Maximum likelihood estimation of a low-order building model

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***Abstract*—The aim of this paper is to investigate the accuracy of the estimates learned with an open loop model of a building whereas the data is actually collected in closed loop, which corresponds to the true exploitation of buildings. We propose a simple model based on an equivalent RC network whose parameters are physically interpretable. We also describe the maximum likelihood estimation of these parameters by the EM algorithm, and derive their statistical properties. The numerical experiments clearly show the potential of the method, in terms of accuracy and robustness. We emphasize the fact that the estimations are linked to the generating process for the observations, which includes the command system. For instance, the features of the building are correctly estimated if there is a significant gap between the heating and cooling setpoint.**

I. Introduction

Buildings and their thermal behaviour are one of the major leverages in designing fighting strategies against climate change. Indeed, according to the European Commission, buildings are responsible for 40% of the energy consumption and 36% of the CO_2 emissions in the European Union. Most of these emissions are due to heating, for instance 77% of them in France in 2013 [1]. One obvious aim is naturally to reduce the consumptions of the building sector. Beyond this, the role of the buildings in the new energy policies is two-fold, because they could be key interactive actors in the flexibility of the emerging smart grids, helping to integrate renewables and to strike a supply-demand balance.

Several tools and ideas exist in order to adress these new challenges: load prediction, estimation of load shift potentials, ultimately optimal control, etc. They have in common the need for a good model of the building, able to reproduce its thermal behaviour. This article is concerned with the identification procedure of such models.

Before identification, a class of models ought to be chosen. We focus here on *grey-box* modeling, because it has both the advantage of cheap computational cost and nice physical interpretation [2]. Many articles ([2], [3], [4]) highlight the fact that low-order models, are sufficient to accurately describe the dynamics. Authors typically agree on the order 2.

However, it is not clear whether the model should be identified in open loop, or within a control loop, the latter case corresponding to the real exploitation of buildings. On one hand, this discussion is often overlooked in the articles developing open loop approaches ([4], [5], [6]). They evaluate their method by comparison to higher-order *white-box* models or looking at the accuracy of the estimates, but no conclusion can be drawn relatively to their behaviour within a closed-loop system. On the other hand, [7] suggested a closed loop identification, which is designed to apply without the measurement of the indoor temperature and gives promising results. Still, this approach lacks of accuracy when taking into account the saturation of the controller. [2] also identifies a closed loop model, but the controller is strongly simplified regarding the small time step.

With these elements in mind, the purpose of the article is to identify an open-loop *grey-box* model, from which physical inference can be drawn, and study its performances when the data are in fact generated in closed loop. Our object is not to work on defining the best representation of a building. Instead, we consider a given model, namely a R3C2 network, and show how to estimate its parameters and their uncertainties. The method is easily adaptable to refinements of R3C2. We use simulated data that reproduce the real exploitation of a building and illustrate that even in these poor conditions (less informative than open loop), we are able to estimate static and dynamic physical parameters of the buildings with only a few weeks of measured data.

This article is organized as follows. In Section II, we define our model, putting emphasis on the physical interpretation of its parameters. We describe also how to derive the discrete-time linear gaussian state-space representation. Next, in Section III, we introduce the estimation procedure grounded in the EM algorithm, and mention some useful statistical results that enable us to gauge the standard errors. Moreover practicalities are not left behind, and we detail the full estimation protocol. Finally, Section IV focuses on the numerical experiments conducted on simulated data. We mainly investigate whether or not inference can be drawn from an open loop building model learned from data actually generated in closed-loop. The performance of the procedure is assessed in terms of accuracy of the estimated physical parameters, robustness to noise corruption and informative content of the data.

II. Dynamic model

A. RC network

Based upon the analogy between thermal and electrical quantities, buildings as electrical networks are known to be

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Table I
INPUTS AND OUTPUT OF THE R3C2 REPRESENTATION.

Inputs	Output
outdoor temperature heating flux internal gains solar gains	indoor temperature

both lightly parametrized and able to reproduce complex dynamics [2].

