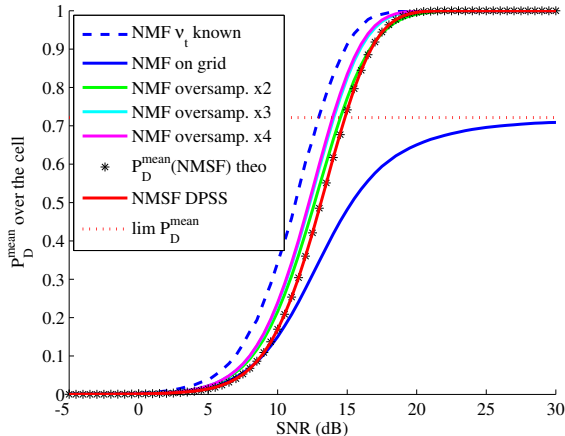


Fig. 2. Mean detection probabilities for the NMF with ν_t known, the on-grid NMF, the NMF-GLRT computed on oversampled grid with oversampled factors ranging 2, 3 and 4 and the DPSS NMSF with $m = 2$. $P_{FA} = 10^{-4}$, K-distributed noise with shape parameter $\gamma = 5$ and $\Gamma = \mathbf{I}_{N_p}$, $N_p = 10$.

in radar. Similar conclusions are observed for different shape parameters γ .

5. CONCLUSION

In this paper, we have shown that the NMF detector is very sensitive to off-grid targets. We have thus proposed two solutions to solve the off-grid target detection problem in the NMF framework. The first solution corresponds to the GLRT; it consists in approximating the angle between the received signal and the steering vector manifold by oversampling the resolution cell and keeping the maximum output; although this solution is the most performant in terms of detection probability, it may be computationally heavy for high oversampling factors, and due to the correlation between the different samples, one cannot compute the theoretical detection threshold. We have thus proposed a second solution that consists in computing the angle between the received signal and a good subspace capturing most of the signal manifold. For Doppler steering vectors, this subspace is provided by a set of whitened Discrete Prolate Spheroidal Sequences. Simulation results permit to demonstrate interesting performance for off-grid targets.

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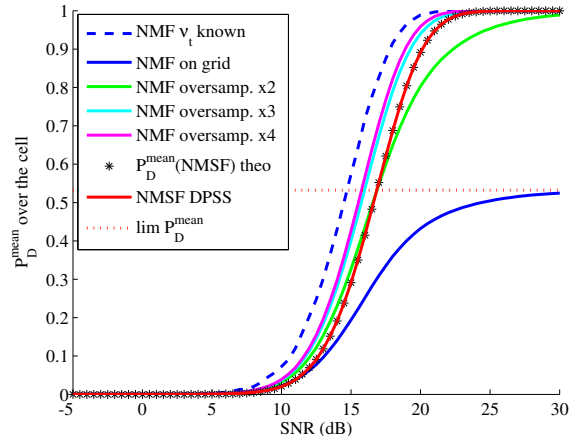


Fig. 3. Mean detection probabilities for the NMF with ν_t known, the on-grid NMF, the NMF-GLRT computed on oversampled grid with oversampled factors ranging 2, 3 and 4 and the DPSS NMSF with $m = 2$. $P_{FA} = 10^{-6}$, K-distributed noise with shape parameter $\gamma = 5$ and $\Gamma = \mathbf{I}_{N_p}$, $N_p = 10$.

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