We consider accordingly a building modeled as a R3C2 network, depicted in Fig. 1. It has five parameters, the two capacitors C_s and C_r which account respectively for the slow and fast dynamics of the building and the three thermal resistors R_f, R_i and R_o. The building is seen as a unique thermal zone, at temperature T_i. The outdoor temperature is denoted T_o. A third node T_s is representative of the temperature inside the envelope of the building. During the winter, the main contribution of the solar gains are the radiations transmitted through the windows: this is the quantity Q_s. The effect of Q_s on the indoor temperature is mediated through the envelope with T_s. Finally, Q_r is the sum of two contributions: the heating flux Q_h and the internal gains Q_i, due to the occupancy of the building. The inputs and output of the model are summarized in Table I.

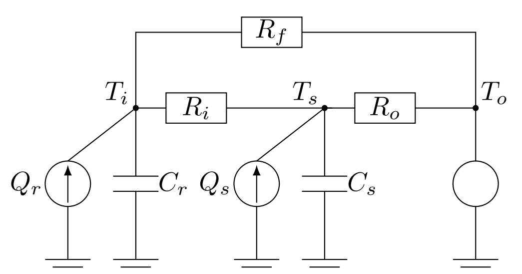

Figure 1. Building model: a R3C2 equivalent network.

In the field of thermal science, the overall heat transfer coefficient UA quantifies the heat loss in a building. However, it is a tedious task to compute it, since it requires the comprehensive knowledge of the geometry of the building, all its materials, etc. One of the benefits of RC networks is that they provide a simple formula for UA in the form of the total conductance of the network, namely

$$UA := \frac{1}{R_f} + \frac{1}{R_i + R_o}. \tag{1}$$

Let us briefly explain the reasoning leading to this formula. Suppose that the system is in the steady-state. Using the representation (3) (introduced later on) at equilibrium, we get

$$Q_r = UA(T_i - T_x) - \frac{R_o}{R_o + R_i} Q_s, \tag{2}$$

where UA is defined as in (1). This indeed corresponds to the definition of UA as the static coefficient for the amount of losses through the envelope of the building per unit of the temperature difference $T_i - T_x$. Likewise, (2) shows that, under the RC network model, $g := \frac{R_o}{R_o + R_i}$ defines the solar energy transmittance. Additionally, two time constants τ_1 and τ_2 can be computed, they characterize the dynamic behaviour of the building. These four parameters provide thus a useful physical interpretation of the network in terms of thermal characteristics of the building.

B. State-space representation

1) Continuous time representation: To derive a state-space representation suitable for the learning process, we write first the Kirchhoff's current law at T_i and T_o:

$$\begin{cases} Q_r &= C_r \frac{dT_i}{dt} + \frac{1}{R_i}(T_i - T_s) + \frac{1}{R_f}(T_i - T_o) \\ Q_s &= C_s \frac{dT_s}{dt} + \frac{1}{R_i}(T_s - T_i) + \frac{1}{R_o}(T_s - T_o) \end{cases}. \tag{3}$$

In this article, we consider that both the indoor temperature and the inputs $U := \begin{pmatrix} T_o & Q_r & Q_s \end{pmatrix}^\intercal$ are indeed measured. The state of the system are then $X = \begin{pmatrix} T_i & T_s \end{pmatrix}^\intercal$ and (3) can be written as a linear continuous time state-space equation:

$$\frac{dX_t}{dt} = A(\theta) X_t B(\theta) U_t + V_t, \tag{4}$$

$$Y_t = C X_t + W_t, \tag{5}$$

where V_t is a model noise and Y_t is a noisy observation of the "true" indoor temperature $T_i = CX_t$. V_t and W_t are independant white gaussian processes of covariances Q and R. The state matrices are

$$\begin{aligned} A &= \begin{bmatrix} -z_r(z_f + z_i) & z_r z_i \\ z_s z_i & -z_s(z_o + z_i) \end{bmatrix} \\ B &= \begin{bmatrix} z_r z_f & z_r & 0 \\ z_s z_o & 0 & z_s \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 \end{bmatrix} \end{aligned} , \tag{6}$$

with $z_r := 1/C_r$, $z_s := 1/C_s$, $z_f := 1/R_f$, $z_o := 1/R_o$, $z_i := 1/R_i$. Finally, the parameter vector is $\theta := \begin{pmatrix} z_r & z_s & z_f & z_o & z_i \end{pmatrix}^\intercal$.

2) Discretization: Applying the constant variation method and the theory of stochastic differential equations (see e.g. [8]), it is standard to discretize the state equation (4) and obtain for a time step δ and at time $k = k\delta$:

$$X_k = A_\delta(\theta) X_{k-1} + B_\delta(\theta) U_k + V_k, \tag{7}$$

with $V_k \sim \mathcal{N}(0, Q_\delta)$, $Q_\delta = \int_0^\delta e^{sA} Q e^{sA^\intercal} ds$, $A_\delta = e^{\delta A}$, $B_\delta = A^{-1}(e^{\delta A} - I)B$. [9] gives an exact computation of Q_δ.

The observation equation (5) is straightforward to discretize In particular, the C matrix remains unchanged. The final model is thus a discrete time-invariant linear gaussian state-space model (LGSSM), in open loop, with five parameters.

III. PARAMETER IDENTIFICATION

A. The Expectation-Maximization (EM) algorithm

We choose the Expectation-Maximization (EM) algorithm (spread by the seminal paper [10]) to estimate the parameters θ of the model, under the assumption that the initial state is normal. In our study case, we are not interested in estimating

Algorithm 1 EM for LGSSM

0. Initialize $\theta = \theta_0$ and set $k = 1$.
1. *E-step:* compute
$$\mathcal{Q}(\theta, \theta_k) := \mathbb{E}_{\theta_k}\left[\ln L_\theta(X, Y) | Y = y_1, \ldots, y_n\right]. \quad (8)$$
2. *M-step:* compute θ_{k+1} :
$$\theta_{k+1} = \arg\max_\theta \mathcal{Q}(\theta, \theta_k). \quad (9)$$
3. If convergence is reached, terminate, otherwise set $k \leftarrow k + 1$ and return to step 2.

the initial state, nor the noise covariances. We merely focus on the estimation of the inverse of the resistors and capacitors. EM is a standard maximum-likelihood approach for estimation in LGSSMs, as described in [11]. To find an estimate $\widehat{\theta}$ of θ, it iterates two steps, sketched in Algorithm 1.

Firstly, the Expectation step calculates $\ln L_\theta(X, Y)$ the log-likelihood of the complete data problem, adding the state to the observations. The states X being actually hidden, the true log-likelihood $\ln L_\theta(Y)$ is approached under the hypothesis θ_k by $\mathcal{Q}(\theta, \theta_k)$ given in (8). This expectation is computed with a Kalman smoother applied to the model parametrized by θ_k. It can be shown, for instance in [12], that if we increase the $\mathcal{Q}$ quantity, then the likelihood increases too: $\mathcal{Q}(\theta, \theta_k) \geq \mathcal{Q}(\theta_k, \theta_k) \Rightarrow L_\theta(y_{1:n}) \geq L_{\theta_k}(y_{1:n})$ (where we have used the generic notation $y_{1:n} := \{y_1, \ldots, y_n\}$). Though it does not prove the convergence of the algorithm, it indicates that at least EM guarantees a non-decreasing likelihood at each step.

Secondly, a closed-form solution to the Maximization step can be derived when no constrain applies to the structure of the state matrices [12]. Yet, our matrices A and B are structurally defined by $\theta = \begin{pmatrix} z_r & z_s & z_f & z_o & z_i \end{pmatrix}^\intercal$, as can be seen from (6). Thus, we use the Matlab routine `fminunc` with the trust-region algorithm, which requires the computation of $\nabla_\theta \mathcal{Q}(\theta, \theta_k)$. We also choose to express the parameters θ as $\theta = \exp(\eta)$ and optimize with respect to η. This is a simple way to make sure that estimated resistors and capacitors are positive and keep the problem unconstrained.

Finally, EM terminates either when a maximum number of iterations (here, 100) is reached or when a stopping criterion is satisfied. We decide that the normalized slope of the likelihood function being under a specified threshold is this criterion.

B. Accuracy

1) Central Limit Theorem: Few of the contributions to building identification ([6]) deal with the evaluation of the uncertainty of the identified parameters. In order to provide confidence intervals, one can use the central limit theorem (CLT). In the general case of a maximum likelihood estimator $\widehat{\theta}_n$ of θ and under some regularity conditions (see [13]), the asymptotic distribution is given by

$$\sqrt{n}(\widehat{\theta}_n - \theta) \underset{n\to\infty}{\longrightarrow} \mathcal{N}(0, \mathcal{I}(\theta)^{-1}), \quad (10)$$

where the Fisher information matrix $\mathcal{I}(\theta)$ is defined by

$$\mathcal{I}(\theta) = \lim_{n\to\infty} \frac{1}{n}\mathbb{E}_\theta\left[-\nabla^2_\theta \ln L_\theta(y_{1:n})\right]. \quad (11)$$

This theorem may be used for our application provided that the data are generated in open loop, but not in closed loop. The numerical experiments in Section IV attest nevertheless that it is still a reasonable approximation in the latter case.

2) Computation of the Fisher information matrix: Using Fisher's identity [14]:

$$\begin{aligned} s_\theta(y_{1:n}) &:= \nabla_\theta \ln L_\theta(y_{1:n}) \\ &= \mathbb{E}_\theta\left[\nabla_\theta \ln L_\theta(X_{1:n}, Y_{1:n}) | Y = y_{1:n}\right], \end{aligned} \quad (12)$$

we see that the score is a by-product of the EM algorithm. Following [15], the information matrix can then be obtained by plugging in the score (12) into the approximation

$$\widehat{\mathcal{I}}(\widehat{\theta}) := \sum_{k=1}^{n}\left[s_{k,\widehat{\theta}}(y_{1:k}) s_{k,\widehat{\theta}}(y_{1:k})\right]^\intercal - \frac{1}{n} s_{\widehat{\theta}}(y_{1:n}) s_{\widehat{\theta}}(y_{1:n})^\intercal \quad (13)$$

of $\mathcal{I}(\theta)$, with $s_{k,\theta}(y_{1:k}) := s_\theta(y_{1:k}) - s_\theta(y_{1:k-1})$.

3) δ-method: Note that the parameters of interest for the application we have in view are $h(\theta) = \begin{pmatrix} UA & g & \tau_1 & \tau_2 \end{pmatrix}^\intercal$ rather than θ itself. The δ-method [13] is a standard tool which only requires the computation of $\nabla_\theta h(\theta)$ to propagate the uncertainty from θ to $h(\theta)$.

C. Procedure

1) Initialization: To avoid convergence of the EM algorithm towards a local minimum, $M > 0$ random initializations are drawn. We obtain M estimators, and select the one with the greatest log-likelihood.

2) Numerical conditioning: [4] emphasizes the impact of numerical conditioning on the estimation procedure. Here we require the state matrix A defined by (6) to be well-conditioned, since its inverse appears both in the descretization and for the resolution of the M-step.

Proposition 1. *Let $\varepsilon := z_s/z_r$. Assuming the three resistors, or equivalently the three conductances, to be of similar magnitudes, then the conditioning number in norm 2 of the matrix A in (6) is $\rho(\varepsilon) = O(\varepsilon^{-2})$. If we denote $\alpha = z_f/z_i$ and $\beta = z_o/z_i$, we have more precisely*

$$\rho(\varepsilon) \underset{\varepsilon\to 0}{\sim} \frac{\kappa(\alpha, \beta)}{\varepsilon^2}, \quad (14)$$

with $\kappa(\alpha, \beta) = \left(\frac{1+(1+\alpha)^2}{(1+\alpha)(1+\beta)-1}\right)^2$.

Proof. The result follows from straightforward calculations omitted here for sake of brevity. □

For any building, the inertia of the structure is (much) greater than that of the air. Hence, it is reasonable to expect $C_r < C_s$ or even $C_r \ll C_s$. With Proposition 1, this means that the problem is ill-conditioned. To adress this issue, [4] uses time-scaled identification on an ARX model. It would be convenient to adapt their method to state-space representation, but here we simply compute the conditioning number at each iteration: above a certain threshold, we stop the iterations and reinitialize to a random θ_0 at step 0. in Algorithm 1.

3) Stability of the filters: We also test that the Kalman filter is stable, which is needed for the asymptotic study of the estimates. If the eigenvalues A_δ are outside the unit circle, we reinitialize the procedure. We will describe the computational burden of the overall procedure in the next section.

IV. Numerical experiments

A. Learning data set

We consider an office building. Our data set is a mixture of measured and simulated data: the internal gains, the outdoor temperature and solar radiations were recorded during 27 days at time-step 10 minutes. The generating process of the heating load and simulate the internal temperature consists in selecting the true values of the five parameters of the model and then

1) either choosing the heating sequence and simulate the discrete LGSSM described in Section 2;
2) or adding a control loop to the building model, and simulate a closed-loop model from the definition of a temperature setpoint sequence.

Note that in both cases, we do identify an open-loop building model. We focused on the second option, because although having the control over the heating sequence enables us to choose a very informative signal, the other method corresponds in contrast to the real exploitation of a building. The controller is a Proportional-Integral one, coupled with a non-linear saturation function and an anti-windup strategy (see [7]), such that the heating power is always positive and bounded. The setpoint temperature has only two levels, alternating between a heating (week days, always equal to 21 °C) and a cooling (nights and week-ends) setpoint.

In the sequel, the base scenario is when the data are generated in closed loop, with a cooling setpoint of 16 °C. $M = 50$ initializations are performed, uniformly in $[0,\, 1]$ for every parameter (all true values are smaller than 1), except for z_s which is drawn between 0 and z_r.

It took 56 minutes to run 10 simulations of this base scenario on a laptop with a IntelCore i7 - 2.80 GHz processor. Most of this time was attributable to the reinitializations, the Kalman smoother and the optimization routine. There are however much less reinitializations as the data length increases.

B. Results

We performed several numerical experiments, in order to illustrate three points: the accuracy of the algorithm, the robustness to noisy data, and the importance of the inputs of the systems.

1) Accuracy of the estimates: An example of point and standard error estimates is given in Tables II. Given the relative small amount of data (14 days), we see that the EM algorithm can perform well on estimating all four physical parameters. UA and g are accurately estimated, whereas the time constants are harder to retrieve, but their order of magnitude is good enough from a thermicist point of view. Table III shows that

Table II
Estimated parameters of the R3C2 network, from the best estimator among $M = 50$ initializations. The data are generated in closed loop, for 14 days at time step 10 minutes.

Parameter	True value	Estimation (± 2SE)	Error (%)
z_r	0.1	0.090 (±0.001)	10
z_s	0.01	0.0070 (±0.0023)	30
z_f	0.2	0.207 (±0.006)	3.5
z_o	0.4	0.430 (±0.080)	7.5
z_i	0.25	0.221 (±0.012)	11.6
UA	0.354	0.352 (±0.009)	0.6
g	0,385	0.346 (±0.040)	10.1
τ_1	33.7	49.9 (±6.0)	48.1
τ_2	3.6	4.2 (±0.2)	16.7

Table III
Estimated standard errors of the parameters, compared to a Monte-Carlo procedure on $N = 100$ data sets. The data are generated in closed loop, for 14 days at time step 10 minutes.

Parameter	CLT	Monte-Carlo
z_r	0.0004	0.0013
z_s	0.0012	0.0015
z_f	0.0030	0.0073
z_o	0.040	0.0871
z_i	0.0058	0.0084
UA	0.0046	0.0059
g	0.0201	0.0496
τ_1	2.98	19.21
τ_2	0.1106	0.127

compared to a Monte-Carlo procedure on 100 experiments, using the CLT gives similar but narrower intervals. It seems therefore legitimate to make the approximation of using the CLT even for closed-loop generated data.

In order to illustrate the peformances of our approach, we also implemented a standard algorithm of the litterature. A widespread approach consists for instance in identifying an open-loop Multi-Input Single-Output (MISO) discrete transfer function by means of the least squares algorithm, as in [5]. The transfer function takes the form

$$H(z^{-1}) = \begin{bmatrix} \frac{b_{11}z^{-1}+b_{12}z^{-2}}{1+a_1z^{-1}+a_2z^{-2}} \\ \frac{b_{21}z^{-1}+b_{22}z^{-2}}{1+a_1z^{-1}+a_2z^{-2}} \\ \frac{b_{31}z^{-1}+b_{32}z^{-2}}{1+a_1z^{-1}+a_2z^{-2}} \end{bmatrix}. \tag{15}$$

In such case, the physical parameter UA and the time constants can be defined from the coefficients $a_1, \dots, b_{32}$. Indeed, the two time constants of the system are classically obtained from the poles of the transfer function, whereas the parameter UA is the inverse of the static gain with respect to the input Q_r evaluated in 1. Confidence intervals can also be constructed for the identified parameters of the transfer function (see e.g. [16]) The $\delta-$method is used to derive the confidence intervals for UA, τ_1 and τ_2 from those of $a_1, \dots, b_{32}$. The different approaches shall therefore be compared through the accuracy of the estimated physical parameters.

We performed the identification with this approach on the same data set generated in closed loop. The point estimates

Table IV
ESTIMATED PARAMETERS (± 2 SE) AND THEIR CORRESPONDING ABSOLUTE ERRORS FOR THE EM AND THE LEAST SQUARES ALGORITHMS. THE DATA ARE GENERATED IN CLOSED LOOP, FOR 14 DAYS AT TIME STEP 10 MINUTES.

Parameter	EM	Transfer function
UA	0.352±0.009 (0.6%)	0.318±0.005 (10%)
τ_1	49.9±6.0 (48.1%)	6.2±13.0 (82%)
τ_2	4.2±0.2 (16.7%)	0.18±0.04 (95%)

and twice their estimated standard errors are given in Table IV, together with the estimation errors. It turns out that this simpler approach gives good results regarding the accuracy of the static parameter UA. However, it fails to reproduce the dynamic behaviour of the system, the two time constants being largely underestimated.

A second algorithm is the closed loop identification from the load curve developped in [7]. This algorithm aims at identifying the building parameters in the closed-loop case It combines Kalman filtering and Gibbs sampling in a Bayesian setting. We refer to [7] for a thorough description. The building is represented by a R3C2 network, but the algorithm requires additionally to model the control loop. We have chosen a simple PI controller, in order to keep the model linear, as in [7]. On the other hand, it is not necessary to measure to indoor temperature, the observation being the load curve.

The numerical simulations are carried out on the same dataset, with the same initialization procedure for the prior distribution of the parameters. There were 5,000 iterations of the Gibbs sampling, and the estimates are the averages over the last 500 iterations. Whenever the indoor temperature data is not used, as in [7], the algorithm typically overestimates the slow time constant ($\tau_1 \sim 70$h), whereas the other physical parameters are of correct magnitude ($UA \sim 0.3$, $\tau_2 \sim 4$).

This second comparison suggests thus that it is preferable to avoid modelling linearly the control loop, and either modelling the saturation function or working in the open-loop case instead.

2) Robustness to noisy data: In order to evaluate the robustness of the algorithm to noise corruption of the model and observations, we simulated the indoor temperature from the identified model, in closed loop. It is compared to the true indoor temperature, simulated from the true model with the same setpoint but without model nor observational noise. The simulation error is displayed in Fig. 2. It is of similar magnitude that the good models identified in [4], even if we have used much less data (2 weeks at rate 10 minutes vs 1 year at rate 1 hour), which suggests that our method is also robust to noise corruption.

3) Role of the inputs: It is well-known that a necessary condition for the identification to be efficient is that the inputs of the system comprise as much information as possible. In this view, we explored the influence of the data length and of

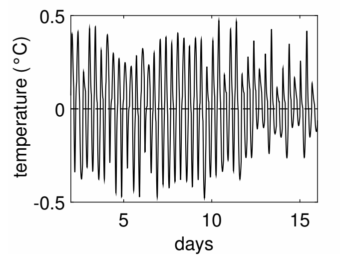

Figure 2. Simulation error for the model estimated with noisy data but simulated in closed loop with the noise-free inputs. The setpoint alternates between 15 °C and 21 °C every 4h.

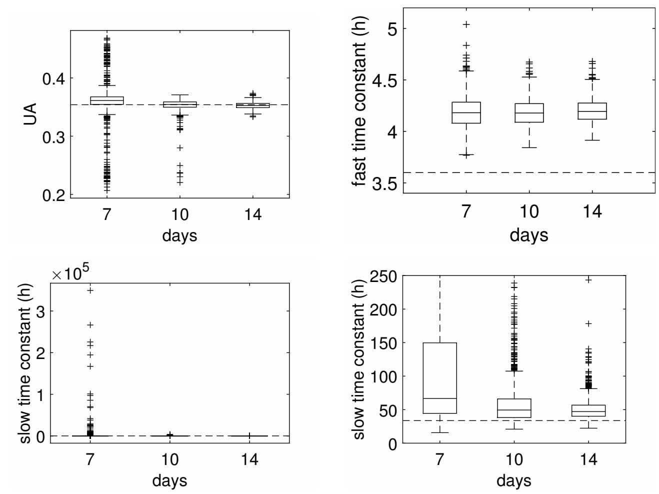

Figure 3. Boxplots of the $N = 1000$ estimations of UA (top left corner), the fast time constant (top right corner) and the slow time constant (bottom line), against the number of days in the learning data set at sample rate 10 minutes. The true values are the horizontal dashed lines.

the gap between the cooling and heating setpoint. The intuition behind the latter point is that under nonlinear control, the bigger the gap is between the two setpoints, the more useful information there is, and the better the time constants are estimated. Fig. 3 and 4 confirm this intuition, since at least 10 days of data sampled at rate 10 minutes and a gap greater than 2 °C are needed to estimate the slow time constant. Whereas UA and g (not displayed) are robustly estimated, the slow time constant estimate may indeed explode when violating these conditions. Table II reinforces the fact that it is difficult to get a very accurate estimate of this parameter in any case, while a small bias remains for the fast time constant. The time-scaled method [4] is a promising track to adress this issue.

V. CONCLUSION

In this article, we suggest that RC networks, typically of order 2, are suitable to model the dynamic thermal behaviour of single-zone buildings. Indeed, when the model is learned via the EM algorithm, the numerical experiments that we conducted indicate that inference can be drawn about the main physical parameters of the building from one or two weeks of recorded data. The static parameters UA and g are specifically robustly estimated. Moreover, the chosen framework has the

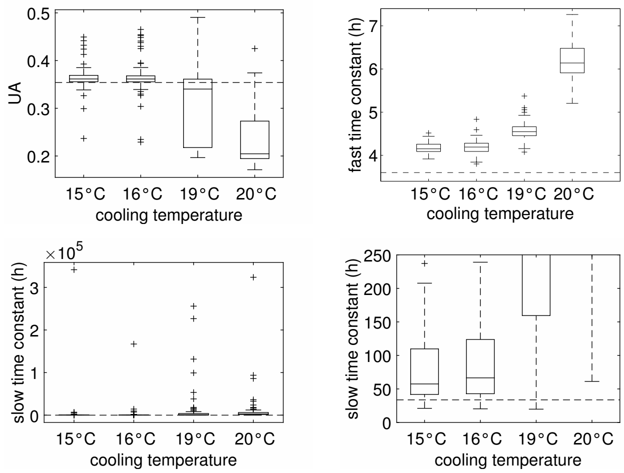

Figure 4. Boxplots of the $N = 100$ estimations of UA (top left corner), the fast time constant (top right corner) and the slow time constant (bottom line), against the cooling setpoint. The heating setpoint was 21 °C, and the data length is 7 days. The true values are the horizontal dashed lines.

advantage of relying on theoretical statistical foundations. This allows us to provide confidence intervals of the parameters of interest. For an open loop model, we use the CLT to do so. Even though the CLT does not remain true when the data are generated in closed loop, the numerical experiments show that it is a sensible approximation. A forthcoming article will demonstrate the theoretical validity of this approach for our application. Finally, we emphasize the dependency of the quality of the estimates, in particular of the time constants, on the informative content of the data.

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